Optimization of a two-stage distribution network with route planning and time restrictions

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Abstract

Location and route planning are implemented independently in most distribution networks. In the majority of cases, low-quality solutions are obtained if sequential methods, e.g. locate depots first and plan routes second, are used. In location-routing problems (LRPs), location and routing are solved simultaneously. The underlying basic problems are the well known facility location problem and the vehicle routing problem, which have been studied intensively over the last decades. This work extends a single-stage LRP to a two-stage distribution network, taking into account route planning and time restrictions, thus covering more realistic aspects, especially timing aspects, of many real-world problems. We present a tabu search approach for solving large-scale instances and compare its performance with a sequential approach.

1. Introduction

Europe’s logistics market is one of the most important economic factors, with a market volume of €930 billion in 2010 [14]. The largest part of the market volume is represented by transportation and warehousing, with a share of 42% and 26%, respectively. The increasing market competition and the service focus of customers force courier-express-parcel service providers to re-evaluate and to continuously improve their networks. Worldwide, the necessity to improve the service quality is rapidly growing. Especially time aspects, such as delivery to the destination within the next day, are key performance indicators for service quality. To reduce costs and increase service quality, facility location and route planning strategies are crucial choices. Both of the underlying basic problems - the facility location problem (FLP) and the vehicle routing problem (VRP) - have been studied and solved intensively over the last decades as individual models. To reduce the complexity of the combined problem, both problems are usually tackled independently and in a sequential manner (locate first and route second). In particular, aspects of future route plans are neglected during the strategical location planning level of most distribution systems, and only approximated. This approximation needs, however, a priori knowledge of transport services, and research has shown that this strategy often leads to suboptimal solutions, as shown by Salhi and Rand [24]. Exact methods for the capacitated location-routing problem include column generation [1] and branch-and-cut [3]. Although some instances with 100 customers remain unsolved, these methods are able to solve instances with up to 200 customers. However, real problems consist of large-size instances with up to 1000 customers and hundreds of potential depots. To handle this kind of instance, several metaheuristics have been proposed (e.g. [9], [18], [20], [21], [26], and [28]). In recent years, the attention has increased, and many of the published works deal with real problems. For example, military [17], the paper industry [22], and postal logistics (e.g. [5], [12], and [27]) to name just a few. An overview is provided by Nagy and Salhi [19].

In this paper we consider a two-stage location-routing problem with time windows (TSLRP) and capacity restrictions at customers and at depots. In some real problems, external truck companies perform the routing, and wish to implement route plans for a long period of time whereby, of course, time windows of customers are respected. Executing routes for a long period of time improves the ability of external truck companies to schedule their staff and vehicle fleet more efficiently and increases planning security. Furthermore, it leads to cost reductions, from which also clients of truck companies can benefit during contract negotiations. Thus, the routing in these cases is tactical rather than operational. The considered problem is derived from real applications in urban areas of Germany.

In Germany a next-day delivery is possible if letter and bulk mail are posted before approximately 5 p.m. An early reception of mail enables law chambers, offices, and agencies to deal with their customers’ business in the morning and afternoon, and to post the return mail before 5 p.m.
Thus, they are able to respond quickly without having to use additional express services if the delivery is early in the morning and the pickup up is before 5 p.m. In fact, a daily time window exists from approximately 6 a.m. to 8 a.m. for delivery and from 3 p.m. to 5 p.m. for pickup. In Fig. 1 the network structure is depicted. The mail is delivered from sorting centers to small depots, and vice versa. A related problem was presented by Burks [6] and Guenduez [11]. In the case of delivery, depots are used as breakup sites, otherwise they are used as consolidation sites. Since the delivery is operated separately, we focus for the sake of simplicity on the delivery case. Finally, the questions of how to transport the mail quantity on the first stage, where to locate the depots, and how to deliver customers’ quantities lead to the TSLRPTW.

This paper is organized as follows. Section 2 introduces the required notation, defines the problem, and proposes an integer linear optimization model. The subproblems are presented in Sect. 3, followed by the explanation of the proposed hybrid tabu search heuristic in Sect. 4. Computational results are presented in Sect. 5. We close with some concluding remarks.

