Selling Random Wind

Eilyan Bitar, Kameshwar Poolla, Pramod Khargonekar
UC Berkeley
{eitarian, poolla}@berkeley.edu

Ram Rajagopal, Pravin Varaiya, Felix Wu
Stanford Univ
ramr@stanford.edu {varaiya, ffwu}@eecs.berkeley.edu

ABSTRACT

Wind power is inherently random, but we are used to 100 percent reliable or ‘firm’ electricity, so reserves are used to convert random wind power into firm electricity. The cost of these reserves is frequently a hidden subsidy to wind power producers. We propose an alternative: package random wind power into electricity with different levels of reliability and sell them at different prices. This variable-reliability market is more efficient than the current firm-electricity market, and may require lower subsidy. However, we have to think of electricity differently. We also explore interesting differences between the variable-reliability and related real-time markets.

I. INTRODUCTION

Many countries have set ambitious targets for renewable energy. In his 2011 State of the Union speech, US President Obama set the goal of 80% electricity from clean energy by 2035. California’s goal is 33 percent by 2020. The cost to reach these goals will affect how well they are met.

The goals are promoted by carrots-and-sticks policies. The biggest stick is the “renewable portfolio standard” or RPS. RPS regulation generally requires electricity suppliers to produce a specified fraction of their electricity from renewables. There are several kinds of carrots. The first takes the form of a thirty percent federal government investment subsidy. The second is the “feed-in tariff” (FIT) which requires small producers of electricity from renewables to be paid at a fixed rate. In California, the FIT is between 8 and 19 cents per kWh, depending on time of day.

Renewable generation resources like wind and solar are inherently non-dispatchable, intermittent, and uncertain in their power output – characteristics that we encompass by the term variable generation [11]. In order to maintain balanced operation of the electric power grid, the independent system operator (ISO) is responsible for procuring a suitable portfolio of reserve generation capacity to absorb such variability. The third form of subsidy shelters renewable energy providers from this incremental cost of reserve generation needed to compensate the attendant variability in their power output. Such subsidies vary by state. In California it is implicit. Specifically, the Participating Intermittent Renewable Program (PIRP) legislation compels the California Independent System Operator (CAISO) to accept all produced wind power subject to certain contractual constraints. Essentially, the renewable energy provider is exempted from the ex-post penalty for the mismatch between forecasted power production (say 24 hours ahead) and what they actually deliver. This amounts to a system take-all-wind scenario in which wind power is treated as a negative load and the subsequent increase in the variability of net-load is absorbed by a portfolio of reserve generation capacity, whose cost is socialized among the load serving entities (LSE) and is ultimately passed on to the consumers – a hidden subsidy to renewable energy suppliers.

The inherent variability of wind and solar power increases the need for operational reserves, including load following and regulation. However, unlike the explicit subsidies, this added cost of variability compensation is not available in public accounts. It is sometimes obtained as a by-product of simulations that estimate the increased need for operational reserves [7], [8], [11], [3], [4], [9]. For example, recent studies in California [4] illustrate the impact on necessary reserves of 20% renewable energy penetration by 2012 and 33% by 2020. These studies [10] project that the spring time maximum up regulation capacity needed will increase from 277 MW in 2006 to 512 MW in 2012 and to 1,135 MW in 2020. Similar increases are projected in down regulation capacity needed. Maximum load following capacity requirements increase from 2,292 MW in 2006 to 3,207 MW in 2012 and to 4,423 MW in 2020. Such increases in operating reserves will have negative cost impacts and will decrease the net greenhouse gas benefit of renewable energy, as regulating reserves are normally supplied by fast-acting, fossil fuel based thermal generators such as natural gas turbines. Clearly, the current strategy of integrating wind and solar energy cannot scale.

Compensating renewable generation variability with operating reserve capacity is consistent with conventional system operations in which dispatchable generation is tailored to counteract variability in load. However, there exists substantial flexibility in load that is currently underutilized. Moving forward, we argue the need for a transition to a modus operandi in which flexible consumers can adapt to renewable generation variability through a market for price-differentiated quality of supply. Such an approach would significantly diminish the need for thermal reserve generation capacity as a balancing resource, as the risk of insufficient power is borne by the consumers who purchase lower-cost, less-reliable electricity.

