Durable Goods Pricing in the Presence of Volatile Demand

Xiaobo Zheng
University of Rochester
xiaobo.zheng@simon.rochester.edu

Vera Tilson
University of Rochester
vera.tilson@simon.rochester.edu

Abstract

Lead time and demand volatility affect manufacturer production and pricing decisions. A manufacturer must make production decisions based on economic forecasts of demand. After the production is complete and economic conditions are better understood, a monopolistic manufacturer can choose to dynamically adjust prices to react to the revealed demand. Durable goods monopolists face an additional complication – the cannibalization of sales via the second-hand market. Using a model of an infinite horizon game between a monopolistic durable goods manufacturer and heterogeneous consumers, we investigate how lead time and demand volatility affect the monopolist’s production and pricing decisions, as well as his expected profit and consumer welfare.

1. Introduction

Substantial operations literature is devoted to problems of production planning under demand uncertainty. We add to this literature by considering production planning under uncertainty in the context of durable goods that deteriorate over time. Durable goods production planning problem, and related to it, perishable goods production planning, is interesting because even though a single type of product is produced by the manufacturer, product’s deterioration over time results in a market where previous production competes with the more recent.

Similar to operations literature on production planning under uncertainty, durable goods literature is extensive. A search on “durable goods” in JSTOR returns more than 7500 articles. Primary issues of interest to researchers fall into four categories: durability and planned obsolescence, time inconsistency, channel design and coordination, and second-hand markets and adverse selection. Choice of durability is a natural problem uniquely related to durable goods and was a significant theme in some of the earliest literature. Papers that model durability choice, such as Levhari [1] and Bulow [2], generally conclude that a monopolist chooses durability that is lower than the social optimum. This is due to the fact that limiting durability is a way for the monopolist to reduce competition (from older units) in the future by reducing their value. The competition of durable goods sold in different periods lead to time inconsistency problem first discussed by Coase [3], who conjectured that a monopoly producer of durable goods may not be able to extract monopoly rents due to the inter-temporal substitutability of consumption. The Coase conjecture has been proved formally by Bulow [4], Stokey [5] and others. Coase further conjectured that the problem of time inconsistency can be remedied if a durable goods monopolist leases the product instead of selling it. Following Coase, a comparison of the leasing and selling options, and the optimal combination of these two different distribution channels, has received extensive exploration. Adverse selection in the context of durable goods was studied in a seminal paper by Akerlof [6], who shows that adverse selection is an important source of inefficiency in trade in the second-hand market of the durable goods. Hendel and Lizzeri [7] later expanded Akerlof’s analysis by incorporating the interaction of new and second-hand markets of a durable good.

Models’ predictions and recommendations vary based on various dimensions of durable goods and their markets. In this work, we are interested in a particular dimension -- demand uncertainty. Although durable goods literature is vast, the treatment of demand uncertainty in the context of durable goods is relatively rare. The scarcity of such research was previously noted by Goering [8]. He pointed out that firms do not have the luxury of knowing future demand due to business cycles, production innovation, etc. At best, the firm may only have information about the distribution of demand and prices. Desai, Koenigsberg and Purohit [9] later argued that production lead time further complicates firms’ decisions on production and pricing in the presence of demand uncertainty.

A consumer decision to purchase a product is related to his/her utility for using the product. A consumer’s utility is affected by product features as
well as by factors such as income, demographics, and individual preferences. Extensive marketing research has been conducted to model consumers’ market participation behavior by analyzing both consumer and product characteristics. Some of the characteristics are not constant or easily predictable. For example, individual income is highly influenced by economic conditions. Both theoretical and empirical models demonstrate that economic volatility (e.g. uncertainty associated with income and cost of capital) impacts consumer expenditures on durables. Barrett and Slovin [10] use twenty five years of aggregate demand data to show that the high economic volatility during the period from 1973 to 1982 had negative impact on durable goods expenditures by US consumers.

We provide generalized models to study the production and pricing decision problems faced by a monopolistic manufacturer of durable goods. We construct a dynamic infinite-horizon model to analyze the pricing and production decisions of a durable goods manufacturer selling to forward-looking consumers. To capture the randomness of demand in different periods, a factor that represents the economic condition is assigned to the consumers’ valuation function. Volatility in the willingness to pay of individual consumers results in uncertain aggregate demand. Our model incorporates the heterogeneity of individual consumer valuations and derives demand by aggregating individual purchasing behavior. The interaction between the manufacturer and consumer is modeled as a game with alternating moves. Both the manufacturer and consumer seek to maximize their expected total profit and utilities over the time horizon.

