Supporting problem-solving in Mathematics with a conversational agent capable of representing gifted students' knowledge

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Abstract
This paper describes a conversational agent designed to support problem solving in Mathematics. The agent's knowledge base has been structured to represent gifted students' problem solving strategies. These were students who won the Brazilian Mathematics Olympics for Public Schools, and the idea here has been to elicit and represent their formal and heuristic knowledge for problem solving. The paper describes the method for capturing the cognitive processes of gifted students in solving Math problems and the structuring of this knowledge for the conversational agent. The paper also presents the results achieved, showing that the students who used the agent to solve Math problems were fully engaged in the tasks proposed and had a better performance than when not using the agent.

1. Introduction
In the mid 1980's, Benjamin Bloom [1] demonstrated that one-to-one tutoring could have a significant advantage when compared with more traditional class instruction. He showed that students that were taught with the one-to-one approach achieved results that were 2 standard deviations higher (2 sigma) than other students following a more traditional approach. However, because of the difficulty in implementing such a method, different strategies have been proposed along the years to bridge the 2 sigma gap. Especially in the area of computers in Education, Bloom's work has served as a parameter for researchers to design tools and methods to provide individualized care to students' needs. The area of intelligent tutoring systems (ITS), in particular, relied heavily on the idea that these systems could give learners individualized attention, based on a deep understanding of the students' behavior [2]. Research in the area has successfully developed adaptive tools and techniques to provide individualized support to students in many different domains [3]. However, delivering individualized assistance to students brings difficulties that are hard to overcome, such as the representation of ill-defined domain knowledge and the modeling of students' behaviors and mental states.

In this paper we propose a mechanism for the representation of mathematical knowledge using the language AIML (Artificial Intelligence Markup Language). A conversational agent using this mechanism to store knowledge from gifted students has been developed. Our main research question has been whether this agent would be able to give individualized assistance to students and help them solve problems they would not be able to on their own. The agent's approach to helping learners in solving the math problems was to present them with similar solutions developed previously by the gifted students. These students use analogy, they organize data and identify patterns in a particular way when solving mathematical problems [4]. The possibility of capturing and storing their reasoning strategies and using this knowledge to support other students' work can be seen as another initiative towards bridging the 2 sigma gap.

2. Conversational Agents
In the 1980s, intelligent tutoring systems started to emerge, with the main purpose of monitoring and guiding students in their learning processes. Despite the advances produced in all of the research developed in the area, the complexity of the structure of these systems greatly limited their practical application. Developing an intelligent tutoring system has always been a difficult task, and this was due to the complexity of the technology needed for knowledge representation, cognitive modeling, qualitative processing and causal modeling processes, in addition to the difficulty involved in domain knowledge elicitation and representation [5]. Some of these systems used conversational agents to interact with users based on the idea that the human figure
could promote system-learner interaction by better simulating social contexts [6]. These agents often exploited natural language technologies to interact with users in different tasks for a broad range of applications [7].

In Education, research on conversational agents has demonstrated that the introduction of animated life-like characters may have a positive effect on students’ perception of their learning experience [8]. By simulating social interaction, these agents try to improve student’s engagement and learning, whether the users are just kids or adults. It also has been demonstrated that students considered a topic of study significantly less difficult and the presentation more entertaining in the presence of a virtual character [9]. In the same experiment most of the students stated that the assistants helped them pay attention to the most important details. According to Vygotsky’s socio-cultural perspective, social interaction is the main condition for learning to happen [10]. Critical thinking and collaborative skills can be promoted by learners interaction. In this context, teacher-student interaction can be seen as a highly social activity, a premise that has founded the development of different conversational agents. Autotutor [11], for instance, employs conversation patterns of human tutors, engaging students in collaborative dialogues that lead learners to the construction of their own answers to a particular problem. Kim & Baylor [12], on the other hand, investigated how virtual characters could be used not as tutors, but as learning companions.

It has also been claimed that animated pedagogical agents provide an opportunity to improve the communication among students and computers by using paralinguistic cues, such as gestures and facial expressions, which are familiar to human beings [13].

Other research has advocated that agent-human communication has the potential to establish emotional and social bonds with the student, a feature that can facilitate learning [14] and reduce the feelings of isolation that is common in distance learning settings [15], [16].