Research supported in part by EPRI and CERTS.
The paper is organized as follows. In §II we present a simple model to bound the integration cost under current market and system operations. In §III we propose an alternative market in which a wind power producer offers to sell power of different reliability levels, with zero integration cost. In §IV we compare this alternative market with a real-time spot market. An alternative formulation of the variable-reliability market is posed in §V.

II. COST OF INTEGRATION

![Diagram](image)

Fig. 1. Variable wind power is ‘firmed’ by adding reserves.

As mentioned in the Introduction, the operational reserve capacity needed to cope with wind power variability will increase substantially with increased penetration of wind energy at or beyond 20%. It will rapidly become unsustainable to continue the socialization of the added reserve costs. Hence, it is likely that the wind power producer (WPP) will have to bear the cost of the added reserve margins [6]. This transfer of financial burden to the WPP has already begun to emerge in the Pacific Northwest. The Bonneville Power Administration (BPA) in cooperation with Iberdrola Renewables, has deployed a pilot program in which the WPP is responsible for self-supplying its balancing services from owned or contracted dispatchable generation capacity [1].

In an effort to gain analytical insight into the individual effects of intermittency and uncertainty of wind power on integration costs, we consider a stylized single-bus model of a two-settlement market system in which reserve capacity procurement is co-optimized with the sale (scheduling) of wind power. Our objective is not to provide exact estimates of the cost of variability, but rather, using such a model, we aim to obtain upper and lower bounds on the subsidy for variability costs. A more nuanced analysis would require systemic considerations accounting for the potential of variability reduction through aggregation of spatially diverse resources as well as synergies with load variability.

The top left panel in Fig. 1 is a 5-min plot of wind power production, \( X(i) \), in BPA during Feb. 2009. The histogram of this plot can also be expressed as the ‘generation availability curve’ on the top right,

\[
G(x) = \max\{p : \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(X(i) \geq x) \geq p\}.
\]

\( G(x) \) is the fraction of the \( n \) 5-min intervals during which power production exceeds \( x \). A thermal plant with 5 percent failure rate produces firm power \( X(i) \equiv M \) with the generation availability curve on the bottom right.

With such a characterization of variable wind power \( X(i) \), what is the necessary procurement of reserve capacity \( S(i) \) such that the aggregate output is firm — i.e., \( X(i) + S(i) \geq \hat{X}(i) \) for all \( i \). The answer certainly depends on how well we can predict the wind power and the cost of reserves. We consider the two extremes of ‘full’ and ‘no’ information depicted in Fig. 2.

![Diagram](image)

Fig. 2. The vertically hatched area is the minimum reserve capacity needed: full information (top), no information (bottom).

Full information

In this case (Fig. 2, top right) \( X(i) \) is known in advance, so the reserves need only compensate for the intermittency of wind, but not its unpredictability. The needed reserve capacity is given by the vertically hatched (red) region on the top right of Fig. 2. Suppose the wind power producer sells in advance \( M \) MW of firm power at a price \( c \) per MW (i.e., \( X(i) \equiv M \)), and suppose the per MW price of reserve capacity and reserve energy is \( c_1 \) and \( c_2 \), respectively.

In the full information case, if the wind power is \( x \), the producer will purchase the same amount of reserve capacity and energy, namely, \( [M - x]_+ \) (notation: \( z_+ = \max\{z, 0\} \)). (Note: the ‘excess’ wind power, \( [x - M]_+ \), given by the horizontally hatched (blue) region is spilled or wasted.) So his maximum expected profit is

\[
\pi_1 = \max_{M} cM - (c_1 + c_2) \int_{0}^{M} [M - x]_+ (-G_x)dx,
\]

in which \( M \) is the wind power producer’s capacity. Setting to zero the derivative with respect to \( M \) of this expression gives

\[
F(M) = \frac{c}{c_1 + c_2},
\]

1932
in which \( F(x) = 1 - G(x) \) is the wind power cdf.