We first analyze manufacturer’s decision problems given that new goods price can be adjusted to react to revealed demand. This pricing practice, widely received as dynamic pricing policy, has been studied in earlier OM literature in the past three decades, see Elmaghraby and Keskinocak [11]. However, very little attention has been paid to forward-looking behavior of consumers and the effects of substitutability between new and used goods. Using numerical experiments we demonstrate that in an infinite-horizon model if the state of the economy in each period doesn’t depend on the state of the economy in previous period, there exists a Markov Perfect Equilibrium (MPE) where the manufacturer produces a fixed quantity of new products every period and sells all that was produced. In this equilibrium consumers fall into three categories: those who always use new goods, those who always use old goods, and those who do not participate in the market.

We later concentrate on the pricing practice where the monopolist manufacturer selects sales quantities so that new and old good prices remain constant. We find that an increase in demand uncertainty will induce the manufacturer to lower the prices for both new and old goods but increase the production quantity. Moreover, manufacturer’s expected profit diminishes under a more volatile demand. We examine a constant price policy because in some of the durable goods markets, the prices of new and used products stay relatively steady. Manufacturers do not change the prices but rather the production quantity of products in different periods. A good example comes from electronics market. Apple Inc. introduces newer versions of iPods every year. A new version generally has more features and offers more functions than the old version. However, the price of a specific model doesn’t change over the years. For instance, the iPod nano started at $149 when the second generation was introduced in 2006, and since then the starting price has remained $149. Apple Inc. is using the similar price strategy for its other product lines, such as laptops, iPad, iPhone and etc.

Automobile market serves as a good example as well. Historically, manufacturer suggested retail price (MSRP) have shown little fluctuation. Figure 1 gives Toyota’s best selling mid-size sedan Camry’s starting MSRP from 1990 to 2011. The price exhibits an upward trend. However, if we consider the inflation adjusted price, it is relatively flat over the years. In the recent late 2008 to 2009 economic crisis, the demand of new automobiles declined. The automobile manufacturers were able to adjust to the demand fluctuation by managing the output of their factories and sales of vehicles.

![Figure 1. Toyota Camry MSRP from 1990 to 2011](image)

The paper is organized as follows. In Section 2, we describe the model setting and formulate the problem. Because the durable goods literature is so vast we choose to start with the description of the model, and then in Section 3 provide a review of literature relevant to our model. In Section 4, we solve for equilibria and present our analysis for infinite-horizon models with both dynamic prices and constant prices. In Section 5, we conclude the paper.

2. Model
Time is measured discretely. The product is durable but has a finite life. To capture the dynamic interaction among market participants while retaining tractability, longevity of goods is restricted to two periods: when the good is newly manufactured it has two periods of service remaining, one period after it is manufactured, no matter being used or not, it has one period of service remaining, and two periods after manufacturing the good is not usable. We assume that unsold new goods can be stored and sold as perfect substitutes for used goods on the second-hand market in the next period. That is a reasonable assumption for some of the durable goods with severe economic obsolescence, e.g. electronics.

The manufacturer is assumed to be a monopolist who produces a single type of product and has no capacity constraints. A constant marginal cost of $c$ is incurred in production and marketing. Both goods that have one period of service remaining and goods that have two periods of service remaining are sold to retail consumers.

### 2.1. Consumer’s problem formulation

The retail demand faced by the manufacturer comes from individual retail consumers. The state of the economy is random and modeled with a single variable $\alpha$. Recall that earlier empirical work has shown that economic conditions affect the consumers’ expenditures on durable goods. Below we describe our assumptions of how $\alpha$ influences individual consumer valuation. As in [12] and [13], we assume that consumers are risk neutral and make decisions to maximize their discounted profit over a decision horizon. We further assume that consumers are heterogeneous in the value they place on the service they receive from a good. Consumers of different types are distributed in the population according to a density distribution function $f(\theta)$ with support on [0,1]. We assume that the size of the population is constant and normalized to one. Each consumer owns at most one unit of goods in each period. In a period where the state of the economy is described by $\alpha$, a consumer of type $\theta$ places value of $u(i, \alpha, \theta)$ on per-period use of a good with $i$ periods of service remaining. So consumer $\theta$ places value $u(2, \alpha, \theta)$ on using a new good for one period when the state of the economy is $\alpha$, and the same consumer places value $u(1, \alpha, \theta)$ on using an old good for one period when the state of the economy is $\alpha$.

In every period a consumer makes a decision whether or not to sell an old good (if he owns one) or to purchase a new or used one (if he does not own one, or sold the one he owned at the start of the period to another consumer on the second-hand market). We formulate a consumer’s decision problem as a dynamic programming problem. The state of the consumer before he makes the decision is given by $(x, \alpha, p^2, p^1)$, where $x \in \{0,1\}$ is the number of period of service remaining in the good he owns, $\alpha$ is the state of the economy, and $p^i$ is the price of a good with $i \in \{1,2\}$ periods of service remaining.