2. Problem and mathematical formulation

First, we introduce some notations before we describe the problem and give a mathematical formulation. The set $S$ contains sorting centers, $D$ potential depot sites, and $I$ customers. Each potential depot has a capacity $Q_d$, opening hours $[open_d, close_d]$, and fixed costs $f_d$. Further, each customer has a demand $q_i$ and has to be served during the time window $[a_i, b_i]$. For service purposes, two homogeneous fleets $K^1$ and $K^2$ of vehicles with capacity $C^1$ and $C^2$ are available for the first and second transportation stage, respectively. Any subset of $K^1$ can be placed at any sorting center and any subset of $K^2$ can be placed at any depot site. We assume that each customer’s demand will be provided at a predefined available time $avt_i$, $i \in I$, by a unique sorting center which is known in advance. Therefore, we define a set of orders $O$. An order $o = (s, i) \in O$ consists of a customer $i \in I$ and the sorting center $s \in S$ providing that customer’s demand. Further, an order path $p_o = (s, d, i)$ is a time-feasible transportation from the sorting $s \in S$ via a depot $d \in D$ to customer $i \in I$ (and back to the depot) of the order $o \in O$. $P_o$ is the set of all order paths of order $o$ and $P$ is the union of all order paths sets $P_o$. With $(p)_1$, $(p)_2$, and $(p)_3$ we describe the sorting center, the depot, and the customer on the path $p \in P$. Path variables in location models are commonly used to denote the fraction of the demand $q_i$ being routed via a path $(o, d, i)$. A path variable formulation allows situations to be modeled where the cost and time feasibility depend on both the source node $s$ and the sink node $i$ (see [15]). Further, if time restrictions are tight, many of the $|S| \times |D| \times |I|$ variables can be fixed to the value 0 or neglected in the model if the path is time-infeasible. This is the main purpose of our path formulation.

We still have to explain the term “time feasible transport for order paths”. First, we introduce transportation time $t_{ld}^i$, $s \in S$, $d \in D$, for the first transportation stage (including service time at $s$), $t_{ld}^{i, o}$, and $t_{o, d}^{i, o}$ for the second transportation stage (including service time at $i$ or $d$). To calculate the time feasibility of an order path $p$, some backward calculations are necessary. First, we start with the closing time $close_d$ of depot $d = (p)_2$ and determine the latest arrival time (LAT) at customer $i = (p)_3$ on path $p$ with the following calculation:

$$\text{LAT}_i = \min \{b_i, close_d - t_{id}^2\}$$

(1)

If $\text{LAT}_i < a_i$ holds, then the path $p$ is time-infeasible, and further checks are unnecessary. Otherwise, the return to depot $d$ from customer $i$ on path $p$ is time-feasible. Now, we continue with $\text{LAT}_i$ and calculate the latest arrival time at depot $d$:

$$\text{LAT}_d = \min \{close_d, \text{LAT}_i - t_{di}^2\}$$

(2)

If $\text{LAT}_d < open_i$ holds, then the path $p$ is time-infeasible, and further checks are unnecessary. Otherwise, the direct transport from depot $d$ to customer $i$ on path $p$ is time-feasible. In a final step, we calculate the latest arrival time at sorting center $s$ on path $d$, which should be interpreted as the latest availability time for the associated order $o$:

$$\text{LAT}_s = \text{LAT}_d - t_{id}^1$$

(3)

If $\text{LAT}_s < avt_i$ holds, then the path $p$ is time-infeasible. Otherwise, the direct transport from sorting center $s$ to depot $d$ on path $p$ is time-feasible, and thus the whole path $p$ is time-feasible. In order to reduce the time complexity and to have some consolidation effect on the first transportation stage, we discretize the time horizon $T$ in disjoint intervals $[t_{i-1}, t_i]$. With $t$ we describe an element of the interval set $T$. Further, a static buffer time $u > 0$ is used and for all orders $o = (s, i)$. If the original availability time is within the interval $[t_{i-1} - u, t_i - u]$, then $avt_i$ will be fixed to $t_i$. We also use the notation $(o, i)$ to describe how $avt_i$ is fixed to $t_i$. Thus, all mail volume with availability time.
Each customer is assigned exactly to one open depot during a preprocessing phase. Finally, we introduce transportation costs $c_{sd}$, $s \in S$, $d \in D$, for the first transportation stage, $c_{ij}$, $c_{il}$, and $c_{d}$, $i, j \in I$, $d \in D$, for the second transportation stage.

A summary of all introduced sets and further parameters for the mathematical formulation is given in Tables 1–4.

Table 1. Description of sets

<table>
<thead>
<tr>
<th>$S$</th>
<th>set of sorting centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>set of customers</td>
</tr>
<tr>
<td>$D$</td>
<td>set of potential depot sites</td>
</tr>
<tr>
<td>$V$</td>
<td>$I \cup D$</td>
</tr>
<tr>
<td>$O$</td>
<td>set of orders</td>
</tr>
<tr>
<td>$K^1$</td>
<td>homogeneous vehicle fleet 1st stage</td>
</tr>
<tr>
<td>$K^2$</td>
<td>homogeneous vehicle fleet 2nd stage</td>
</tr>
<tr>
<td>$T_s$</td>
<td>set of availability time points at $s \in S$</td>
</tr>
<tr>
<td>$L$</td>
<td>index set of equidistant points in time</td>
</tr>
</tbody>
</table>

