Design of Consumer Review Systems and Product Pricing

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Abstract

Consumer review systems have become an important marketing communication tool through which consumers share and learn product information. This paper analyzes firms’ review system design and product pricing strategies. We show that firms’ optimal strategies critically depend on contextual characteristics such as product quality, product popularity, and consumer misfit cost. Our results suggest that firms should choose a low rating scale for niche products and a high rating scale for popular products. Different pricing strategies should be deployed during the initial sale period for different product types. For niche products, firms are advised to adopt lower-bound pricing for high-quality products to take advantage of the positive word of mouth. For popular products, firms are advised to adopt upper-bound pricing for high-quality products to enjoy the direct profit from the initial sale period, even after taking into account the negative impact of high price on consumer reviews.

1. Introduction

With the prevalence of the internet and the success of e-commerce, consumers increasingly resort to the Web to gather information about the products of interest before making their purchasing decisions. According to a recent study by the E-tailing Group [16], 71% of online shoppers stated that consumer reviews have the greatest impact on their product researching experience.

Several factors contribute to consumers’ preference for reviews from fellow consumers when researching products online. Information presented in consumer product reviews is more credible, trustworthy, and relevant than merchant-provided product descriptions [2]. Consumer reviews are more user-oriented while merchant-provided descriptions and professional reviews are more product-oriented [3]. Consumer reviews generate empathy and a sense of community among readers [2; 14]. There is abundant empirical evidence that consumer reviews affect product sales in various domains including movies [6; 12], books [4; 15], beer [5], TV shows [11], video games [17], and so on.

In response, many firms have adopted consumer review systems as a marketing communication tool to facilitate consumer sharing and learning about their products [3]. These consumer review systems are usually integrated with firms’ e-commerce sites. Customers are invited to rate the products after their purchases and promotional incentives are often provided to encourage these post-purchase evaluations. Firms often present rating statistics and aggregate consumer reviews on their product pages. Both firms and consumers benefit from such consumer review systems. Prospective customers can read these product reviews from fellow consumers to learn more about the products in order to make more informed purchasing decisions. Consumer review systems also help firms improve customer service, lower return rate, increase conversion from browsers to buyers, and better respond to consumer needs through product improvement, logistic planning, assortment, and price adjustment [10; 13].

Advanced Web technologies enable firms to obtain precise control over consumer review systems as they design, host, and monitor such review systems. Firms make design decisions regarding what type of information to solicit from customers, what rating scale to use, how to aggregate product reviews, and what type of information to display in consumer review systems. Different review system design choices are observed in popular online consumer review systems. For example, IMDb allows customers to rate a movie on a scale of one to ten. Many retailers such as Amazon, L.L.Bean, Macy’s, Target, and Wal-Mart let customers rate a product on a scale of one to five. These firms also ask customers if they would like to recommend the product to other customers or not. Similarly, YouTube’s “like/dislike” button enables a binary positive or negative rating.

Observing the diversified design features in existing consumer review systems, we are interested in exploring what accounts for these design differences and further examining the optimal design of review systems. In this paper, we focus on firm-managed consumer review systems and study the impact of review system design features on current...
customer ratings and future customers’ purchasing decisions. We propose a formal analytical model to address a series of research questions: How do consumers rate a product after purchase? How do prospective customers interpret the product reviews and make their purchasing decisions accordingly? What is the impact of a firm’s review system design and pricing decisions on consumer reviews? What are the firm’s optimal design choices for its online consumer review system and how do these design choices interact with its pricing decision? How do product and consumer characteristics moderate the firm’s design and pricing strategies?

We model one specific design feature of consumer review systems – the scale of overall rating. Once a firm decides the rating scale for its review system, the corresponding available rating levels are embedded in the user review page from which a reviewer can choose the one that best matches her evaluation of the product.

Based on the resulting product ratings, prospective customers update their beliefs about the product and further make their purchasing decisions. In addition, contextual factors such as product popularity, consumers’ misfit cost, and product quality moderate the impact of online review system design on consumers’ purchasing decisions and therefore the firm’s optimal pricing and review system design decisions are contingent on these factors.

There is an extensive literature studying online word of mouth in the fields of information systems and marketing. Researchers have identified different roles of online word-of-mouth systems. One research stream views online word-of-mouth systems as a reputation mechanism [1; 7] and papers in this research stream focus on building trust and reducing seller uncertainty. Another research stream views online word-of-mouth systems as a marketing communication tool [3; 8; 11] and papers in this research stream focus on communicating product information to consumers and reducing their uncertainty about products. This paper takes the product information view of online word-of-mouth systems and studies one particular type of such systems – online product review systems.

Our work builds upon the existing literature of consumer reviews and addresses the design issues of consumer review systems that have not yet been answered in the literature. Existing literature in online consumer reviews either does not consider the design of consumer review systems or treats the system design as exogenously given. In this paper, we endogenize the firm’s design choice by modeling the rating scale of consumer review systems. We systematically study the impact of rating scale on consumer rating and learning behavior and derive the firm’s integrated optimal review system design and pricing decisions while accounting for effects of different product and consumer characteristics.

2. The Model

2.1. Firm and Consumers

Consider a firm selling a product through the online channel. Consumers value both the quality of the product and the fit of the product (e.g., how well the product fits their tastes). The quality of the product \( v \) is the firm’s private information. Before purchase, consumers are uncertain about the true value of \( v \) but they share a common belief that the product quality \( v \) is uniformly distributed on \([\bar{v}, \bar{v}]\), where \( \bar{v} > v > 0 \). Therefore consumers’ pre-purchase expectation of the product quality is \( \hat{v} = (v + \bar{v})/2 \).