We denote the decision to own a new good for one period with $k = 2$, the decision to own an old good for one period with $k = 1$, and a decision to own nothing with $k = 0$. Interpreting $u(k, \alpha, \theta)$ as willingness to pay, the one period surplus for customer $\theta$ who takes action $k \in \{0,1,2\}$ in state $(x, \alpha, p^2, p^1)$ is given by

$$R(x, \alpha, p^2, p^1; k; \theta) = u(k, \alpha, \theta) - p^k + \alpha p^1$$

(1) with $p^0 = 0$. A consumer decision problem at time $t$ is the solution to

$$V_t(x, \alpha, p^2; p^1; \theta) = \max_{k} \{R(x, \alpha, p^2, p^1; k; \theta) + \gamma E[V_{t+1}(\max(k-1,0), \alpha, p^2, p^1; \theta)|a, p^2, p^1]\}$$

(2)

where $\gamma$ is the consumer discount factor.

### 2.2. Manufacturer’s problem

The manufacturer profit comes from selling goods in each period and the manufacturer determines quantities of goods of different vintage sold on the market. We assume a deterministic relationship between the period’s demand and the price of goods in that period and this relationship will result from the actions of individual retail consumers. Let $p_t^i$ be the price of a good with $i$ periods of service remaining. Let $s_t^i \equiv s(i, \alpha, p_t^2, p_t^1)$ be the number of goods in the market in period $t$ with $i$ periods of service remaining. Following Desai et al. [9], we assume that there is a one-period lead time so that the manufacturer has to decide and complete production for the next period in the current period. When the selling season begins, the manufacturer is not able to produce any more goods for the current selling season. Such assumption is also widely employed in newsvendor models. The manufacturer’s profit over $T$ periods is given by

$$\sum_{t=0}^{T} (\gamma_M)^t (p_t^2 \cdot s_t^2 + p_t^1 \cdot (s_t^2 - s_{t-1}^2) - c q_t)$$

(3)

where $\gamma_M$ is the discount factor, and $q_t$ is the production quantity that will be available for sale at the start of the period $t + 1$, and $s_t^2 \equiv 0$ for $t < 1$ and $i \in \{1,2\}$. The term $p_t^2 \cdot s_t^2$ represents revenue from new goods sold in period $t$. The total number of old goods on the market in period $t$ is $s_t^2$, i.e. $s_t^1$ is an echelon variable. The number of old goods sold by the manufacturer in period $t$ is $(s_t^2 - s_{t-1}^2)$, thus manufacturer’s revenue from old goods sold in period $t$ is $p_t^1 \cdot (s_t^2 - s_{t-1}^2)$. We assume that unsold goods can be stored for one period, and for simplicity assume that the holding costs associated with storage are 0. Thus
each period the manufacturer makes a decision on \( q_t \), how many goods to manufacture for future sales, as well as on \( p^2_t \) and \( p^1_t \) which determine the sales in the current period. The manufacturer problem can be formulated as a dynamic programming equation for \( t > 0 \) as follows.

\[
V^M(a_t, q_{t-1}, s^2_{t-1}, q_{t-2}) = \max_{q_t, p^2_t, p^1_t} \left\{ R^M(a_t, q_{t-1}, s^2_{t-1}, p^2_t, p^1_t) + \gamma_M E[V^M_{t+1}(a_{t+1}, q_{t+1}, s^2_{t+1}, q_{t+1})] \right\}
\]

where

\[
R^M(a_t, q_{t-1}, s^2_{t-1}, p^2_t, p^1_t) = \left( p^2_t - \gamma_M E[p^2_{t+1}(a_{t}, p^2_t, p^1_t)] \right) \cdot s^2_t + p^1_t \cdot s^1_t - \left( \frac{c}{\gamma_M} \right) q_{t-1}
\]

where (5) is derived from (3) by combining \( s^1_t \) that have the same subscript \( t \). Recall that \( s^1_t \) are functions of prices. Decision variables \( p^2_t \) and \( p^1_t \) are subject to constraints

\[
s^2_t \leq q_{t-1}
\]

(6)

\[
s^2_{t-1} \leq s^2_t \leq q_{t-2}
\]

(7)

\[
0 \leq s^2_t + s^1_t \leq 1.
\]

(8)

and appropriate boundary conditions. The first constraint ensures that new goods sales cannot exceed the amount of new goods available in each period. The second constraint requires that the total amount of old goods in the market should be no less than what was sold new in the previous period and no more than the total of old goods available. The third constraint guarantees that the total sales of both new and old won’t exceed the total population.

3. Literature review

In this section we briefly review previous papers that model demand uncertainty in the durable goods literature.

Bhatt [14] considers channel selection problem for a risk-averse durable goods monopolist in the presence of demand uncertainty. Using a two-period model Bhatt shows that the monopolist has an incentive to offer sales contracts to transfer risk to consumers. Bhatt’s conclusion contrasts the finding of previous durable goods models, which predicted monopolist preferences for leases.