The area of pedagogical agents has also seen alternative approaches along the years, such as the use of agents as virtual pupils to be taught by students [17], or the introduction of these agents in virtual worlds to provide a programmable stage for agents to interact with students in a distance learning context [18]. Here, however, we have focused on a more traditional approach in which the agent developed has the goal to help learners in solving mathematical problems, providing individualized support for each student. Such an approach relies on the premise that the placement of a virtual character in an interactive learning environment can have a strong positive effect on the students’ view of their learning experience [19]. This notion has been called the “persona effect”, a precept that has been discussed by several researchers in the human-computer interaction community [20], [21], [22].

In the early 1990s ALICE (Artificial Linguistic Internet Computer Entity) was an original and innovative project in the field of artificial intelligence [23]. It was an example of a conversational agent, with an open source system maintained by an active community. The system has been operational until today and is composed of two parts: a conversational machine and a knowledge base constructed using AIML (Artificial Intelligence Markup Language). The language has a specific structure composed of categories, which consist of at least two elements: pattern and template, as in the example below.

```xml
<category>
<pattern>possible user input</pattern>
<template>agent's response</template>
</category>
```

The operation of agents using AIML is based on a stimulus-response model, where the stimuli (user input) is compared with patterns and when one or more pattern matches occur, an associated response is triggered [23]. All of these actions in terms of seeing the appropriate pattern and showing the related template are loaded by the data treatment machine.

Different conversational agents have been built using AIML. Cybelle is an agent that interacts in Portuguese, and also in English and French [24]. It gives information about other agents, such as ALICE. Professor Elektra is an educational agent whose main goal is to serve as a complementary learning tool for students doing distance education courses [25]. CHARLIE is an agent responsible for interacting with students and an intelligent educational system, presenting course contents and asking about the learning material [4].

The wide range of conversational agents developed with interactive engines based on the ALICE project has been an important factor for this choice of engine for the research presented in this paper.
3. Conversational Agent Blaze

The basis of the conversational agent Blaze was structured in AIML with knowledge extracted from gifted students during the course of solving math problems. These students were medalists from the Brazilian Mathematics Olympics for Public Schools who participated in particular research project.

The software used for the development of the conversational agent Blaze was hosted on a public server (Pandorabots.com). The agent was capable of responding to students who interacted with it through its AIML conversational mechanism. Blaze could not solve the problems himself, but he could serve as a trained and reliable assistant during the problem-solving process. Through keywords or questions, students could talk to Blaze, who provided tips for solving new math problems. An example of a dialogue between Blaze and a student is presented in Fig. 1.

![Figure 1. Blaze's response with a video on magic squares](image)

In this example, translated here from Portuguese, the agent answers a student's query on "what a magic square is", displaying a definition and a video containing explanations and examples about magic squares.

In another example, the student enters the search terms "divisibility by 5", and Blaze gives the following answer: "A number is divided by 5 when its last digits are 5 or 0. More details can be found in the link: http://pessoal.sercomtel.com.br/matematica/fundam/naturais/divisibilidade.htm (in Portuguese)"

Blaze’s AIML mechanism enables it to answer queries that are written in natural language, or simply as search terms. The optional context of a search category allows the agent to remember a previous statement. This feature, together with the possibility of launching particular programs when a certain pattern is found, makes the AIML communication mechanism very distinct from a simple retrieval of questions and answers from a database.

4. Using the Agent Blaze

The development and evaluation of agent Blaze's knowledge base involved two main research phases. The first phase (A) took place in the region of Campos, in Rio de Janeiro, Brazil, involving gifted students from the Brazilian Public Schools Mathematics Olympics. Its goal was to capture and represent the cognitive processes of these gifted students during the solving of math problems. In the second phase (B), it has been our goal to determine whether this knowledge could be used to assist math students' in problem-solving tasks, by providing them with personalized assistance with the agent Blaze.

**Phase A: Research with Gifted Students**

The 11 students who participated in this part of the research were medalists of the Brazilian Mathematics Olympics for Public Schools. Nine of these students were in elementary school, and two were in high school (age range between 10 and 17 years old).