From the consumers’ perspective, the difference between the two bounds of perceived product quality \((\bar{v} - v)\) measures the degree of quality uncertainty.

Consumers are heterogeneous in terms of their tastes for the product. The firm knows its product information but not individual consumer’s taste preference. Consumers, on the other hand, know their taste preferences, but do not know product information and how the product fits their own tastes before consumption. We use a unit line to represent consumer taste preference and denote \( t \) as the misfit cost parameter which represents consumer unit misfit cost. Without loss of generality, we assume that the product is located at point zero. Thus, a consumer’s taste location on the unit line also represents the misfit of the product for the focal consumer. For example, a consumer located at \( x \in [0,1] \) has a product misfit of \( x \) and incurs a misfit cost of \( tx \). A consumer with a higher \( x \) incurs a higher misfit cost. Without product information, consumers are uncertain about their product misfit and their belief of the product misfit can be represented by a random variable with a density function \( f(x) \).

Consumers’ belief of the product misfit is consistent with the true consumer taste distribution. We assume that the true consumer taste distribution has density function of \( f(x) = \theta + 2x(1-\theta) \), where \( x \in [0,1] \) represents a consumer’s taste location on the unit line and \( \theta \in [0,2] \) represents the popularity of the product. This general density function \( f(x) \)
represents a series of products with different popularity levels. When $\theta \in [0,1]$, there are relatively fewer consumers located close to the offered product and therefore these cases correspond to niche products. When $\theta \in (1,2]$, there are relatively more consumers located close to the offered product and therefore these cases correspond to popular (or mass) products. When $\theta = 1$, the density function $f(x) = 1$ represents a uniform distribution and corresponds to neutral products. Thus, $\theta$ is the product popularity parameter, with a higher $\theta$ indicating a higher popularity.

We assume the density function $f(x)$ is public knowledge. Before purchase consumers do not know exactly how well the product fits their own tastes and they share a common belief that the product misfit follows the density function of $f(x)$. Therefore consumers’ pre-purchase expectation of the product misfit is $\hat{x} = \int_0^1 f(x)dx = (4 - \theta)/6$. Consequently, consumers have a higher expected misfit cost for niche products and a lower expected misfit cost for popular products. When the firm charges a price $p$ for the product, consumers’ pre-purchase expected net utility is $\hat{u} = \hat{v} - \hat{x} - p$.

2.2. Consumer Rating and Interpretation Processes through Review Systems

The firm hosts an online product rating system to facilitate information sharing among its customers. We study the firm’s review system design choice and its pricing strategy in a two-period model. Consumers arrive independently in each period and each consumer has unit demand for the product. The total number of consumers in each period is normalized to 1. At the beginning of the first period, the firm makes its pricing and review system design decisions. In the first period, consumers are uncertain about their valuations of the product quality and how well the product will fit their tastes. First-period consumers make their purchasing decisions based on their expected valuation of the product quality and their expected misfit cost. After consuming the product, consumers learn the true product quality and the true product fit. Based on their realized net utility, first-period consumers rate the product in the review system. In the second period, consumers learn more about the product quality and the product fit from the posted reviews. Second-period consumers then update their beliefs for the product and make their purchasing decisions accordingly.

We use $p_i$ to represent the product price, $\hat{v}_i$ to represent consumers’ expected valuation of the product quality, and $\hat{x}_i$ to represent their expected misfit in period $i$, where $i = 1, 2$. In the first period, consumers’ expected misfit is $\hat{x}_1 = 2(1-\theta)/6$ and their expected valuation on product quality is $\hat{v}_1 = (\bar{v} + \bar{v}_n)/2$. The first-period consumers’ pre-purchase expected utility is given by $\hat{u}_1 = \hat{v}_1 - \hat{x}_1 - p_1$.

After purchase, consumers learn the true product quality and how well the product fits their tastes, and they rate the product based on their realized utility, given by $u(x) = v - \hat{x} - p$.

There are two key modeling components regarding online consumer product rating systems. The first component is the product rating process. In other words, after first-period customers consume the product, how do they rate the product? The second component is the rating interpretation process. In other words, how do product ratings affect consumers’ expectations of the product in the second period?

We start with the product rating process. We use $s$ to denote the rating scale of the review system. To simplify the exposition, we normalize product ratings to a vertical unit line segment where the highest rating is 1, the lowest rating is 0, and other rating levels are evenly spaced out along the unit line segment. Thus, in a system with $s$ rating levels, the available rating levels correspond to points 0, $1/(s-1)$, ..., $(s-2)/(s-1)$, and 1 on the vertical line. We next introduce a rating function $R(\cdot)$, which maps a customer’s post-purchase utility to one of the available rating levels. We assume that consumers rate the product truthfully based on their post-purchase net utility. We first transform consumers’ post-purchase utility $u(x) \in R$ to a utility score $w(x) \in (0,1)$ according to a logistic function

$$w(x) = \frac{e^{u(x)}}{e^{u(x)} + 1} = \frac{1}{1 + e^{u(x) - \bar{v}}}. $$

This transformation converts consumers’ post-purchase utility to a utility score which has the same scale as the product ratings. Consumers then rate the product by matching the converted utility score to a rating level according to the rating function $R(x)$ defined below:

$$R(x) = \begin{cases} \frac{r}{s-1}, & \text{if } \min \left\{ \frac{i}{s-1} - w(x), i = 0, \ldots, s-1 \right\} \leq \varepsilon, \\ \text{does not rate, otherwise} \end{cases}$$
where \( r = \arg\min_i \left\{ \frac{i}{s-1} - w(x) \leq \epsilon, \ i = 0, \ldots, s-1 \right\} \) is the rating level chosen by consumer \( x \), \( w(x) = \frac{1}{1 + e^{e x}} \) is the utility score for consumer \( x \), and parameter \( \epsilon \) measures consumers’ propensity to review the product. The rating function \( R(x) \) is defined such that the consumer selects the rating level closest to her utility score. As a result, this rating function creates a mapping between consumer misfit and product rating. In this paper, we capture the overall rating results by the mean \( \mu(p_i,s) \) and volume \( n(p_i,s) \) of customer ratings and focus on the case when all ratings exist. The resulting review volume and mean rating can be characterized as:

\[
n(p_i,s) = \int_0^{w_{(i,s)}} f(x) \, dx + \sum_{i=1}^{s-1} \left[ \int_{w_{(i,s)}}^{w_{(i+1,s)}} f(x) \, dx \right]
\]

and

\[
\mu(p_i,s) = \frac{1}{n(p_i,s)} \left[ (1) \int_0^{w_{(i,s)}} f(x) \, dx + \sum_{i=0}^{s-1} \left( \int_{w_{(i,s)}}^{w_{(i+1,s)}} f(x) \, dx \right) \right].
\]

These two key rating results, \( n(p_i,s) \) and \( \mu(p_i,s) \), can be further simplified and detailed derivations are provided in Appendix A.

Next, we model the rating interpretation process and demonstrate how second-period consumers learn from the product review results. In the second period, customers observe the product ratings and update their beliefs on quality \( \nu \) accordingly. The numerical overall rating serves as a signal of the product quality. Given a review scale level \( s \) and observing the mean rating \( \mu(p_i,s) \) and the volume of reviews \( n(p_i,s) \), the second-period consumers form their expected valuation on product quality as follows:

\[
\hat{\nu}_2 = n(p_i,s) \left[ \nu + \mu(p_i,s)(\overline{\nu} - \nu) \right] + \left[ 1 - n(p_i,s) \right] \left( \frac{\overline{\nu} + \nu}{2} \right).
\]

Second-period consumers’ updated belief of the product quality is a weighted average of the review-based belief and the no-review belief. Without considering the review results, consumers’ expected product quality is \( (\overline{\nu} + \nu)/2 \). If consumers form their belief of the product quality purely based on the review results, then the perceived product quality is \( \nu + \mu(p_i,s)(\overline{\nu} - \nu) \), which increases in the mean product rating \( \mu(p_i,s) \). When the mean rating is low, consumers’ perceived product quality is close to \( \nu \). When the mean rating is high, consumers’ perceived product quality is close to \( \overline{\nu} \). We assume that consumers only partially depend on product reviews to learn product quality. Specifically, when updating their belief, the weight that consumers put on the review results is the review volume. In other words, consumers rely on the review results more if there are more reviews. Prior studies have identified several reasons for this type of consumer behavior such as awareness [9, 12], as well as credibility and trust [10].

2.3. Design of Consumer Review Systems

In this subsection, we model rating scale as the firm’s design choice of online product review systems. We use \( s \in \mathbb{Z} \) and \( s \geq 2 \) to denote the number of rating scale levels. For example, \( s = 2 \) corresponds to the “like/dislike” or “recommend/not recommend” case and \( s = 5 \) corresponds to the 5-star rating case. As the number of scale levels increases, it becomes more costly for consumers to compare and select the most appropriate rating level that reflects their evaluations of the product and it eventually becomes overwhelming for consumers to rate. Thus, we only observe limited options of rating scales in practice. In other words, rating levels are bounded by consumers’ capacity to evaluate the product\(^1\). Therefore we examine the firm’s optimal choice of the rating scale from a finite set \( s \in \{2,3,\ldots,\overline{s} \} \), where \( \overline{s} \) is the maximum number of rating levels.


In this section, we analyze a review system that solicits and displays consumers’ overall product ratings. The firm chooses the rating scale of the review system and sets the product price to maximize its profit.

3.1. Formulation

In the first period, anticipating consumers’ pre-purchase expected utility function, the firm sets its

\(^1\) Commonly observed scale levels include 2, 5, and 10.
price \( p_t \leq \tilde{p}_i = \hat{v}_i - \xi_t \),

\[ \frac{\hat{v}_i + v}{2} - \left( \frac{2}{3} - \frac{\theta}{6} \right) t \]

to achieve a positive sale, where \( \tilde{p}_i \) is the maximum price the firm can charge such that first-period consumers will purchase. The maximum price, \( \tilde{p}_i \), is determined by consumers’ pre-purchase expected gross utility. We assume that the expected quality is higher than the expected misfit cost, i.e.,

\[ \tilde{p}_i = \frac{\hat{v}_i + v}{2} - \left( \frac{2}{3} - \frac{\theta}{6} \right) t \geq 0. \]

This assumption ensures that it is feasible for the firm to set a positive price and make a positive sale. As a result, all consumers purchase the product in the first period and the firm’s profit is \( \pi_i = p_i \). The result of serving all consumers in the first period is due to the assumption that first-period consumers share the same belief about the quality and fit of the product. This assumption simplifies the model analysis and helps us to focus on the impact of the firm’s review system design and pricing decisions on past consumers’ review behavior and future consumers’ purchasing behavior.