Goering [8] analyzes the influence of demand variability on a monopolist’s choice of product durability. Using a two-period model Goering shows that larger variance in demand may lead to a more durable product. Both Bhatt and Goering share common modeling approach. Each uses a two-period model. Demand uncertainty is represented by a parameter embedded in the demand function, and a simple inverse demand function is used where price in each period is decreasing in the total available stock in that period. The drawback of this simple aggregate demand function is that it is not able to capture the interaction of new and used products because available products in the market are priced at the same level. In addition, instantaneous production is assumed, and no production volume decisions are considered.

Desai, Koenigsberg and Purohit [9] incorporate demand uncertainty, interaction of new and used goods markets, and production and pricing decisions in their two-period model of a strategic monopolistic manufacturer and forward looking consumers. They are specifically considering the scenario of new product introduction where demand is uncertain only in the first period. Production lead time is considered because goods cannot be produced instantaneously. They assume that the manufacturer has to complete production before the start of the selling season without knowing the exact demand. In their two-period model, demand uncertainty is driven by randomness of heterogeneous individual valuations on products of different vintages and is captured by a parameter in consumer’s utility function. This enables them to model the interaction of new and second-hand markets. The possible mismatch between supply and demand in the first period results in shortage and overage of the durable product. Inventory unsold at the end of first period (if any), can be sold as new goods in the second period. They find that manufacturer’s optimal inventory level is U-shaped in the durability of the goods.

Our model builds upon Desai et al.’s two-period model but has a number of differences. In our work, we consider demand uncertainty in every period as well as production lead time. Another difference is that the unsold inventory in period \( t \) is carried over to period \( t+1 \) as used goods and can be sold by the manufacturer in the second-hand market. As a result, the manufacturer potentially has the power to control part of the second-hand market and coordinate the two markets to react to demand uncertainty and maximize the total profit. This different assumption on treatment of unsold products models durable goods markets where goods suffer from economical obsolescence due to fast technological advancements and periodic introduction of new versions. For instance, electronics manufacturers introduce new versions of products with more built-in features; automobile manufacturers following a “model year” approach launch new vehicles with improved specifications each year; publishers bring in new editions of textbooks. New versions of the durable goods make old versions less desirable to consumers. Introducing older products into the second-hand market a producer balances the profits received from the sale of older goods against the effects of cannibalization. Both the prices of new and used goods are reduced because there are more used goods in the second-hand market. The manufacturer
thus needs to identify which of the two effects dominates when he makes decisions. Our model also differs in modeling the decision problems over an infinite time horizon.

Two-period discrete time models such as [14], [8], and [9] are widely used in durable goods studies. A drawback of two-period model and other finite-horizon multi-period models is the existence of end-of-period effects. These effects arise because used goods still yield utility to owners after the end of the periods being studied. For example, Desai et al. [9] in their two-period model assume that new goods sold in the second period are of no use at the end of the period. This assumption could affect the conclusions about the new good prices in the last period. These effects can be mitigated by assigning salvage values to used goods at the end of the period, but nonetheless the salvage values cannot capture the market characteristics due to a lack of interaction between new and used goods. It is even more difficult to deal with these end-of-period effects in the presence of demand uncertainties since the yielded utilities are affected by the randomness of the individual valuations. Thus, our model employs a discrete time infinite horizon setting with infinitely lived durable goods to capture the dynamic interaction of new and used goods.

When solving the infinite horizon problem, we adopt the solution concept of a Markov Perfect Equilibrium (MPE) which has been employed in other durable goods models (e.g. [15], [13]). The advantages of using MPE as a solution concept for dynamic games are discussed in Maskin and Tirole[16], who write that “Markov strategies prescribe the simplest form of behavior that is consistent with rationality”.

Our main contribution is that we present an infinite horizon dynamic model that captures strategic interaction between individual consumer and manufacturer in the presence of demand uncertainty in each period. In particular, analyzing how demand uncertainty affects the manufacturer’s production and pricing decisions, as well as profits and consumer surplus, sets our paper apart from the existing literature.

4. Analysis

This section consists of five subsections. In 4.1 we derive the relationship between aggregate demand and market prices of new and used goods. In section 4.2 we consider the manufacturer production and pricing problem over the infinite horizon. Using numerical experiment we found that there exists a steady-state equilibrium in which the manufacturer produces the same quantity and sells all that was previous produced in each period. We conjecture the form of the optimal policy based on extensive numerical experiments. In section 4.3 we solve the infinite-horizon model when constant price policy is adopted. In section 4.4 we provided sensitivity analysis of the results from constant price policy model. In the last section we compare the results from dynamic price policy and constant price policy.