For example, suppose wind power is uniformly distributed over \([0, M]\), so \( F(x) = (x/M \wedge 1) \wedge \min \) (\wedge = \min). The optimum value of \( M \) from (2) and the maximum profit from (1) for this example are

\[
M = \frac{c}{c_1 + c_2} M, \quad \pi_1 = \frac{1}{2} \frac{c^2}{c_1 + c_2} M.
\]

(3)

If the producer is not required to purchase any reserves (as in current CAISO operations), he will sell all the wind power for an expected profit of \( 1/2 \times cM \). Hence the cost of integration in the full information case is

\[
CI_1 = \frac{1}{2} cM - \frac{1}{2} \frac{c^2}{c_1 + c_2} M = \frac{cM}{2} \left[ 1 - \frac{c}{c_1 + c_2} \right],
\]

which as a fraction of the revenue from CAISO can be regarded as a subsidy of

\[
\sigma_1 = \frac{CI_1}{1/2 cM} = \left[ 1 - \frac{c}{c_1 + c_2} \right].
\]

(4)

No information

The producer has no prior information about future wind power production, other than its distribution. So if he sells \( M \) MW of firm power in advance, he must purchase \( M \) of reserve capacity (there is a chance that zero wind power is available), and \([M - x]_+ \) of reserve energy when \( x \) is the actual wind power realized. Thus the maximum expected profit is

\[
\pi_2 = \max M cM - c_1 M - c_2 \int_0^M [M - x]_+ (-G_x) dx,
\]

(6)

which gives the optimum \( M \) as

\[
F(M) = \frac{c}{c_1}.
\]

(7)

For the example, in place of (3) one gets

\[
M = \frac{c - c_1}{c_2} M, \quad \pi_2 = \frac{1}{2} \frac{(c - c_1)^2}{c_2} M.
\]

(8)

Once again, if the producer is not required to purchase any reserves he will sell all his power (as in the full information case) for an expected profit of \( 1/2 \times cM \). Hence the cost of integration in the no information case is

\[
CI_2 = \frac{1}{2} cM - \frac{1}{2} \frac{(c - c_1)^2}{c_2} M = \frac{cM}{2} \left[ 1 - \frac{(c - c_1)^2}{cc_2} \right],
\]

(9)

which as a fraction of the revenue from CAISO is the subsidy

\[
\sigma_2 = \frac{CI_2}{1/2 cM} = \left[ 1 - \frac{(c - c_1)^2}{cc_2} \right].
\]

(10)

We can get a rough idea of these integration costs by taking plausible values (from CAISO data) of \( c = 55 \), \( c_1 = 10 \), \( c_2 = 60 \) per MW. These values give

\[
\sigma_1 = [1 - 55/70] = 21\%, \quad \sigma_2 = [1 - (45^2/55 \times 60)] = 39\%. \quad (11)
\]

That is, the hidden integration cost amounts to a subsidy as a fraction of total wind power revenue of 21 percent in the case of full information and 39 percent in the case of no information. When wind power is partially predictable, the subsidy will be between 21 and 39 percent. This subsidy seems large. The next section proposes a market where this cost is eliminated.

III. SELLING RANDOM WIND

If we are to realize the deep levels of renewable energy penetration called for by various renewable portfolio standards with minimal subsidy, it will become essential to transition to a modus operandi in which the flexibility in demand-side consumption is utilized to counteract the variability in renewable supply – potentially attenuating the conventional thermal reserve capacity requirement.

In this section, we propose a market-based mechanism that allows the wind power producer to sell its energy at various price-differentiated levels of reliability to consumers who are willing to bear the risk of insufficient power production in exchange for a lower price for energy. Such a variable-reliability market requires no reserves.

Later, we compare the variable reliability power market with well-known demand response mechanisms like real-time spot pricing.