After purchase, customers learn about their realized utility given by

\[ u(x) = v - \alpha x - p_i \]

and rate the product according to the rating function \( R(x) \).

Note that the firm may charge a price lower than \( \tilde{p}_i \) such that customers will have higher post-purchase utility, which in turn will positively impact the reviews by first-period customers. Lemma 1 summarizes the properties of consumers’ rating results – rating volume \( n(p_i, s) \) and mean rating \( \mu(p_i, s) \). The proofs of all lemmas and propositions are delegated to Appendix B.

**Lemma 1 (Properties of rating volume and mean rating):**

(a) Mean rating, \( \mu(p_i, s) \), decreases in the first-period price \( (p_i) \) and increases in the product quality \( (v) \) regardless of the product type.

(b) For niche products, rating volume, \( n(p_i, s) \), increases in the first-period price \( (p_i) \) and decreases in the product quality \( (v) \); for popular products, rating volume, \( n(p_i, s) \), decreases in the first-period price \( (p_i) \) and increases in the product quality \( (v) \); and for neutral products, rating volume, \( n(p_i, s) \), is independent of the first-period price \( (p_i) \) and product quality \( (v) \).

When the first-period price decreases, or the product quality increases, consumers’ post-purchase utility increases and therefore the mean rating \( \mu(p_i, s) \) increases. As a result, a higher mean rating signals a better-quality product to second-period consumers.

Lowering the first-period price and increasing the product quality have similar impacts on the review volume and their impacts depend on the product type. Lowering the first-period price and increasing the product quality both result in higher post-purchase utilities for all consumers. As a result, more consumers located close to the product on the taste line give the rating 1, and fewer consumers located close to 1 on the taste line give the rating 0. For popular products, because more consumers are located close to the product and fewer are located close to 1, the net result is that the total rating volume increases. In contrast, for niche products, more consumers are located close to 1, and the net result is that the total rating volume decreases with fewer consumers giving low ratings. For neutral products, because the consumer taste density is a constant, the first-period price and product quality have no impact on rating volume for neutral products.

In the rating interpretation process, second-period consumers update their beliefs on product quality based on rating results. Lemma 2 describes the properties of consumers’ updated belief on quality.

**Lemma 2 (Properties of the second-period consumers’ expected valuation on quality):**

The second-period consumers’ expected valuation on product quality \( (\hat{v}_2) \):

(a) increases in the true product quality \( (v) \) and decreases in the first-period price \( (p_i) \);

(b) increases in the rating scale \( (s) \) for popular products, decreases in the rating scale \( (s) \) for niche products, and is independent of the rating scale \( (s) \) for neutral products; and

(c) increases in the product popularity \( (\theta) \) and decreases in the unit misfit cost \( (t) \).

As shown in Lemma 1, a higher product quality or a lower price leads to a higher mean rating which signals a higher product quality. Lemma 2 shows that second-period consumers observe this signal and their perception of the product quality increases. The firm can manipulate the first-period price to influence first-period consumer reviews, and therefore, second-period consumers’ expected valuation on quality.

Interestingly, for a given product quality level and a given price, consumers’ perception of the product quality is negatively related to the unit misfit cost. This relationship is due to the fact that consumer ratings reflect their evaluations of both the
quality and the goodness-of-fit of the product. When unit misfit cost decreases, first-period customers enjoy a higher post-purchase net utility and thus rate the product better. As a result, new consumers observe an increased overall rating and therefore form a higher expectation for the product quality. Since consumers incur a lower overall misfit cost for popular products, their perception of the product quality is higher for popular products when everything else remains the same.

Since consumers cannot infer any information regarding the goodness-of-fit of the product from the overall rating score, second-period consumers face the same level of fit uncertainty as first-period consumers, i.e., $\hat{x} = \hat{x}' = 2/3 - \theta/6$. Thus the second-period consumers’ pre-purchase expected utility is $\hat{u} = \hat{v} - \hat{f}x - p_2$. In response, the firm sets the second-period price at $p_2 = \hat{v} - \hat{f}x$. We focus on the more interesting case in which the true quality is high enough that $\hat{v} > \hat{f}x$. As a result, all consumers will purchase in the second period and the second-period profit is given by $\pi_2 = p_2 - \hat{v} - \hat{f}x$.

Therefore the firm’s overall decision problem can be specified as:

$$\max_{p_1, s} \pi(p_1, s) = \pi_1 + \pi_2 = p_1 + \hat{v} - \hat{f}x$$

s.t. $0 \leq p_1 \leq \bar{p}_1$,

$$2 \leq s \leq \bar{s}, s \in \mathbb{Z}$$

The firm sets the first-period price and selects the rating scale for the consumer review system to maximize its total profit of the two periods.

### 3.2. Optimal Design of the Rating Scale $s$

Proposition 1 delineates the firm’s optimal choice for the rating scale of the review system.

**Proposition 1 (Optimal rating scale level):**

(a) For a popular product ($\theta > 1$), it is optimal for the firm to offer $s^* = \bar{s}$, the maximum number of rating levels;

(b) For a niche product ($\theta < 1$), it is optimal for the firm to offer $s^* = 2$, the minimum number of rating levels;

(c) For a neutral product ($\theta = 1$), rating levels have no impact on the firm’s profit.