4.1. Derivation of aggregate demand

We assume that all consumers value new goods more than old goods

\[ i_L \leq i_H \Rightarrow u(i_L, \alpha, \theta) \leq u(i_H, \alpha, \theta) \lor \theta \alpha \quad (9) \]

and \( i_L, i_H \in \{0, 1, 2\} \)

Further, we assume that consumers of higher type place higher value on goods of all qualities

\[ \theta_L \leq \theta_H \Rightarrow u(i, \alpha, \theta_L) \leq u(i, \alpha, \theta_H) \lor i, \alpha \quad (10) \]

An additional assumption that high-type consumers have higher marginal value for quality than lower type consumers allows for consumer segmentation

\[ i_L \leq i_H, \theta_L \leq \theta_H \Rightarrow u(i_H, \alpha, \theta_L) - u(i_L, \alpha, \theta_H) \leq u(i_H, \alpha, \theta_L) - u(i_L, \alpha, \theta_H) \lor \alpha \quad (11) \]

Similar to other analytical literature that considers infinite horizon models (e.g. [15], [13]), we assume that consumers are infinitely lived and maximize their surplus over infinite horizon. We employ infinite horizon MPE solution concept and restrict consumers to Markov strategies. We therefore drop the time subscript in the consumer’s value function. We denote the optimal market participating decision of an individual consumer as

\[ k^*(x, \alpha, p^2, p^1; \theta) = \arg \max_k (R(x, \alpha, p^2, p^1; k; \theta) + \gamma E[V(max(k - 1, 0), \theta, p^2, p^1; \theta)\theta, p^2, p^1]) \quad (12) \]

The following lemmas characterize optimal action:

**Lemma 1:** Action \( k^* \) is optimal in state \( (1, \alpha, p^2, p^1) \) if and only if action \( k^* \) is optimal in state \( (0, \alpha, p^2, p^1) \).

**Corollary 1:** \( V(1, \alpha, p^2, p^1; \theta) - V(0, \alpha, p^2, p^1; \theta) = p^1 \)

In the next lemma we prove that consumers with higher willingness to pay select to own goods with more periods of service remaining, which leads to consumer segmentation.

**Lemma 2:** Optimal policy \( k^*(x, \alpha, p^2, p^1; \theta) \) is non-decreasing in \( \theta \).

Lemma 2 implies consumer segmentation: for a given value of \( (\alpha, p^2, p^1) \) there exist two thresholds: \( \theta^1 \) and \( \theta^2 \) such that \( 0 \leq \theta^1 \leq \theta^2 \leq 1 \). Consumers who do not own a functioning good at the start of a decision period, do not buy anything if their type \( \theta \) is in the interval \( [0, \theta^1) \), buy a good with one period of service remaining if their type \( \theta \) is in the interval \( (\theta^1, \theta^2) \) and

---

1 Due to the page limit, all proofs in this paper are available upon request.
buy a good with two periods of service remaining if their type is in the interval \((\theta^2, 1]\). Similarly, consumers who own a good at the start of a decision period, sell it and do not buy any replacement if their type \(\theta\) is in the interval \([0, \theta^2]\), keep it if their type is in the interval \((\theta^2, \theta^3)\) and replace it with a good with two periods of service remaining if their type is in the interval \([\theta^3, 1]\).

A customer of type \(\theta^1\) is indifferent between actions \(k = 1\) and \(k = 0\), when the state is given by \((x, a, p^2, p^1)\), therefore the threshold \(\theta^1(a, p^1)\) is found from the equation

\[
u(1, a, \theta^1) - u(0, a, \theta^1) = p^1 \tag{13}\]

Similarly we can derive an analytical expression for customer \(\theta^2\).

**Lemma 3:** Let \(\theta^2(a, p^2, p^1)\) be a customer for whom in state \((x, a, p^2, p^1)\) actions \(k = 1\) and \(k = 2\) are equivalent and weakly dominate action \(k = 0\), then

\[
u(2, a, \theta^2) - u(1, a, \theta^2) = p^2 - p^1 - \gamma E[p^1|a, p^2, p^1] \tag{14}\]

To continue with the derivation of the aggregate demand, we assume a particular form for the consumer’s one period utility function. In multiple papers including [17], [18], [19], and [9], the consumer utility function is modeled as

\[
u(2, a, \theta) = a\theta^1, u(1, a, \theta) = \delta a\theta, u(0, a, \theta) = 0 \tag{15}\]

where \(\delta > 0\), \(\delta < 1\), is the deterioration factor. The parameter \(\delta\) can also be thought of as a substitutability factor [13], describing how close of a substitute for a new good an old good is. This form of the utility function satisfies assumptions (9) through (11). Combined with the above utility function, equation (13) and (14) imply that

\[
\theta^1(a, p^2, p^1) = \frac{p^2}{\delta a} \\
\theta^2(a, p^2, p^1) = \frac{p^2 - p^1 + \gamma E[p^1|a, p^2, p^1]}{(1 - \delta a)} \tag{16}\]

To complete the derivation of the aggregate demand, we need to make an assumption about the distribution of consumer types in the population. For simplicity we assume uniform distribution of consumer types, so that

\[
f(\theta) = 1 \tag{17}\]

This assumption is widely used in economic modeling literature, see for example [17], [9], [13], etc.