During this first phase, a "think aloud" protocol was used to elicit the cognitive processes of the student’s mathematical reasoning when solving problems. In this method, it is suggested that the student freely and spontaneously verbalize, out loud, all the thoughts that occur during the execution of the task. These were recorded in audio and a few times in video, after which the responses were analyzed and categorized.

For example, one of the problems given to students involved right triangle trigonometry:

**Triangle ABC has sides** **AB = \sqrt{12} , BC = 4 and CA = \sqrt{20} .** **Calculate the area of triangle ABC.**

The gifted student solved the problem and verbally presented the method used, through a dialogue with the teacher, as follows:

**Student S1:** I plotted the height and then separated the triangles. I used 4 as a base, one side I called b and the other 4 – b.

**Teacher:** 4?

**Student S1:** The sides were \sqrt{12} , \sqrt{20} and 4. I set it up like that. Then I used the Pythagorean Theorem with each one. First in the one with and after in the one with. Then one goes hooking up with the other. I found that 20 = h2 + h2 replaces the other. I found
the height to be equal to $\sqrt{11}$, as required by the area, which is equal to the base four times the height $\sqrt{11}$ over 2. And 4 divided by 2 equals 2.

Apart from the verbal presentation, students also represented in writing how they solved the problem, as can be seen in Fig. 2. A written representation was necessary because this problem with right triangle trigonometry requires a geometric construction of the triangle which, along with the other information given in the question, allows for a better understanding of the process used for solving it.

The different representations are complementary, since capturing the cognitive thinking of students while solving a problem only through words may not clearly reflect the process used. Often, the decisive idea that solves a problem is linked to a well-chosen word or sentence [26]. The basic reasoning mechanisms adopted by gifted students in the problem-solving process focus on retrieving concepts and, also, on combining ideas to reach the solution. The reasoning of students is based on past experience, that is, on prior knowledge which is a powerful means for people to solve problems. Apart from that, talent and high intelligence are associated with selective comparison (effectiveness in recalling similar problem situations that were previously resolved) or analogical reasoning [27]. At school, as well as in daily life, new problems arise. Solutions often come from prior learning and experiences acquired through solving similar problems.

![Figure 2. Representation of the right triangle trigonometry problem](image)

Therefore, in order for new problems to be solved by students who did not have the knowledge base and experiences of gifted students, a set of cases and respective solutions was coded into Blaze’s AIML knowledge base. Kojima and Miwa (2006) [28] also exploited the idea of using previous cases to support learning in Mathematics. Here, however, we have focused on the representation of these cases in a way that could used by the agent Blaze in its interaction with the students. This approach allowed more inexperienced students to improve their problem-solving skills with tips from "experts", in this case the gifted students.

**Phase B: Student Interaction with Blaze**

In this phase, 13 undergraduate Science students went to the computer lab to solve math problems with the support of agent Blaze (students' age ranging between 18 and 22). During this stage, students were given a handout with guidelines on Polya’s heuristic method for problem solving [26] which involves five steps: understanding the problem, representing it, devising a plan for solving it, carrying out the plan and verifying the solution.

In addition, an already-solved math problem was provided showing ways students could interact with agent Blaze, i.e. tips on how to ask Blaze questions. This procedure was adopted in order to supply directions and facilitate interaction between students and Blaze, since it was the first time they were interacting with the agent.

By providing students with explanations on how to solve specific math problems, Blaze attempted to support students learning and improve their performance, in a scaffolding self-regulated approach analogous to [29].

During the solving of math problems with the support of agent Blaze, students are inserted into a self-regulated learning process with four main features: self-motivation, planning or automatization, self-awareness concerning performance results and skill within the learning environment [30].

In the experiment conducted, after solving the problems that were posed, students answered a questionnaire (in Portuguese) containing several questions. The next section presents the main results of the experiment.

**5. Results and Discussion**

These results are related to the experiment carried out with the 13 undergraduate Science students who solved a series math problems with the support of agent Blaze (phase B). The first three questions of the questionnaire checked the frequency of computer use, with the goal of verifying if there would be students participating in the experiment that did not use the computer regularly. Results showed that all the participants were very familiar with computers and the Internet. This was important as we did not want the lack of familiarity with technology to influence the results regarding the capacity of agent Blaze to help students in solving Math problems. The work done with the support of agent Blaze used the
concept of cognitive scaffolding, providing orientation to help learners while solving problems. Scaffolding guides students to make predictions, experiment, reflect, write explanations, collaborate, contribute to online discussions and participate in classroom discussions [31]. Within the context of this project, therefore, the purpose of scaffolding was to supply new learners with a learning environment with limited complexity and gradually remove the limits until students became more skilled, as proposed by Young [32].