Table 2. Description of order path sets/parameters

<table>
<thead>
<tr>
<th>$p_o$</th>
<th>order path of order $o \in O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_o$</td>
<td>set of time-feasible order paths of order $o \in O$</td>
</tr>
<tr>
<td>$P$</td>
<td>set of all time feasible order paths</td>
</tr>
<tr>
<td>$q_o$</td>
<td>mail volume of order $o \in O$</td>
</tr>
<tr>
<td>$q_p$</td>
<td>mail volume of order path $p \in P$</td>
</tr>
</tbody>
</table>

Table 3. Description of parameters

<table>
<thead>
<tr>
<th>$q_i$</th>
<th>demand of customer $i \in I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>capacity of 1st stage vehicles</td>
</tr>
<tr>
<td>$C_2$</td>
<td>capacity of 2nd stage vehicles</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>capacity of potential depot sites</td>
</tr>
<tr>
<td>$f_d$</td>
<td>fixed costs of potential depot sites</td>
</tr>
<tr>
<td>$c_d$</td>
<td>transportation costs between $s \in S$ and $d \in D$</td>
</tr>
<tr>
<td>$M$</td>
<td>large number ($\infty$)</td>
</tr>
</tbody>
</table>

Table 4. Description of time parameters

<table>
<thead>
<tr>
<th>$[a_i, b_i]$</th>
<th>time window of customer $i \in I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[a_d, b_d]$</td>
<td>opening times of depot $d \in D$</td>
</tr>
<tr>
<td>$t_{ij}$</td>
<td>transportation time between $i \in V$ and $j \in V$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>point in time, $i \in L$</td>
</tr>
<tr>
<td>$T$</td>
<td>disjoint union of time intervals $T = \bigcup_{l \in L} (t_{l-1}, t_l]$</td>
</tr>
<tr>
<td>$T(p)$</td>
<td>earliest arrival/availability time at ($p$), $j = 1, 2, 3$</td>
</tr>
</tbody>
</table>

Table 5. Description of integer variables

<table>
<thead>
<tr>
<th>$AT_i$</th>
<th>arrival time at customer $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i$</td>
<td>waiting time at customer $i$</td>
</tr>
<tr>
<td>$DPV_k$</td>
<td>departure time of vehicle $k$ at depot $d$</td>
</tr>
<tr>
<td>$RTV_k$</td>
<td>return time of vehicle $k$ at depot $d$</td>
</tr>
<tr>
<td>$\Gamma_{sd}$</td>
<td>number of transports from $s$ to $d$ at point in time $t_l$</td>
</tr>
</tbody>
</table>

The task of the TSLRPTW is to find a subset of the potential depots, a transportation plan of the first stage, and a route plan of the second transportation stage such that all time and capacity constraints are satisfied, total costs, consisting of depot fixed costs and transportation costs on both stages, are minimized, and the following constraints are satisfied:

- Each customer is assigned exactly to one open depot and served by exactly one vehicle during its time window. A waiting time $w_i$ at customer $i$ is allowed.
- Each vehicle is used once at most.
- Each vehicle route begins and ends at the same open depot during opening hours.
- The vehicle load does not exceed the vehicle capacity on both transportation stages.
- The total demand of the customers assigned to an open depot does not exceed the depot capacity.

To give a mathematical formulation of the TSLRPTW, the following binary decision variables are now introduced:

$$ y_d = \begin{cases} 1, & \text{if depot } d \text{ is open} \\ 0, & \text{otherwise} \end{cases} $$

$$ z_{kl} = \begin{cases} 1, & \text{if customer } i \text{ is assigned to depot } d \text{ on vehicle } k \\ 0, & \text{otherwise} \end{cases} $$

$$ x_{ij} = \begin{cases} 1, & \text{if } j \in V \text{ is directly visited after } i \in V \text{ by vehicle } k \\ 0, & \text{otherwise} \end{cases} $$

$$ \lambda_p = \begin{cases} 1, & \text{if order path } p \text{ is used} \\ 0, & \text{otherwise} \end{cases} $$

Note that $\lambda_p$ implies the allocation of a customer to a depot but does not contain any information about the assignment of the customer to a vehicle or that customer’s position on the tour. Therefore, binary variables $z_{kl}$ are still necessary. On the other hand, the introduction of path variables is useful to give a compact formulation of the location-allocation subproblem in Sect. 3.1. Further, we introduce the integer decision variables in Table 5. A mixed integer program can now be stated as follows:

$$ \min \sum_{d \in D} f_d \cdot y_d + \sum_{i, j \in V} c_{ij} \cdot x_{ij} + \sum_{s \in S, d \in D} \sum_{l \in L} c_{sd} \cdot \Gamma_{sd} $$

$$ (4) $$
subject to

\[ \sum_{k \in K} \sum_{j \in V} x^k_{ij} = 1 \quad \forall j \in I \]  
(5)

\[ \sum_{j \in V} x^k_{ij} - \sum_{j \in V} x^k_{ji} = 0 \quad \forall k \in K, i \in V \]  
(6)

\[ \sum_{i \in I} \sum_{k \in K} x^k_{ij} \leq 1 \quad \forall k \in K \]  
(7)