Let \(s^2(a, p^2, p^1)\) and \(s^1(a, p^2, p^1)\) be the number of new and old goods in the market when the economic condition is \(a\) and the prices for new and old goods are \(p^2\) and \(p^1\). Then the market clearing condition implies

\[
s^2(a, p^2, p^1) + s^1(a, p^2, p^1) = 1 - \frac{p^1}{\delta a} \\
s^2(a, p^2, p^1) = 1 - \frac{p^2 - p^1 + \gamma E[p^1|a, p^2, p^1]}{(1 - \delta a)} \tag{18}\]

Adding in the time subscripts, the prices at time \(t\) can be expressed as

\[
p^1_t = \delta a_t(1 - (s^1_t + s^2_t)) \\
p^2_t = a_t(1 - s^2_t - \delta s^1_t) + \gamma E[p^1_{t+1}|a_t, p^2_t, p^1_t] \tag{19}\]

### 4.2. Infinite-horizon model with dynamic prices

For tractability we assume that consumers and manufacturer share the same discount factor. By employing the aggregate demand functions (19) we change the decision variables in the manufacturer’s problem to \(s^2_t, s^1_t\) and \(q_t\) and re-write the immediate reward function (5) as

\[
R^M(a_t, q_{t-1}, s^2_t, s^1_t) = \alpha_t(s^2_t(1 - s^2_t) + \delta s^1_t(1 - s^1_t) - 2\delta s^1_t s^2_t - (c/y) q_{t-1} \tag{20}\]

Notice that the \(E[p^1_{t+1}|a_t, p^2_t, p^1_t]\) terms cancel out. With this formulation, and with discretizing the state space it is possible to compute an equilibrium numerically for both cases where \(\alpha_{t+1}\) is independent of \(\alpha_t\) and where it is dependent on \(\alpha_t\).

To explore analytical results, we assume that each period the economic condition \(a\) can take on only one of two possible values, \(\alpha_H\) with probability \(\rho\) or \(\alpha_L\) with probability \(1 - \rho\). Based on extensive numerical experiments where we discretized the state space and computed optimal policy using value iteration, we make the following conjecture for the case where \(\alpha_{t+1}\) is independent of \(\alpha_t\).

**Conjecture:** Assuming

\[
\gamma_M = \gamma, c/E[\bar{a}] \leq 1 + \delta \tag{21}\]

in equilibrium the manufacturer produces

\[
q_d = \frac{(1 + \delta) - c/E[\bar{a}]}{2(1 + 3\delta)} \tag{22}\]

every period and sells his entire production in the following period. Prices of new and used goods depend on the realized value of \(\bar{a}\) and are given by

\[
p^2_t = \alpha_t(1 - (1 + \delta)q_d) + \gamma \delta (1 - 2q_d)E[\bar{a}] \\
p^1_t = \delta a_t(1 - 2q_d) \tag{23}\]

The expected discounted profits over the time periods is

\[
E[V] = \frac{[(1 + \delta)E[\bar{a}] - q_d]^2}{4(1 - \gamma)(1 + 3\delta)} \tag{24}\]

Both numerical examples and the conjectured forms above indicate that the optimal production quantity and expected profit are independent of the variance of the demand distribution but do depend on expected value of demand. Prices for new and used goods are time independent in each period but do depend on the revealed economic condition \(\bar{a}\) in that period. The pricing policy segments consumers into three groups: those who use a new good every period, those who use an old good every period, and those who stay out of the market.

### 4.3. Infinite-horizon model with constant prices

The aggregate consumer demand for different goods is determined by current and expected future used good prices and revealed economic conditions. In
this section we consider a scenario where prices of goods do not change over time, so the number of new and used goods in the market in a particular period depends only on the revealed economic conditions. If new and used goods prices are set by the monopolistic manufacturer upfront and stay constant over the time horizon, the demand function has slight changes. To be consistent with the assumption and mathematical formulations in prior section, the assumption is stated as

\[
p^1 = p^1, p^2 = p^2
\]  

(25)

Recall that the monopolistic manufacturer is able to control the used good price since (a) he can release stored unused goods on the second-hand market, and (b) the prices of old goods are also dependent on the quantities of new goods on the market.

Define \( s(2, \alpha_L) \) and \( s(1, \alpha_L) \) as the number of new goods and old goods in the market in each period when the state of the economy is \( \alpha_L \). Since the manufacturer starts with no inventory and consumers start with no goods on hand, the market will take time to reach its equilibrium. We write down the manufacturer’s expected profit starting from the period \( \tau \) when the market is stabilized as

\[
\sum_{t=\tau}^{\infty} \gamma^{t-\tau} \left( (p^2 - \gamma p^1)E[s(2, \alpha_L)] + p^1E[s(1, \alpha_L)] - c_q \right)
\]

(26)

where

\[
E[s(x, \alpha)] = \rho s(x, \alpha_H) + (1 - \rho)s(x, \alpha_L)
\]

(27)

To find \( p^2 \) and \( p^1 \) that maximize the manufacturer profit, we solve a one-period optimization problem as follows.

\[
\max_{p^1, p^2} [c_q + (p^2 - \gamma p^1)E[s(2, \bar{\alpha})] + p^1E[s(1, \bar{\alpha})]]
\]

(28)

subject to the feasibility constraints from section 2. The model is further simplified by the assumption that consumers and the manufacturer have the same discount factor. Proposition 1 given below characterizes manufacturer’s decisions on prices and production quantity.