The following series of questions, presented in Table 1, tried to establish the interest/engagement of the student during the study. These questions, adapted from [33], were necessary to assess the engagement level of the participants when carrying out the activities proposed with the support of agent Blaze. The concept of engagement is directly related to the motivation that a participant has in truly performing a task, for which outside reward is not needed [34].

It can be noted in table 1 that students were highly engaged in solving the math problems. Concerning the evaluation of students’ engagement with using a computer system, the use of a human figure can result in an increase in engagement due to factors such as student identification with the character and novelty [33]. In Table 1, 31% of students strongly agreed and 54% partially agreed that they were conscious of their decisions for reaching solutions to the problems. These represent the actions and behavior of learners, made with self-control. This is metacognition, which is conscious and involves reflection. In Table 1, 61% of students agreed that they were in control of the situation and 54% agreed that they were feeling good about themselves, which denotes self-evaluation, a process of metacognition, understood as an internal mental process by which learners themselves are aware of the different stages and aspects of their cognitive activity [35].

Table 1. Results regarding student engagement

<table>
<thead>
<tr>
<th>On a scale of 1 (strongly agree) to 5 (strongly agree), select the appropriate answer for each situation.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Abstain</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) While solving the problems, I was concentrated.</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>8 (62%)</td>
<td>5 (38%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>b) I was very conscious of my decisions in reaching the solution.</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>1 (8%)</td>
<td>7 (54%)</td>
<td>4 (31%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>c) I was in control of the situation.</td>
<td>1 (8%)</td>
<td>4 (31%)</td>
<td>0 (0%)</td>
<td>6 (46%)</td>
<td>2 (15%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

The experiment that was conducted showed a level of student engagement equivalent to 4.15 on a scale of 1 to 5. This result demonstrated that students were very involved in the realization of the proposed activity with agent Blaze. Table 2 presents another set of questions that sought to establish whether agent Blaze provided support to the students in solving problems. The results show that, among the students surveyed, 85% agreed that Blaze's assistance enabled them to obtain solutions to the mathematical problems.

Table 2. Results regarding Blaze's capacity to help students

<table>
<thead>
<tr>
<th>On a scale of 1 (strongly agree) to 5 (strongly agree), select the appropriate answer for each situation.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) The agent made suggestions that helped recall relevant information for solving the problems.</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>4 (31%)</td>
<td>9 (69%)</td>
</tr>
<tr>
<td>b) Interaction with Blaze respects the student's own pace.</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>2 (15%)</td>
<td>3 (23%)</td>
<td>8 (62%)</td>
</tr>
<tr>
<td>c) Blaze offers students individualized help.</td>
<td>0 (0%)</td>
<td>2 (15%)</td>
<td>0 (0%)</td>
<td>2 (15%)</td>
<td>9 (69%)</td>
</tr>
<tr>
<td>d) Interaction with agent Blaze enabled using new ways to solve the problems.</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>1 (8%)</td>
<td>2 (15%)</td>
<td>9 (69%)</td>
</tr>
<tr>
<td>e) Interaction with the agent made for an improvement in the chain of ideas for solving the problems.</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>0 (0%)</td>
<td>9 (69%)</td>
<td>3 (23%)</td>
</tr>
<tr>
<td>f) I will be able to use the type of assistance provided by Blaze, even without being asked by the professor.</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>3 (23%)</td>
<td>9 (69%)</td>
</tr>
<tr>
<td>g) Blaze's assistance enabled solutions to be reached for solving mathematical problems.</td>
<td>0 (0%)</td>
<td>2 (15%)</td>
<td>0 (0%)</td>
<td>7 (54%)</td>
<td>4 (31%)</td>
</tr>
<tr>
<td>h) I think this kind of support should also be given for problems in other fields, such as Physics, Chemistry and Biology.</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>12 (92%)</td>
</tr>
<tr>
<td>i) I recommend Blaze's assistance to my fellow students for solving mathematical problems.</td>
<td>0 (0%)</td>
<td>2 (15%)</td>
<td>0 (0%)</td>
<td>5 (38%)</td>
<td>6 (46%)</td>
</tr>
</tbody>
</table>