\[ \sum_{k \in K \in S \in (V \setminus S)} \sum_{j \in V} x^k_{ij} \geq 1 \quad \forall S \subseteq I, 2 \leq |S| \]  
(8)

\[ \sum_{m \in V} (\lambda_{dm} + \lambda_{ma}) - z^k_{di} \leq 1 \quad \forall d \in D, i \in I, k \in K \]  
(9)

\[ \sum_{p \in P} \lambda_p = 1 \quad \forall o \in O \]  
(10)

\[ \lambda_p - \sum_{k \in K} z^k_{di} \leq 0 \quad \forall d \in D, i \in I, \quad p = (., d, i) \in P \]  
(11)

\[ z^k_{di} \leq y_d \quad \forall d \in D, i \in I, k \in K \]  
(12)

\[ \sum_{k \in K \in d \in D} z^k_{di} = 1 \quad \forall i \in I \]  
(13)

\[ \sum_{k \in K} q_i \cdot z^k_{di} \leq Q_d \quad \forall d \in D \]  
(14)

\[ \sum_{i \in I} \sum_{j \in V} q_{ij} x^k_{ij} \leq C_i^k \quad \forall k \in K \]  
(15)

\[ \frac{\sum_{p = (s, d, i) \in P} \lambda_p \cdot q_{pi}}{C_i^k} \leq \Gamma_{sdti} \quad \forall s \in S, d \in D, i \in I \]  
(16)

\[ |AT_j + w_i - T_{ij}^1| \leq M(1 - x^k_{ij}) \quad \forall i \in I, j \in I, k \in K \]  
(22)

\[ |RT_d^k - AT_i - w_i - T_{ij}^1| \leq M(1 - x^k_{ij}) \quad \forall i \in I, d \in D, k \in K \]  
(23)

\[ |AT_j - DT_d^k - T_{ij}^1| \leq M(1 - x^k_{ij}) \quad \forall i \in I, j \in I, k \in K \]  
(24)

The objective function (4) minimizes the sum of depot fixed costs, tour costs (second-stage), and transportation costs (first-stage). Constraints (5) and (6), known as ‘degree constraints’, guarantee the uniqueness and continuity of a route performed by a vehicle.

Each vehicle is used once at the most through constraints (7). Subtours consisting of customers only are eliminated by constraints (8). Constraints (9) ensure that a customer is only served by a vehicle assigned to the same open depot. For each order exactly one order path is used through constraints (10). An order path can only be used if the depot on the path is open and the customer on the path is uniquely assigned to that depot by a vehicle (see constraints (11)-(13)). Capacity constraints of the open depots and the used vehicles are satisfied through inequalities (14) and (15). Constraints (16) determine the number of transports needed from sorting center s to depot d at a given point in time t_i.

While constraints (17) and (18) imply that the arrival time (with additional waiting time) at a customer is within that customer’s time window, constraints (19) and (20) guarantee that each vehicle starts and ends at a depot during the depot’s opening time. Further, the departure time of a vehicle k must be greater or equal to the maximum availability time at the depot of all customers’ demands which are assigned to the depot and the vehicle (see constraints (21)). The arrival time, the departure time, and the return time on a route performed by a vehicle are determined by inequalities (22)-(24). If x^k_{ij} = 1 holds, then inequalities (22) reduce to the equation AT_j = AT_i + w_i + t_{ij}, otherwise to the relaxed inequality −∞ ≤ AT_j − AT_i − w_i − t_{ij} ≤ −∞. Similarly, the same holds for (23)-(24). For the purpose of time variables description, the time horizon is coded as integer values. Obviously, the number of constraints is exponentially growing with the number of customers and potential depots (see constraints (8)). Moreover, the number of variables is dominated by the multiplication of number of customers by the number of potential depots. Therefore, only very small instances of this formulation can be solved with commercial solvers. Thus, state-of-the-art approaches must be applied and investigated to solve real world instances. This work is considered as a first step to providing good solutions for large-scale TSLRPTW instances, and therefore a hybrid heuristic approach is presented.

3. Subproblems

If we neglect the routing aspect and instead perform direct transportation on the second stage, the TSLRPTW will be reduced to a two-stage capacity facility location problem with time windows (TSCFLPTW). Furthermore, if the location decision is known, the arising problem will be reduced to an order-path allocation problem. Finally, if location and allocation decisions are known, the problem will be reduced to a vehicle routing problem with
availability times and time windows for multiple depots.