**Proposition 1**: When the exogenous parameters \( \alpha_H, \alpha_L, \delta, \rho, \) and \( c \) satisfy

\[
c/\alpha_H \leq (1 + \delta)(1 + 2(1 - \rho)(1 - \alpha_L/\alpha_H)) \quad \text{and} \quad c/\alpha_H \geq (1 - \delta)(1 - \alpha_L/\alpha_H) - 2\delta \alpha_L/\alpha_H
\]

(29)

In equilibrium the manufacturer produces

\[
q = \frac{(1 + \delta)(1 + 2(1 - \rho)(1 - \alpha_H/\alpha_H)) - c/\alpha_H}{2(1 + \delta)(1 + \rho)(1 - \alpha_H/\alpha_H)\alpha_H/\alpha_H}
\]

(30)

every period. Profit maximizing prices of new and used goods are

\[
p^1 = \frac{\alpha_L c}{\alpha_H} \frac{\alpha_H + 2\delta \alpha_H - (1 - \rho)(1 - \delta)(1 - \alpha_L/\alpha_H)}{(1 + \delta)(1 - \alpha_H/\alpha_H)\alpha_H},
\]

\[
p^2 = \frac{1}{2} \left( 1 + 2\rho + \delta \right) p^1 + (1 - \delta)\alpha_H
\]

(31)

### 4.4. Sensitivity analysis for the constant price policy

In this section we examine how economic volatility (as measured by a change in \( \alpha_L \)) and substitutability between new and used goods (as measured by the exogenous parameter \( \delta \)) affect the manufacturer’s decision on pricing and production as well as his profit. We also look at how these two parameters affect consumer surplus.

In the previous section we derived the expressions for the equilibrium profit, prices, and quantities. A change \( \alpha_L \) is able to capture the changes of coefficient of variation (CV) of the demand. If we hold \( \alpha_H \) constant and decrease \( \alpha_L \), the coefficient of variation increases. The Figure below illustrates CV of \( \bar{\alpha} \) versus the ratio \( \alpha_L/\alpha_H \) for \( \alpha_H = 1 \).

![Figure 2. CV versus ratio \( \alpha_L/\alpha_H \)](image)

**Proposition 2**: In equilibrium, new and used goods prices and the manufacturer profit are increasing in \( \alpha_L \), while the quantity manufactured every period is decreasing. The more substitutable new and used goods are, the higher the prices of both, and the higher the manufacturer’s profit. Quantity manufactured every period is decreasing in \( \delta \).

Higher \( \alpha_L \) implies lower demand volatility. By pricing higher and producing less, the manufacturer is able to extract more profit since he won’t lose many sales from low demand periods. When used goods are closer substitutes for new goods, goods provide more utility to consumers who purchase or own used goods. Thus the willingness to pay for the used goods increases, which in turn drives used goods prices higher. Since the prices are set by the manufacturer, he is encouraged to set the used goods price high to extract more profit from selling used goods on the second-hand market. When it comes to the new goods prices, there is a tradeoff when the substitutability increases. One on the one hand, increased substitutability makes the goods more valuable to the consumers, thus driving the new goods prices higher. On the other hand, the increased substitutability makes the used
goods more valuable and thus more competitive to new goods and the cannibalization of new and used goods drives down the new goods prices. However, our results indicate that the new goods price increase with substitutability, which shows that the first effect denominates the second. Optimal production quantity decreases with increased substitutability. The intuition is that the prices of new and used goods increase as the goods become close substitutes. Although an increase in substitutability increases consumers’ willingness to pay, but the increased prices defer these individuals from purchasing new products. The aggregate effect of the increased substitutability is to reduce the demand, which in turn result in a lower production quantity.

Spurred by the work done by Desai et al. [9] we study the inventory in our model. The inventory from their two-period model is essentially the unsold inventory after the first period due to the demand uncertainty in the first period. Our model also gives us expected unsold inventory because of the uncertain demand. And our results indicate that our findings differ. Denote I as the expected unsold inventory at the end of each period. I can thus be expressed as

\[ I = q - E[s(2, \tilde{a})] \]  

(32)

Based on numerical experiments, Proposition 3 generalizes our results on average inventory

**Proposition 3:** In equilibrium, the expected unsold inventory \( I \) decreases in \( \alpha_L \) and decreases in substitutability \( \delta \).

The results above are in accordance with the real industrial practice. If the demand volatility is low compared to average demand, the manufacturer will be able to better predict the demand such that the mismatch between supply and demand is mitigated. The recent economic recession provides a proper period to check our results. In 2009, the days of supply of inventory of almost all automobile manufacturers increased and GM led the league of inventory at 161 days of supply.