Consider the two-period setup of Fig. 3. In period 1,

\[
\{\{\rho_k, p_k\}\} \quad \{k(t), d(t)\} \quad S(\omega) \quad R_\omega(t)
\]

Fig. 3. In period 1, the supplier offers variable-reliability contracts \((\rho_k, p_k)\) and customer \( t \) buys \( d(t) \) units of contract \( k(t) \). In period 2, \( S(\omega) \) of wind power is realized, and allocated among the customers. Knowing only the generation availability curve \( G(x) \) of Fig. 1, the wind power supplier offers for sale contracts \( \{\rho_k, p_k\}, k = 1, \cdots, n \). Contract \( k \) stipulates that if a customer purchases \( d \) units of this contract, he must pay \( p_k d \) in period 1, and in period 2 he will receive \( d \) MW of power with probability \( \rho_k \) and 0 MW of power with probability \((1 - \rho_k)\).

Suppose, facing this commodity choice, customer \( t \) buys \( d(t) \) units of contract \( \langle \rho_k(t), p_k(t) \rangle \). For mathematical convenience, the total number of customers (say \( N \)) is normalized to 1. Customers are indexed by the continuous variable \( t \in [0, 1] \), so \( N dt \) of them have index in \([t, t + dt]\).

In period 2, a random event \( \omega \) occurs and the supplier realizes \( S(\omega) \) MW of wind power. (The complementary cdf of \( S \) is \( G \).) The supplier must now select which customers to ration. This selection is represented by the rationing function \( R_\omega(t) \in \{0, 1\} \), with \( R_\omega(t) = 1 \) or 0 accordingly as customer \( t \) receives \( d(t) \) or 0 MW of power.

The allocation \( \{d(t), \rho(t) = \rho_k(t), R_\omega(t)\} \) is feasible if

\[
P_\{\omega | R_\omega(t) = 1\} = \rho(t), \quad \text{all } t \quad (12)
\]

\[
\int_0^t d(t) R_\omega(t) dt \leq S(\omega), \quad \text{wp } 1 \quad (13)
\]

\[
R_\omega(t) \in \{0, 1\}, \quad \text{all } t, \omega. \quad (14)
\]
The contracts stipulate (12) and (14); (13) stipulates that the total energy delivered cannot exceed the available supply.

The supplier’s non-random revenue received in period 1 is

\[ \pi = \int_0^1 d(t)p(t)dt. \]  

(15)

**Maximum welfare allocation**

We suppose that customer \( t \) who consumes \( d(t) \) units of \( \rho(t) \)-reliability power gets a utility whose monetary value is

\[ \rho(t)U(d(t)) - (1 - \rho(t))L(d(t)). \]  

(16)

The first (second) term above is the expected benefit (loss) from consumption (no consumption). We assume that \( U(0) = L(0) = 0 \), and \( U \) (\( L \)) is a strictly concave (convex) increasing function. We define the social welfare \( W \) of a consumption profile \( (d, \rho) \) by adding up everyone’s utility,

\[ W(d, \rho) = \int_0^1 [\rho(t)U(d(t)) - (1 - \rho(t))L(d(t))]dt. \]  

(17)

**Fig. 4.** Consumption \( (d(t), \rho(t)), (d(t'), \rho(t')) \) of customers \( t, t' \) (left); optimum \( (\hat{d}_i^*, \hat{\rho}_i^*) \) for \( k = n = 4 \) (right).

We first ask which feasible allocation maximizes social welfare. The answer was given in [14] for the case that the random wind power \( S \) takes \( n \) values: \( 0 < s_1 < \cdots < s_n \), with \( P\{S(\omega) = s_i\} = \pi_i > 0 \), \( \sum \pi_i = 1 \).

Observe that a feasible consumption profile is represented by arbitrary disjoint \( (d(t), \rho(t)) \) strips, one for each customer \( t \), located in the feasible region \( \{(p, x) : p \leq G(x)\} \). The left pane of Fig. 4 exhibits strips for two customers, \( t, t' \). However, as will be seen, there are only \( n \) different optimum consumption strips, illustrated in the right pane.