An initial solution is obtained by a sequential approach. First, an appropriate two-stage capacitated facility location problem with time windows (TSCFLPTW) is solved with the commercial solver CPLEX. Afterwards, a multi-depot vehicle routing with time windows (MDVRPTW) for the open depots and the assigned customers is solved. The MDVRPTW is performed with construction and improvement heuristics. Feasible routes are created with the savings method of Clarke and Wright and improved with arc-exchange operations. We propose a tabu search heuristic to solve large-scale instances of the TSLRPTW. It is based on add, drop, and shift moves, and restricted to defined regional and catchment areas. To create these reasonable areas, a neighborhood of depots is introduced and applied. The tabu search starts with the initial solution and applies add, drop, and shift moves. After each move, an appropriate order path allocation problem (OPAP) and afterwards an MDVRPTW is solved again for the defined regional area. To improve the solution quality, tabu lists are used to avoid an immediate return to a local optimum and to explore a big variety of the solution space.

3.1. Two-stage capacitated facility location problem with time windows

Without the first transportation stage and without routes on the second transportation stage, the reduced problem would be similar to the capacitated facility location problem, first introduced by Balinski [2], with additional time restrictions for the latest arrival time at a customer. The task of the TSCFLPTW is to determine the number and location of open depots and the order paths with the minimum sum of depot fixed costs, first-stage transportation and second-stage assignment costs, such that the following constraints hold:

- Each customer’s order is performed exactly on one order path of the associated order.
- The total demand of order paths using an open depot does not exceed the depot capacity.
- All arrival times at customers or depots are before the time window end (waiting times are allowed, therefore, the time window begins is not restrictive).

We use the notation in Sect. 2. A mathematical formulation of the TSCFLPTW can be stated as follows:

\[
\begin{align*}
\min \sum_{d \in D} f_d \cdot y_d &+ \sum_{p \in P} (c_{(p)2}(p)_3 + c_{(p)3}(p)_2) \cdot \lambda_p \\
&+ \sum_{s \in S, d \in D, i \in L} c_{i, d} \cdot \Gamma_{s, d, i}
\end{align*}
\]

such that constraints (10), (16), and (26) hold. The objective function (25) minimizes the above mentioned costs. Constraints (26) ensure that an order path can only be used if the depot on the path is open. Further, the volume on the order paths using the same depot must not exceed the depot capacity. Time-feasibility constraints are not needed, because they are implicitly included through the order path variables. Therefore, the formulation of order paths is very useful to get a compact mathematical formulation of this subproblem. Note that all time-feasibility checks can be outsourced to a preprocessing phase with calculations (1)-(3) and checks described in Sect. 2. This preprocessing also reduces the number of order path variables. Large-scale instances of this model with 400 customers and 50 potential depots have been solved to optimality with the commercial solver CPLEX [8].

An order path allocation problem (OPAP) occurs if open depots, however, are determined. Then, order paths are limited to open depots, the variables \(y_d\) can be omitted, and the number of constraints (26) is limited to the number of open depots. Thus, the first term of the objective function (25) is constant and can be omitted, too.

3.2. Capacitated vehicle routing problem with time windows and availability times

The (multi-depot) vehicle routing problem with time windows (MDVRPTW) occurs as a subproblem if depot locations and a feasible assignment of customers are known. Furthermore, the arrival time of customers’ demands at the depots can differ. Thus, availability times of customers’ demands at the depot must be respected at the start of each route. Then, the task is to construct routes for each depot and its assigned customers with the following constraints:

- Each customer is served by exactly one vehicle during the customer’s time window (waiting time is allowed).
- Each route begins and ends at the depot during the opening hours.
- Each route respects the availability times of the assigned customers.
- The vehicle load does not exceed the vehicle capacity.

The goal is to minimize the overall transportation costs. We omit a description of a full mathematical formulation and refer to Toth and Vigo [25]. To construct ini-
tial routes, we use the savings heuristic of Clarke and Wright [7] and obtain an initial solution. Each customer is served individually by a separate route. Combining two routes, serving customers \(i\) and \(j\), results in cost savings \(s_{ij} = c_{id}^{*} + c_{jd}^{*} - c_{ij}^{*}\) with \(d\) as the serving depot. We link customers \(i\) and \(j\) with maximum positive savings such that the combined route is time- and capacity-feasible. Further, we restrict the linking of customers to those who retain the traversing order of the previous two routes, because reversing a route order leads in most cases to time infeasibility. The savings procedure is applied iteratively. Local search methods are used to improve feasible solutions. Arc-exchange operators are applied to find neighboring solutions. The following arc-exchange moves are applied: 2-opt, 2-opt*, Or-opt, relocate, exchange, and cross-exchange. We refer to Braeysy and Gendreau [4] for more details.