The interesting finding in our model is that the expected inventory decreases with substitutability. Our finding is inconsistent with the one in Desai et al.’s work that argues that the inventory is U-shaped with respect to \( \delta \) (the variable plays the same role as in our model, but is called by Desai and co-authors durability). Desai et al. present an example of a good with high \( \delta \) and high inventory the Chevrolet Prizm. The model had high inventories in 2000, which Desai et al. explained by high durability. According to our model, high \( \delta \) corresponds to low inventories, and we believe that the reasons for high inventory of Chevrolet Prizm might be due to the fact that it turned out not to be a popular model.

After examining sensitivity of manufacturer profit and decisions to exogenous parameters, we turn our attention to consumer surplus. The previous section on consumer decisions indicates that consumers are segmented in to three groups according to their purchasing behaviors. These three groups are identified as purchasing new goods, purchasing used goods (or keep used goods), and staying out of the market. The surplus of all consumers who buy used goods in a given period is given by

\[ E \left[ \int_0^{\alpha_L} (\delta \alpha_L \theta - p_1) d\theta \right] \]  

(33)

The one period expected surplus of individuals who buy new goods is given by

\[ E \left[ \int_0^{\alpha_H} (\alpha_L \theta - (p_2 - \gamma p_1)) d\theta \right] \]  

(34)

We focus on one period consumer surplus, since in equilibrium the expected consumer surplus does not change from period to period. Based on numerical experiments, the following proposition describes how consumer surplus changes with exogenous parameters.

**Proposition 4:** In equilibrium consumer surplus is decreasing in \( \alpha_L \), and increasing in substitutability \( \delta \).

The relationship between consumer surplus and economic volatility is counterintuitive because consumer surplus increases when the economic condition becomes more volatile. One explanation is that the manufacturer becomes conservative when he makes pricing and production decisions upfront. The manufacturer sets lower new and used goods prices and produces more goods each period. The direct result of his decisions is that consumers are paying less for the goods whenever the economic condition is high or low. Since we are focusing on the expected consumer surplus in each period, the decreased prices and unchanged expected economic condition lead to an increase in the consumer surplus on average.

4.5. Comparison of the dynamic price and constant price policies

In this section we present a comparison of the dynamic price and constant price policies through conducting computational study to gain insights into the effect of economic volatility (captured by ratio \( \alpha_L/\alpha_H \)) on manufacturer’s production and pricing decisions and expected profit in both policies. We only present a subset of our numerical observations, but our findings are verified through several experiments to characterize the effect of economic volatility on optimal decisions and profits in these two policies. We fixed substitutability to \( \delta = 0.7 \), discount factor \( \gamma = 0.9 \), production cost to \( c = 0.5 \), probability of having \( \alpha_H \) to \( p = 0.5 \), and expected value of \( \tilde{\alpha} \) to \( E[\tilde{\alpha}] = 3 \). By varying the value of \( \alpha_L/\alpha_H \) from 0.75 to 1, we study the effect of \( \alpha_L/\alpha_H \) on various decisions and expected profits in each policy. To make the graph concise, we denote DPP as dynamic price policy and
CPP as constant price policy. Figure 3 and Figure 4 characterize the effects.

Figure 3. Price of new and old product

Figure 4. Production quantity and expected profit

Figure 3 shows how the optimal prices for both new and old goods in dynamic price policy depend on revealed economic conditions. When economic condition is high (low), the optimal prices are high (low). The optimal prices for both new and old goods in constant price policy lie between the two optimal prices in dynamic price policy. Moreover, all the prices converge when there is no demand uncertainty. Figure 4 shows that the production quantity is lower and expected profit is higher under dynamic price policy. Furthermore, there two policies converge as demand volatility diminishes.

5. Conclusions

In this paper we investigated the production and pricing decisions of a durable goods monopolistic manufacturer facing volatile demand. We constructed infinite-horizon dynamic model to study the interaction of strategic consumers and manufacturer. By using the infinite-horizon model, we are able to counter the end-of-period effect which is commonly seen in two-period models and to show that this is a steady state equilibrium where the manufacturer produces and sells the same amount of new goods in each period when he faces identical demand distributions. Moreover, the expected discounted profit won’t be affected by the uncertainty demand.

We also studied the effect of demand uncertainty on a manufacturer who uses constant price policy in which new and old goods prices are predetermined upfront. Our results indicate that the demand uncertainty will impel the manufacturer to set the prices lower and produce more new goods in each period. Furthermore, the expected discounted profit diminishes as a result of uncertain demand.

Opportunities for future work are wide. An interesting extension is to model the future demand to be dependent on observed current demand. We have had preliminary numerical result which shows that there might not be steady state equilibrium. Another extension is to relax the constant price model to accommodate the case where only new good price is predetermined by the manufacturer but not old goods price.

6. References


and Transaction Costs.