We find that the firm’s optimal design for rating scale is contingent on the popularity of the product. For a popular product, a higher rating level, $s$, has a positive effect on the second-period consumers’ perception of the quality of the product (as shown in Lemma 2), which leads to a higher overall profit for the firm for a given first-period price. Therefore, it is optimal for the firm to offer the maximum number of rating levels. In contrast, for a niche product, a higher rating level, $s$, has a negative effect on the second-period consumers’ perception of the quality of the product. Therefore it is optimal for the firm to offer the minimum number of rating levels. For neutral products, the firm’s profit is independent of the rating level, and therefore the optimal rating level could be any integer between 2 and $\bar{s}$.

### 3.3. Pricing Strategy

Second-period consumers learn about the quality of the product through the review system and their uncertainty about the product fit remains the same. In other words, consumers have the same expected misfit cost in both periods. The review system only reduces consumer product quality uncertainty and has no impact on product fit uncertainty.

The firm’s first-period pricing has two countervailing effects on its overall profit. Increasing the first-period price directly increases the firm’s first-period profit but indirectly decreases its second-period profit through its impact on consumer reviews. The consumer review system provides a mechanism for the firm to manipulate second-period consumers’ perception of the product quality. Specifically, the second-period consumers’ updated belief on the quality of the product is at its maximum when the firm offers the product for free ($p_1 = 0$), and it is at its minimum when the firm sets the price to the maximum price such that first-period consumers will purchase ($p_1 = \bar{p}_1$).

The firm aims to balance these two effects of first-period pricing to maximize its total profit and the resulting optimal pricing strategy depends on product quality, product popularity, and misfit cost.

**Proposition 2 (Optimal pricing in the first period):**

(a) For a popular product ($\theta > 1$), if the true product quality is high with $v > \bar{p}_1 + \frac{i^2 - \theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)}$, it is optimal for the firm to charge $p_1 = \bar{p}_1$ in the first period; if the true product quality is medium with $\frac{i^2 - \theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)} \leq v \leq \bar{p}_1 + \frac{i^2 - \theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)}$, it is optimal to charge $p_1^* = v - \frac{i^2 - \theta(\bar{v} - v) - i^2}{2(\theta - 1)(\bar{v} - v)}$; if the true product quality is low with $v \leq \frac{i^2 - \theta(\bar{v} - v)}{2(1 - \theta)(\bar{v} - v)}$, it is optimal to offer it for free in the first period.
(b) For a niche product (\( \theta < 1 \)), if the true product quality is high with \( v > \frac{p_1}{\theta} + \frac{t^2 - \theta(v - \bar{v})}{2(1 - \theta)(\bar{v} - v)} \), it is optimal for the firm to offer it for free in the first period; if the true product quality is low with \( v \leq \frac{p_1}{\theta} + \frac{t^2 - \theta(v - \bar{v})}{2(1 - \theta)(\bar{v} - v)} \), it is optimal to charge \( p_1' = \bar{p}_1 \).

(c) For a neutral product (\( \theta = 1 \)), if the misfit cost is high relative to the product uncertainty \( t > \bar{v} - v \), it is optimal for the firm to charge \( p_1' = \bar{p}_1 \); otherwise, it is optimal to offer the product for free in the first period.

The direct effect of the first-period price on the first-period profit is straightforward—the first-period profit increases linearly in the first-period price for all products (\( \partial \pi_1 / \partial p_1 = 1 \)). The indirect effect of the first-period price on the firm’s second-period profit is more nuanced. Overall the second-period profit decreases in the first-period price for all products (\( \partial \pi_2 / \partial p_1 < 0 \)). However, product characteristics (product popularity \( \theta \) and product true quality \( v \)) moderate the magnitude of this indirect effect (\( |\partial \pi_2 / \partial p_1| \)). Specifically, increasing the true product quality amplifies the indirect effect for niche products (\( \frac{\partial |\partial \pi_2 / \partial p_1|}{\partial v} > 0 \) when \( \theta < 1 \)) while it diminishes the indirect effect for popular products (\( \frac{\partial |\partial \pi_2 / \partial p_1|}{\partial v} < 0 \) when \( \theta > 1 \)). For neutral products, the true product quality has no impact on the indirect effect (\( \frac{\partial |\partial \pi_2 / \partial p_1|}{\partial v} = 0 \) when \( \theta = 1 \)) and the magnitude of the indirect effect is determined by product misfit and quality uncertainty (\( |\partial \pi_2 / \partial p_1| = \frac{1}{(v - \bar{v})/t} \)). The firm balances the direct and indirect effects of the first-period price on the firm’s profit and adopts three possible pricing strategies.

The lower-bound pricing strategy refers to offering the product for free either through charging zero price or through providing coupons, rebates, and other promotional benefits. It is optimal for the firm to adopt lower-bound pricing for high-quality niche products, low-quality popular products, and low-misfit neutral products since the negative indirect effect of the first-period price on the second-period profit dominates the positive direct effect of the first-period price on the first-period profit. The upper-bound pricing strategy refers to charging the maximum price at which consumers still participate. It is optimal for the firm to adopt upper-bound pricing for low-quality niche products, high-quality popular products, and high-misfit neutral products since the positive direct effect of the first-period price on the first-period profit dominates the negative indirect effect of the first-period price on the second-period profit. The interior pricing strategy refers to charging a price between the lower and upper bounds. It is optimal for the firm to adopt interior pricing for medium-quality popular products only and the price is set at such a level that the positive direct effect equals the negative indirect effect.