Applying relocate or exchange operations to routes assigned to different depots allows us to change the assignment of customers. But then, volume flows on the first transportation stage have to be redirected, and availability times at the depots of the changed assignments must be updated. To check and guarantee the time feasibility (respecting also the availability times), we use resource extension functions generalized to segments, as introduced by Irnich [13]. These functions also allow the feasibility of the operations to be checked in constant time instead of linear time by iterating each changed route. We denote the savings on the second transportation stage by \(\text{sav}\). If an exchange operation is applied to two customers \(i_3\) and \(i_4\) assigned to different depots \(d_1\) and \(d_2\), as in Fig. 2, then the following cases occur:

- The volume flow from \(s_1\) to \(d_1\) is reduced by \(q_{i_3}\) at \(t_1(\text{avt}_{i_3})\). If then the number of vehicles is reduced by one, we set \(\text{sav}_{1} = c_{s_1d_1}^{1}\), otherwise to 0.
- The volume flow from \(s_2\) to \(d_2\) is reduced by \(q_{i_4}\) at \(t_1(\text{avt}_{i_4})\). If then the number of vehicles is reduced by one, we set \(\text{sav}_{2} = c_{s_2d_2}^{1}\), otherwise to 0.
- The volume flow from \(s_1\) to \(d_1\) is increased by \(q_{i_3}\) at \(t_1(\text{avt}_{i_3})\). If then the number of vehicles is increased by one, we set \(\text{sav}_{3} = -c_{s_1d_1}^{1}\), otherwise to 0.
- The volume flow from \(s_2\) to \(d_1\) is increased by \(q_{i_4}\) at \(t_1(\text{avt}_{i_4})\). If then the number of vehicles is increased by one, we set \(\text{sav}_{4} = -c_{s_2d_1}^{1}\), otherwise to 0.

The exchange operation is only applied if and only if \(\text{sav} + \text{sav}_1 + \text{sav}_2 + \text{sav}_3 + \text{sav}_4 > 0\) holds. Then, a positive saving is found, and the overall transportation costs can be reduced. When using the relocate operation for a customer, the pair \(\text{sav}_1\) and \(\text{sav}_3\) or \(\text{sav}_2\) and \(\text{sav}_4\), respectively, can be omitted. Algorithm 1 gives an overview of how the MDRVPTW is solved in our work.

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4. A hybrid tabu search approach

‘Locate first and route second’-type heuristics are sequential methods for LRP. In our work, we refer to the more suitable approach of nested methods, presented by Nagy and Salhi [18], and we use the sequential method as the initial solution and for comparison purposes. An alternative approach could be that of iterative methods (e.g. [23]), where the location problem and the routing problem are treated equally. These methods iterate between the location and routing phases until a stop criterion is met. The drawback of iterative methods is that the location solution space cannot be searched intensively, as in the nested methods, due to an intensive routing phase for the whole distribution system after each location and allocation decision.

First, we define a neighborhood structure of the TSLRPTW by the moves add, drop, and shift, introduced by Kuehn and Hamburger [16]. ‘Add’ means opening a closed depot, ‘drop’ means closing an open depot, and ‘shift’ is a simultaneous add and drop move. The change of a location mostly influences a connected area of closely located depots with their assigned customers and the changed depot and its customers. Thus, we restrict the three moves to a region and a catchment area of an open

---

Algorithm 1 Multi-depot VRPTW

1: for each depot and its assigned customers do
2: construct routes with the savings method
3: end for
4: for each depot and its routes do
5: improve routes by operations in the following sequence: 2-opt, 2-opt*, Or-opt, relocate, exchange, cross-exchange
6: if an operator improves a solution then
7: stop, update the route(s), and go to 4
8: end if
9: end for
10: for each pair of depots and their routes do
11: improve routes by operations in the following sequence: relocate, exchange
12: if an operator improves a solution then
13: stop, update the route(s), and go to 10
14: end if
15: end for
16: if an operator in 11 improved the solution at least once then
17: go to 4
18: end if

---

Figure 2. Illustration of flow changes
depot, where after each move an MDVRPTW is solved. In addition, we define a neighborhood relation between two depots to specify the term closely in this context. The following definitions are taken from Nagy and Salhi[18].

**Definition 1** Two depots $d_1$ and $d_2$ are neighbors, if and only if at least one customer $i$ exists, such that $d_1$ and $d_2$ are the nearest two depots to customer $i$.

**Definition 2** The region $R(d)$ of an open depot $d$ consists of the depot $d$ itself, its customers, neighbor depots and their assigned customers.

**Definition 3** The catchment area $CA(d)$ of an open depot $d$ is the smallest rectangle comprising depot $d$ and its customers.

The neighborhood relation, the region of a depot $d$, and its catchment area can be created easily. Moreover, we restrict add and shift moves to the catchment area of a considered depot $d$. Closed depots too far from depot $d$ are not considered, because they have only a slight direct influence on each other.

The model given in the Sect. 2 very quickly becomes computationally intractable for commercial MIP solvers for instances with 400 customers and over 50 potential depots. As a first step, the problem was solved by means of a two-stage heuristic approach. We start by constructing a feasible initial solution, which is then improved by a tabu search procedure. For details of tabu search we refer to Glover and Laguna [10].