**Theorem**

The optimum allocation \( (d^*(t), \rho^*(t), R^*_n(t)) \) has the following form. There are numbers \( 0 = t_0^* < \cdots < t_k^* = 1 \), \( k \leq n \), such that for \( t \in [t_{i-1}^*, t_i^*) \):

\[ d^*(t) = d_i^* = \frac{s_i - s_{i-1}}{t_i^* - t_{i-1}^*}, \]  

(18)

\[ \rho^*(t) = \rho_i^* \pi_i + \cdots + \pi_{n-1}, \]  

(19)

\[ R^*_n(t) = 1, \text{ if } S(\omega) \in \{s_i, \cdots, s_n\}. \]  

(20)

An allocation of this form is optimal iff there are prices \( p_1^* > \cdots > p_k^* \geq p_{k+1}^* = \cdots = p_n^* = 0 \) such that for contracts \( \{(\rho_i^*, p_i^*), i = 1, \cdots, k\} \), consumer surplus is maximized by purchasing \( d_i^* \) units of \( (\rho_i^*, p_i^*) \) i.e.,

\[ d_i^* = \arg \max_d [\rho_i^*U(d) - (1 - \rho_i^*)L(d) - p_i^*d]; \]

moreover, consumers are indifferent among the different contracts since they all yield the same surplus:

\[ \rho_i^*U(d_i^*) - (1 - \rho_i^*)L(d_i^*) - p_i^*d_i^* = h \text{ (constant), all } i. \]

Hence the maximum social welfare is

\[ W^* = \sum_i (t_i^* - t_{i-1}^*)[\rho_i^*U(d_i^*) - (1 - \rho_i^*)L(d_i^*)] = h + \sum_i p_i^*[s_i - s_{i-1}]. \]  

(21)

Lastly, if the producer uses his supply \( G(x) \) to offer variable-reliability commodities for sale in a market with commodities \( \{(\rho_i^*, p_i^*), i = 1, \cdots, k\} \), his profit is maximized if he produces \( D_i^* = d_i^*(t_i^* - t_{i-1}^*) \) units of the \( i \)th commodity, and his maximum profit will be \( \sum_i p_i^*[s_i - s_{i-1}] \). Thus, the maximum welfare \( W^* \) is the sum of consumer and producer surplus.

**Remarks**

If the wind producer can also supply firm power, this can be included in \( S(\omega) \). If \( s_0 \) is the firm power, then \( s_1 \) should be replaced by \( s_0 + s_1 \).

\( G(x) \) need not have the ‘staircase’ shape of Fig. 4. If \( G \) is the true generation availability curve, one can always approximate it by any staircase shape \( G \), with \( G(x) \leq G(x) \).

The optimum rationing function is easy to implement: If the realized supply \( S(\omega) = s_i \), the producer should deliver electricity to everyone who bought contracts \( 1, \cdots, i \), and nothing to those who purchased contracts \( i + 1, \cdots, k \). So the optimum contracts are also priority contracts [12], [5], as contracts of type \( i \) are fulfilled ahead of contracts of type \( j > i \).

In the ISO-guaranteed firm electricity market of \( \S \), the risk of insufficient wind power is absorbed through reserves, whose cost is socialized. The variable-reliability market requires no reserves, and the risk of insufficient wind power is borne by the customers who purchase lower-priced, less-reliable electricity. Although a customer can purchase firm power with reliability \( \rho_1^* = 1 \), in fact identical customers are equally well off purchasing any reliability level.

In practice, customers are not identical in at least two respects. First, they may have different utility functions. This does not present theoretical difficulty since the theorem above extends to the case of heterogeneous customers [13]. Second, some consumers may either have greater flexibility in the pattern of their electricity consumption or they may be able to take countermeasures (e.g. access to storage or local generation) to more comfortably withstand a period with no electricity. Those customers will benefit more from cheaper, less reliable electricity, which, in turn, will also reduce the aggregate demand and hence the price for more reliable electricity. This differentiation is not possible when
only firm electricity is sold. It is also not possible in the ‘real-time’ spot market considered in the next section.

In addition, there is a practical question. How can someone who purchases electricity with reliability $\rho$ be assured that the probabilistic requirement (12) is actually met? If there is a large number of consumers, the producer could translate this requirement into a customer-checkable rule like $1/n \sum_{i} R_i(t) \geq \rho$ with $1$, in which $R_i(t) = 1$ if customer $t$ receives electricity in period $i$ for at least fraction $\rho$ of $n$ periods in all. (Notation: $R_i(t)$ is the rationing function for customer $t$ in time period $i$.) An even more customer-friendly rationing rule may take the form: the customer will get no electricity for at most (say) 30 min, every Wednesday. Such rules rely on statistical averaging. There is always a small probability that the producer may be unable to comply with such a rule, in which case he would need to compensate the customer.