We find that the firm’s pricing strategies serve different objectives. Through upper-bound pricing, the firm pursues the maximum first-period profit. Here the firm takes advantage of the information asymmetry on quality by charging the maximum possible price in the first period. Through lower-bound pricing, the firm aims to maximize second-period profit by sacrificing its first-period profit. Specifically, the firm sets the first-period price to its lowest possible level to signal a higher quality to future consumers through the review system.

Another interesting finding is that the firm’s optimal design of rating scale and optimal pricing strategy are different for popular and niche products. The firm utilizes a high rating scale for popular products, but a low rating scale for niche products. When the product quality is relatively high, the firm selects upper-bound pricing for popular products, but lower-bound pricing for niche products. When the product quality is relatively low, the firm selects lower-bound pricing for popular products, but upper-bound pricing for niche products.

4. Conclusions

This paper formally models consumer review system design as a firm’s strategic decision. To explore the information role of consumer review systems, we explicitly depict both the product rating and the rating interpretation processes. In addition, we investigate the interaction of a firm’s review system design choices and its pricing strategies. We find that a review system with low scale levels such as like/dislike is optimal for niche products and a review system with high scale levels such as 1-10 is optimal for popular products. Our results suggest different pricing strategies during the initial sale period for different product types. When the firm offers a niche product, it should set a lower price for a better-quality product to take advantage of the impact of positive word of mouth. When the offered product is popular, the firm is able to charge a higher...
price for a better-quality product to enjoy the direct profit from the initial sale, even after taking into account the negative impact of high price on consumer reviews.

5. References


6. Appendix A: Derivations of Important Values

6.1. Derivation of Review Volume \( n(p_1, s) \)

When there are ratings for each of the rating levels, the review volume is given by

\[
n(p_1, s) = \int_0^{w^{-1}(1 - \epsilon)} f(x) \, dx + \sum_{i=1}^{s-1} \int_0^{w^{-1}(\frac{i}{s-1} - \epsilon)} f(x) \, dx + 1 \int_0^{w^{-1}(s) - \epsilon} f(x) \, dx
\]

where \( w^{-1}(\cdot) \) is the inverse function of \( w(\cdot) \). Substituting the individual terms yields \( n(p_1, s) \)

\[
1 - 2 \frac{2(1 - \theta)(v - p_1) + t\theta}{\epsilon^2} \ln \left( \frac{1 - e^{-\theta}}{\epsilon} \right) D_1, \quad \text{where}
\]

\[
D_1 = \frac{1 - (s-1)\epsilon}{[2 - (s-1)\epsilon][2 + (s-1)\epsilon]...[s-2] - (s-1)\epsilon][1 + (s-1)\epsilon][2 + (s-1)\epsilon]...[s-2] + (s-1)\epsilon]
\]

for \( s \geq 3 \). When \( s = 2 \), \( n(p_1, s) \) can be simplified to

\[
1 - 2 \frac{2(1 - \theta)(v - p_1) + t\theta}{\epsilon^2} \ln \left( \frac{1 - e^{-\theta}}{\epsilon} \right).
\]

6.2. Derivation of Mean Rating \( \mu(p_1, s) \)

Mean rating is given by

\[
\mu(p_1, s) = \frac{1}{n(p_1, s)} \int_0^{w^{-1}(1 - \epsilon)} f(x) \, dx + \sum_{i=1}^{s-1} \left( \frac{i}{s-1} \right) \int_0^{w^{-1}(\frac{i}{s-1} - \epsilon)} f(x) \, dx
\]

Let \( m(p_1, s) \) denote the numerator of \( \mu(p_1, s) \), which can be simplified to

\[
m(p_1, s) = \left( \frac{1 - \theta}{\epsilon^2} \right)[v - p_1 - \ln \left( \frac{1 - e^{-\theta}}{\epsilon} \right)]^2 + \frac{\theta}{\epsilon} \left[ v - p_1 - \ln \left( \frac{1 - e^{-\theta}}{\epsilon} \right) - \frac{2(1 - \theta)(v - p_1) + t\theta}{\epsilon^2} \ln D_1 \right]
\]

\[
+ \left( \frac{1 - \theta}{\epsilon^2} \right) \sum_{i=1}^{s-1} \left( \frac{s-1-2i}{s-1} \right) \ln D_2 \ln D_3, \quad \text{where}
\]

\( \text{int}(s-2)/2 \) takes the integer part of \( (s-2)/2 \),
\[ D_2 = \left[ \frac{(s-1-i)-(s-1)\epsilon}{i+(s-1)\epsilon} \right] \left[ \frac{(s-1-i)+(s-1)\epsilon}{i-(s-1)\epsilon} \right], \]
and \[ D_3 = \left[ \frac{i+(s-1)\epsilon}{(s-1)-(s-1)\epsilon} \right] \left[ \frac{(s-1-i)+(s-1)\epsilon}{i-(s-1)\epsilon} \right] \]
for \( s \geq 3 \). When \( s = 2 \), \( m(p_i,s) \) can be simplified to

\[ \frac{1-\theta}{\epsilon^2} \left[ v-p_i - \ln \left( \frac{1-\epsilon}{\epsilon} \right) \right]^2 + \frac{\theta}{\epsilon} \left[ v-p_i - \ln \left( \frac{1-\epsilon}{\epsilon} \right) \right]. \]