A last open question was presented to the students in an attempt to reinforce this finding. Students were asked: Which of the math problems presented would you be able to solve without Blaze's help? Why?
Some of the comments made by students about this question are presented below:

- 31% stated they would not be able to solve the questions. Some of the explanations are given below:
  - I wasn’t familiar with terms that were in the questions, such as what a “magic square” or “golden ratio” is. It was necessary to research the meaning of these terms and only then begin to solve the questions;
  - I was almost unable to solve any question, because I needed another type of help that the robot could not give me. However, without the concepts it gave me, I wouldn’t even be able to start the 1st and 2nd. I also wouldn’t be able to do the 4th, since it requires more research.

- 31% would only be able to solve question 3 and explain why with reasons such as:
  - I used logical thinking;
  - all that students needed to know were the multiples of the numbers requested in order to solve the question;
  - the reasoning was more logical and the trial and error method could be used;
  - because it was a question that depended more on organizing the numbers as opposed to more complex calculations.

- 15% would only be able to solve question 2, explaining why with reasons such as:
  - I used logic to solve it;
  - I used trial and error.

- 15% would be able to solve questions 2 and 3 since:
  - I would be able to solve them using my knowledge of mathematical logic;
  - because I already had previous knowledge about the magic square and the concepts of divisibility and probability.

- 8% did not provide any answer to the question.

The need for an individualized approach in the learning process of Mathematics was emphasized in the response of option (c) (question 5, Table 2). These results demonstrated that Blaze’s assistance was important during problem-solving. The individualized support given to students by the agent Blaze could be compared to a one-to-one tutoring approach.

Results in a different experiment demonstrated that students obtained grades in their tests results that were 2 sigma higher when compared with test grades in which the same students did not have the assistance of the agent. This can be verified in figure 3, which shows a comparative chart of the performance of 19 undergraduate Math students using the agent Blaze and not using the agent to solve problems (students’ age ranging between 18 and 27). Results shown in the chart demonstrate that students either improved or maintained their performance by using the agent.

![Figure 3. Comparative chart with students performance in two Math tests](image-url)

The conversational agent’s help provided a motivating learning environment, which supported the pursuit of strategies for solving problems, thus favoring the acquisition of problem-solving skills. It is also worth mentioning that other studies performed with an experimental group (which used agent Blaze) and a control group (which worked without the help of agent Blaze) revealed situations where the control group could not solve certain problems, while the experimental group was able to obtain the solutions with the help of the agent. In this experiment, only the students that interacted with the agent could solve the questions: "Have you ever heard the expression ‘golden rate’? Do you know what number this is? Can you show a way to find this number? Can you show a real life and a Math application for this number?"

We understand that the students managed to answer these questions by interacting with the agent because it gave them the necessary previous knowledge to enable them to have the basic foundation to find a solution to the problems.
6. Conclusion

This paper presented a conversational agent model capable of representing the problem-solving processes of gifted students in order to provide support to other students for solving mathematical problems. The conversational agent developed was able to interact with students, showing applicable strategies for the process of solving new problems, thereby helping them to develop cognitive skills in a meaningful learning process. According to Ausubel [36], meaningful learning occurs when new concepts are connected with relevant prior concepts in the cognitive structure of students (subsumers). In case the students interacting with the agent would not have these subsumers, the agent could create learning opportunities that could lead to the development of new subsumers which, in turn, would make students more skillful in solving problems. Results obtained from experiments with 13 students showed that students' grades were higher when they counted with the help of the agent to solve mathematical problems, which is an evidence that the agent's assistance is comparable to that of a one-to-one tutoring approach.

In this study, the representation of agent Blaze's knowledge used AIME. This form of representation has been improved along the way in a process where new developments can be anticipated, such as the use of generic search engines combined with the agent's work, as an extension of the body of information stored in its knowledge base. It is intended, in future studies, to implement in agent Blaze a record for storing the students' data, in addition to recording the history of the agent's dialogue with students. This feature could contribute to improve and customize the interaction mechanism between the agent and students.

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8. References