IV. REAL-TIME MARKET

The real-time or ‘spot’ market also requires no reserves. The setup is simpler than in Fig. 3. There is only one period. When wind power $S(\omega)$ is realized, the producer inelastically offers it for sale. Competition among consumers determines the spot price at which the market clears: sum of the demands of all consumers equals the supply $S(\omega)$ [2]. No reserves are needed, but the consumption profile is quite different from that of the variable-reliability market of §III.

As before, suppose $S(\omega)$ takes values $0 < s_1 < \cdots < s_n$. Suppose each consumer $t \in [0, 1]$ derives the same benefit with monetary value $U(d)$ when he consumes $d$ MW of electricity. If $S(\omega) = s_i$ of power is competitively offered for sale, the identical consumers will each purchase the same amount $d_i$ at price $p_i$ given by the equilibrium conditions:

$$d_i = \arg \max_{d} U(d) - p_i d, \quad d_i = s_i,$$

which gives

$$d_i = s_i, \quad p_i = \frac{\partial U}{\partial d}(s_i), \quad i = 1, \cdots, n.$$

The total expected social welfare

$$\sum_{i} \pi_i U(d_i) = \sum_{i} \pi_i [U(d_i) - p_i d_i] + \sum_{i} \pi_i p_i s_i, \quad (22)$$

is the sum of consumer and producer surplus, which may be compared with (21).

Remark

The real time price may exhibit volatility, as its value depends on the realization of wind power. Suppose the price elasticity of demand is $\epsilon$. Suppose wind energy penetration amounts to $\gamma$ percent. As wind power varies between 0 and capacity, the price will fluctuate by $\epsilon \times \gamma$ percent.

V. ALTERNATIVE FORMULATION

The formulation of §III is unrealistic. It is a ‘one-shot’ problem inasmuch as electricity is delivered or not in a single period, period 2. In reality, electric power is produced and consumed continuously. We may assume that energy transactions are conducted (say) every 5-minutes. The consumption decisions over different 5-min intervals are of course interlinked, since a consumer may decide to defer some consumption decisions. The production of wind power is also interlinked, because the wind power process $\{X(i)\}$ (i indexes 5-min intervals) has temporal correlation, which assists its prediction.

Thus a more realistic formulation of the variable-reliability market may be something like this. The wind power producer faces a stochastic wind power process, $\{X(i)\}$. He also receives a time series of sensor measurements, say $\{Y(i)\}$. At each time $i$, he predicts wind power production $k$ intervals into the future, i.e., he calculates the conditional probability distribution, based on available measurements,

$$P\{X(i + k) | Y(j), j \leq i\},$$

for various time horizons $k$.

Based on these predictions, the wind power producer offers hourly firm power contracts, say 24, 12, 6 hours in advance. If sensor measurements indicate that the contract commitments cannot be met, he buys back these contracts in (say) 15 min intervals. The firm power contract together with the buy back constitutes a variable-reliability contract.

We formulate the problem as follows. The wind power producer offers hourly firm power contracts $s_1, s_2, \cdots, s_N$ at times $t_1, t_2, \cdots, t_N$ in advance at prices $p_1, \cdots, p_N$ dollars per MWh. In real time, during the delivery contracts are purchased back in intervals of $\Delta$ minutes at a cost $q$ dollars per MWH. For example, contracts are offered at $t_1 = 14$, $t_2 = 12$ and $t_3 = 6$ hours in advance and unmet contracts are purchased back in 15 minute intervals. This is similar to what CAISO does now with day-ahead and 15-min balancing reserves, except that now the ‘balancing reserve’ takes the form of customers opting to desist from consuming electricity. The total profit from the sale is given by

$$\pi = p_1 s_1 + p_2 s_2 + \cdots + p_N s_N - q \sum_{m=1}^{M} \sum_{k=1}^{N} s_k - X(m)\}_{+} \quad (23)$$

where $M$ is the number of $\Delta$ intervals in an hour. The power producer sells contracts sequentially maximizing his expected profit. But he is able to benefit from the additional information available as time progresses. The proof of the next result will be published elsewhere.