7. Appendix B: Proofs of Propositions and Lemmas

7.1. Proof of Lemma 1

We first analyze the properties of rating volume
\[ n(p_i,s). \]
Since \( \frac{\partial}{\partial \epsilon} \left( \frac{1-\epsilon}{\epsilon} \right) = -\frac{1}{\epsilon^2} < 0 \) and
\[ \frac{\partial}{\partial \epsilon} \left[ \frac{i-(s-1)\epsilon}{i+(s-1)\epsilon} \right] = -\frac{2i(s-1)}{[i+(s-1)\epsilon]^2} < 0, \]
for a given \( s \), the natural logarithm term in \( n(p_i,s) \) decreases in \( \epsilon \) and thus rating volume increases in \( \epsilon \). Since
\[ \frac{\partial n(p_i,s)}{\partial p_i} = \frac{4(1-\theta)}{\epsilon^2} \ln \left( \frac{1-\epsilon}{\epsilon} D_1 \right), \]
and
\[ \frac{\partial n(p_i,s)}{\partial v} = \frac{4(\theta-1)}{\epsilon^2} \ln \left( \frac{1-\epsilon}{\epsilon} D_1 \right), \]
for popular products the rating volume decreases in \( p_i \) but increases in \( v \), whereas for niche products the rating volume increases in \( p_i \) but decreases in \( v \). For neutral products, the rating volume is independent of \( p_i \) and \( v \).

Next, we analyze the properties of mean rating \( \mu(p_i,s) \) which can be rewritten as
\[ \mu(p_i,s) = m(p_i,s)/n(p_i,s). \]
Thus the first derivative of mean rating \( \mu(p_i,s) \) w.r.t. \( p_i \) is
\[ \frac{\partial \mu(p_i,s)}{\partial p_i} = \frac{n(p_i,s)\partial m(p_i,s)/\partial p_i - m(p_i,s)\partial n(p_i,s)/\partial p_i}{n(p_i,s)^2}, \]
where \( \frac{\partial n(p_i,s)}{\partial p_i} \) is given earlier and
\[ \frac{\partial m(p_i,s)}{\partial p_i} = \frac{1}{\epsilon^2} \left[ -2(1-\theta) \left( v-p_i - \ln \left( \frac{1-\epsilon}{\epsilon} D_1 \right) \right) - \theta \right] < 0. \]
For niche and neutral products \( 0 \leq \theta \leq 1 \), \( \partial n(p_i,s)/\partial p_i \geq 0 \) and thus \( \frac{\partial \mu(p_i,s)}{\partial p_i} < 0 \). For popular products \( 1 < \theta \leq 2 \), the numerator of \( \partial \mu(p_i,s)/\partial p_i \) is negative, since \( m(p_i,s) < n(p_i,s) \), \( \partial n(p_i,s)/\partial p_i < 0 \), and \( t \geq v-p_i + \ln \left( \frac{1-\epsilon}{\epsilon} \right) \).

Therefore, the mean rating decreases in \( p_i \) for all product types. Since \( \partial \mu(p_i,s)/\partial v = -\partial \mu(p_i,s)/\partial \epsilon \), the mean rating increases in product quality.

7.2. Proof of Lemma 2

The second-period consumers’ expected product quality \( \hat{v}_2 \) can be simplified to
\[ v + \left( \frac{1-\theta}{\epsilon^2} \left[ (v-p_1) + D_4 - D_5 \right] + \theta (v-p_1) \right), \]
where \( D_4 = \ln \left( \frac{1-\epsilon}{\epsilon} \right)^2 \) and
\[ D_5 = \sum_{i=1}^{\text{int}(s-1)/\epsilon} \frac{(s-1-2i)}{s-1} \ln D_2 \ln D_3 > 0. \]

Since \( \frac{\partial \hat{v}_2}{\partial p_i} = \left( \frac{v-p_1}{D_2} \right) \ln D_2 \ln D_3 - \left( \frac{s-2i}{s} \right) \ln D_2 \ln D_3 \),
the second-period expected quality \( \hat{v}_2 \) decreases in the first-period price. Similarly, \( \hat{v}_2 \) increases in the true quality \( v \), since \( \frac{\partial \hat{v}_2}{\partial v} = -\frac{\partial \hat{v}_2}{\partial p_i} > 0 \).

To check the impact of the rating scale on the second-period expected quality, we need to compare \( \hat{v}_2(p_i,s+1) - \hat{v}_2(p_i,s) \), which can be simplified as
\[ \frac{(\theta-1)(v-p_1)}{\epsilon^2} \left[ \sum_{i=1}^{\text{int}(s-1)/\epsilon} \frac{(s-1-2i)}{s-1} \ln D_2 \ln D_3 \right], \]
where \( D_2 \) and \( D_3 \) are in the same form as \( D_2 \) and \( D_3 \) but replacing \( s \) with \( s+1 \). For a given \( i \),
\[ \left( \frac{s-2i}{s} \right) \ln D_2 \ln D_3 > \left( \frac{s-2i}{s-1} \right) \ln D_2 \ln D_3. \]
Hence the sign of the second-period expected quality difference depends on \( \theta \). Specifically, \( \hat{v}_2(p_i,s+1) > \hat{v}_2(p_i,s) \) for \( \theta > 1 \); \( \hat{v}_2(p_i,s+1) < \hat{v}_2(p_i,s) \) for \( \theta < 1 \); and \( s \) has no impact on \( \hat{v}_2 \) for \( \theta = 1 \).