Algorithm 2 describes the proposed heuristic. The initial solution for the described problem in line 1 is obtained by solving the TSCFLPTW (see Sect. 3.1) first and the MDVRPTW (see Sect. 3.2) second. The solution of the TSCFLPTW serves as input for Algorithm 1. Locations (including volume flows to them) and assignments outside the region $R(d)$ are not affected by the moves in line 4, 6, and 7. All open depots in a region are determined by the moves, so only an OPAP (see Sect. 3.1) has to be solved in these lines. Then, Algorithm 1 is applied to the OPAP solution. Thus, the routing is nested in the location phase. Note that a drop or shift move can lead to an infeasible OPAP instance due to time and capacity infeasibility.

Further, the largest improvement in line 10 can be negative and result in a non-improving solution. In general, the acceptance of non-improving allows us to climb out of a local optimum solution and to explore a big variety of the solution space. To forbid the immediate return to a local optimum, we use a tabu list strategy. In our algorithm, the reverse move of the move in line 10 is made tabu for a given number of iterations from lines 2 to 13. Our stop criterion is met if a maximum number of iterations or of non-improving move selections (line 10) is performed.

---

**Algorithm 2 Tabu Search for TSLRPTW**

1: Solve the TSCFLPTW first and then the arising MDVRPTW to generate an initial solution $IS$ for the TSLRPTW. Calculate total costs $C(IS)$ of $IS$. Set the current solution $CS = IS$ and the best solution $S_{best} = IS$.

2: for each depot $d$ in $CS$ do 3: Calculate the Region $R(d)$. 4: Drop open $d$ from $R(d)$ if this drop move is not tabu. Solve the order-path allocation problem OPAP first and then the MDVRPTW for $R(d) \setminus \{d\}$. Calculate the costs $C(R(d) \setminus \{d\})$ of the drop move.

5: for each closed depot $d$ in the catchment area $CA(d)$ do 6: Add $d$ to $R(d)$ if this add move is not tabu. Solve the order-path allocation problem OPAP first and then the MDVRPTW for $R(d) \cup \{d\}$. Calculate the costs $C(R(d) \cup \{d\})$ of the add move.

7: Shift $d$ with $d$ in $R(d)$ if this shift move is not tabu. Solve the order-path allocation problem OPAP first and then the MDVRPTW for $R(d) \cup \{d\} \setminus \{d\}$. Calculate the costs $C((R(d) \cup \{d\}) \setminus \{d\})$ of the shift move.

8: end for

9: end for

10: Implement the move with the largest cost reduction, update the tabu list, $CS$, and $C(CS)$.

11: if $CS$ is better than the best known solution $S_{best}$, i.e., $C(CS) < C(S_{best})$ then
12: Set $S_{best} = CS$.
13: end if
14: Repeat 2 to 13 until a suitable stop criterion is reached.

---

5. Computational study

The proposed heuristic was coded in C++ and executed on an Intel Core 2.15 Ghz computer with 3.24 GB RAM. CPLEX 9.1 [8] was applied to solve the TSCFLPTW and OPAP. Before the computational results, we give a brief overview of the generated instances.

5.1. Test instances

Since benchmarks for the TSLRPTW are not available, we derived TSLRPTW instances from the extended VRPTW Solomon benchmark. We used the class RC1 with 400 customers, in order to have instances with clustered and randomly distributed customers. The 10 instances of RC1 vary in the time windows of customers. In each instance, the service time, demand, and coordinates are given. To add further potential depots to the existing depot, we take the smallest rectangle including all customers and draw horizontal and vertical lines parallel to the sides of the rectangle with equal distance. The result is a grid and each grid point in the rectangle is a potential depot. Overall, the generated instances consist of 50 potential depots. Open hours of the potential depots are defined as $[\min_{i \in d} a_i, \max_{i \in d} b_i]$, and their service time is set to zero. As fixed costs we used the set of 100, 500, 1,000,
and 3,000 monetary units (MU) combined with depot capacity 250, 500, 1,500, and 3,000 quantity units (QU). The demand of all customers is 7,127 QU and the minimum required number of open depots according to the capacity is given in Table 6. Further, we divided the rectangle into four equal rectangles by drawing a horizontal and a vertical line parallel to the sides at the middle of the height and width. To the center of each of the four rectangles we located a sorting center. Customers’ demands are provided by the sorting center within the same rectangle. We set the availability time of each customer at the associated sorting center such that at least one order path exists. Overall, 160 instances were generated and tested. The vehicle capacity is adopted from the Solomon instances. Further, the transport costs and time match the Euclidean distance multiplied by factor 10.

<table>
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<th>QU [QU]</th>
<th>250</th>
<th>500</th>
<th>1500</th>
<th>3000</th>
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<td>29</td>
<td>15</td>
<td>5</td>
<td>3</td>
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</table>

5.2. Computational results

First, note that the 10 instances with the combination of the mentioned fixed costs and depot capacity are called test group in this section. We use this notation in the remainder of this section to illustrate average results instead of explicit results of all test instances.