When \( p_1 + \ln \left( \frac{1-\epsilon}{\epsilon} \right) < v < p_1 + \ln \left( \frac{1-\epsilon}{\epsilon} \right) \), all rating levels exist. This implies that
\[ \frac{\partial \hat{v}_2}{\partial \theta} = \frac{-v-v}{t^2} \left[ (v-p_i)(t-v+p_i) + D_4 - D_3 \right] > 0. \]

Thus the second-period expected quality increases in the product popularity parameter \( \theta \).

We next evaluate the impact of the unit misfit cost on the second-period expected quality. Since \( \frac{\partial \hat{v}_2}{\partial t} \) can be simplified to
\[ \frac{2(1-\theta)}{t} \left[ (v-p_i)^2 + D_4 - D_3 \right] + \theta t(v-p_i), \]
we know that \( \frac{\partial \hat{v}_2}{\partial t} < 0 \) for niche and neutral products \((\theta \leq 1)\). At \( \theta = 2 \), we have \( \frac{\partial \hat{v}_2}{\partial t} < 0 \). Since the numerator of \( \frac{\partial \hat{v}_2}{\partial t} \) is monotone in \( \theta \), \( \frac{\partial \hat{v}_2}{\partial t} < 0 \) for all popular products.

### 7.3. Proof of Proposition 1

To determine the firm’s optimal choice on the rating scale \( s \), we need to compare its profit level at the rating scale \( s \) with that at \( s+1 \) for any given first-period price level \( p_i \). The firm’s profit function can be simplified as \( \pi(p_i,s) = p_i + \hat{v}_2(p_i,s) - \hat{e}_1 \), and the profit difference is then given by \( \pi(p_i,s+1) - \pi(p_i,s) = \hat{v}_2(p_i,s+1) - \hat{v}_2(p_i,s) \) as shown in Lemma 2, the sign of the profit difference depends on \( \theta \). Specifically, \( \pi(p_i,s+1) > \pi(p_i,s) \) for \( \theta > 1 \); \( \pi(p_i,s+1) < \pi(p_i,s) \) for \( \theta < 1 \); and \( s \) has no impact on profit for \( \theta = 1 \). As a result, the firm selects the maximum rating scale \( s^* = \overline{s} \) for popular products, the minimum rating scale \( s^* = 2 \) for niche products, and any integer between 2 and \( \overline{s} \) for neutral products.

### 7.4. Proof of Proposition 2

The shape of the profit function depends on product popularity since \( \frac{\partial^2 \pi}{\partial p_i^2} = -\frac{2(1-\theta)(v-v)}{t^2} \) and \( \frac{\partial \pi}{\partial p_i} = 1 - \left( \frac{v-v}{t^2} \right) \left[ 2(1-\theta)(v-p_i) + t \theta \right] \). We analyze three cases for popular, neutral, and niche products.

For popular products \((\theta > 1)\), \( \frac{\partial^2 \pi}{\partial p_i^2} < 0 \). Thus the profit function is concave in \( p_i \). Solving first order condition yields an interior solution
\[ p_{i} = v - \frac{t \theta (v-v) - t^2}{2(1-\theta)(v-v)}, \]
which is feasible, if
\[ 0 < v - \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \leq \overline{p}_i. \]
Therefore, if the true valuation is high, i.e., \( v > \overline{p}_i + \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \), then we will have one boundary solution \( p_{i}^* = \overline{p}_i \); if the true valuation is medium, i.e.,
\[ \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \leq v \leq \overline{p}_i + \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)}, \]
we will then have the interior solution
\[ p_{i} = v - \frac{t \theta (v-v) - t^2}{2(1-\theta)(v-v)}, \]
if the true valuation is low, i.e., \( v < \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \), then we will have the other boundary solution \( p_{i}^* = 0 \).

For neutral products \((\theta = 1)\), \( \frac{\partial \pi}{\partial p_i} = \frac{1 - \frac{v-v}{t}}{t} \) which means the profit function is linear in \( p_i \). If \( v-v > t \), then \( \frac{\partial \pi}{\partial p_i} < 0 \) and thus \( p_{i}^* = 0 \). If \( v-v < t \), then \( \frac{\partial \pi}{\partial p_i} > 0 \) and thus \( p_{i}^* = \overline{p}_i \).

For niche products \((\theta < 1)\), \( \frac{\partial^2 \pi}{\partial p_i^2} > 0 \). Thus the optimal \( p_i \) will take a boundary solution and we need to compare \( \pi(0,s) \) and \( \pi(\overline{p}_i,s) \) to determine the optimal price \( p_i \). The profit difference \( \pi(\overline{p}_i,s) - \pi(0,s) \) can be simplified to
\[ \frac{t}{t^2} \left[ (v-v)(1-\theta) \frac{1}{\overline{p}_i + \frac{(v-v)(1-\theta) + t \theta}{t^2}} \right]. \]
Therefore, if \( v \leq \frac{\overline{p}_i}{2} + \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \), then \( p_{i}^* = \overline{p}_i \); if \( v > \frac{\overline{p}_i}{2} + \frac{t^2 - t \theta (v-v)}{2(1-\theta)(v-v)} \), then \( p_{i}^* = 0 \).