For all instances, we aborted the TCFLPTW and the OPAP calculations if the optimality gap dropped to below 1%, in order to speed up the overall procedure. The relative cost reduction between the initial and best solution varies between 1.84% and 54.41% and the number of open depots is reduced by between 1 to 28. For 50% of instances, the relative improvement is between 10.51% and 28.03%, and the depot number reduction is between 4 and 17. Actually, the decrease of depots permits the exploitation of economies of scale in both transportation stages. In all test groups the number of depots varies for the initial as well for the best solution. This indicates that time aspects have an effect on the location decision of the TSLRPTW. The domination of assignment costs on the second transportation stage is decreasing with increasing fixed costs. If in addition only tight depot capacities are available, then the number of depots is close to the minimum required number for both the initial and the best solution. Therefore, the lowest relative cost reductions are within the test group of fixed costs 3000 MU and capacity 250 QU. The relative cost and depot number reduction below the 0.25-quantile is achieved mostly for the test groups with depot capacity 250 QU. This is due to the fact that the depot number cannot be reduced as in instances with more depot capacity. The reverse holds for depots with high capacity. Thus, the results above the 0.75-quantile of the relative cost reduction are achieved mostly for the test groups with fixed costs 1000 and 3000 MU and depot capacity 1500 and 3000 QU. Finally, the results above the 0.75-quantile of the depot number reduction are achieved mostly for the test groups with fixed costs 100 and 500 MU and depot capacity 1500 and 3000 QU. This is due to the fact that the TSCFLPTW model locates a huge number of depots because of the low fixed costs and the dominating assignment costs on the second transportation stage. Hence, the proposed tabu search detects unnecessary depots and is able to close a large number of depots because of the sufficient capacity and to reduce simultaneously the overall costs.

In Table 7 each line corresponds to a test group. Columns ‘# depots’ correspond to the average number of open depots and ‘costs’ to the average overall costs of the initial and best obtained solutions, respectively. Column Δ provides the average percentage reduction of the initial solution costs by the proposed tabu search heuristic. ‘time’ is the average computation time of Algorithm 2, given in hh:mm:ss, and ‘# iter.’ the average number of iterations. The computational results prove that a location-routing approach leads to much better solutions than the sequential approach. The results show that initial solutions open many more depots on average than the best solution obtained by the tabu search approach (see Table 7, column 3 and 5). We also observed that, with increasing capacity for given depot fixed costs, the average percentage costs reduction increases, too. Moreover, the increase is growing stronger with increasing depot fixed costs.

We further analyzed the impact of the obtained initial and best solution to the distribution costs. In Table 8, columns ‘fixed’ correspond to the average fixed costs, ‘1st’ to the average costs of the first transportation stage, ‘2nd’ to the average routing costs of the second transportation stage for the obtained initial and best solution of each test group. By comparing the transportation costs of the first and second stages, we observed that they do not differ as much for the initial solution as for the best solution. For the best solution, the routing costs (second transportation stage) are much higher than the first transportation stage. Moreover, the comparison of the initial and the best solutions shows that in average the first transportation costs are lower (see Table 8, columns 4 and 7) and routing costs are higher (see Table 8, columns 5 and 8) for the best obtained solution in all test groups. This is primarily the result of the fact that the open depot number is reduced. Therefore, an increasing consolidation on the first transportation stage is performed. As mentioned before, the domination of the assignment costs on the second transportation stage of the TSCFLPTW model is the reason for locating more
Table 7. Average computational results of the test groups

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Table 8. Average distribution costs

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Depots. With the higher number of depots, the proximity to customers is reduced and, thus, the assignment costs. In test groups with increasing fixed costs, we have a decreasing domination of assignment costs and if in addition only tight depot capacity is available, then the number of open depots is close to the minimum required number (compare Tables 6 and 7, column 3).

The average computation time (see Table 7, column 10) shows that the instances can be solved in acceptable time. We observed that the TSCFLPTW model is solved quickly in most cases.

6. Conclusion

In this work, location and routing aspects of the two-stage distribution network with time restrictions are tackled together. The problem consists of locating depots, assigning customers to them, and planning the first transportation stage and the routes on the second transportation stage with the consideration of time windows and capacity restrictions at depots and customers.

We tested the proposed tabu search on 160 instances with different combinations of fixed costs and depot capacity, as well as varying time windows. The solutions obtained indicate that the sequential approach considers real transportation costs and time aspects insufficiently. Further, the results show that the proposed tabu search is an appropriate and far better method to solve the TSLRPTW. The sequential method is dominated by the assignment costs of the second transportation stage. As a result, too many depots are selected to reduce the proximity to customers. This leads to less consolidation on the first transportation stage and thus to higher overall transportation costs. In contrast, due to simultaneous location and routing decisions, the proposed tabu search reduces the...
number of open depots, increases the consolidation on the first transportation stage, and accepts the increase of the routing costs on the second transportation stage as long as the overall costs are reduced. Because of the higher number of assigned customers per depot, due to fewer open depots, a more cost-effective routing is possible than in the sequential approach.

References