Theorem

There is a unique allocation of contracts that maximizes the expected profit $E[\pi]$. The sequential profit maximizing allocation of contracts at time $t_r$ is

$$s_r^* = \left[ \phi_r - \sum_{k=1}^{r-1} s_k^* \right]_{+} \quad (24)$$
where the optimal ‘threshold’ \( \phi_r \) is given by \( x \) that solves

\[
\begin{align*}
    p_r - p_{r+1}P(x \leq \phi_{r+1} | Y_r) + \cdots + \\
    - p_N P(x \leq \phi_N, x > \phi_{N-1}, \cdots, x > \phi_{r+1} | Y_r) + \\
    - q \sum_{m=1}^{M} P(x \leq X(m), x > \phi_R, \cdots, x > \phi_{r+1} | Y_r) = 0,
\end{align*}
\]

which uses all the information \( Y_r = \{ Y(j), j \leq r \} \) available up to time \( t_r \). Notice that \( \phi_r \) is random before time \( t_r \).

Several variations are possible. 1. Contracts can be purchased back in advance of the delivery hour or even with advance notice during the delivery hour. 2. Contracts can be offered for different start times spanning different hours or the buy-backs can comprise purchasing back the whole remaining unused contract. 3. Another variation is to offer contract menus with specific reliability guarantees at different times. These extensions can also be shown to satisfy particular threshold rules. 4. Lastly, one may offer variable-reliability contracts in the sequence of forward markets. In this way, the forward prices \( p_t \) would reflect the quantity risk associated with insufficient wind power, eliminating the need to buy back contracts at price \( q \) as in (23), if customers are appropriately rationed according to their reliability levels.

The expected revenue from selling these contracts is

\[
E[r] = p_1E[\phi_1] + p_2E[\phi_2 - \phi_1] + \cdots + \\
+ p_N E[\phi_N - \max_{r \leq N-1} \phi_r] - q E \sum_{m=1}^{M} \sum_{k=1}^{N} s_k - X(m)]
\]

In the above expression, each optimal threshold \( \phi_r \) depends on the information available at that point. This information changes with different realizations of the wind. It depends on how accurately we can forecast wind power and how the error of the forecast reduces with the time horizon. In the simple case where \( M = 1 \), and thus contracts for every 15 min intervals are sold independently, if the forecast error sequence is the sum of independent random variables, it can be shown that the optimal contract level is given by \( \phi_r = W_r + \Delta_r \), where \( W_r \) is the forecast (conditional expectation) of wind at time \( r \) and \( \Delta_r \) is a threshold that can be pre-calculated based on the statistics of the error forecasts.

An important advantage of this type of procedure is that additional resources available can be integrated to improve profits. For example, if the producer has some storage or local generation available he can use these in conjunction with reliable contracts to achieve even higher levels of utilization of the random power being offered. These decisions can be made dynamically and additional constraints on emissions and other target objectives can be added.

The sequential offering procedure highlights an important feature of the problem: a contract is in fact a bundle of quantity, provisioning lag (i.e. reliability or predictability) and cost of buy back (i.e. price of reliability). The greater the provisioning lag, the more expensive power is under a constant buy back price policy. If buy backs are also offered at different times, it will be such that the sooner a buy back has to be announced, the less will be offered for that particular buy back. In fact, the customers that face the smallest cost for power are those most capable of instantaneously consuming (selling back) power since at that point the exact realization of wind power generation is known and the excess (shortfall) can be offered at very low cost.

Flexible consumers are rewarded for their choices. Moreover, the cost of power can be reduced over time since less firm generation is required for deployment as consumer flexibility increases over time. For example, consumers that decide to purchase responsive appliances are immediately rewarded.

Sequential contracts also provide an additional benefit: the generated wind is consumed as far as information allows it to be. In fact, more wind can be consumed if forecasts for wind are improved. Improving forecasts is achievable in the near future with new sensors and new data analysis methodologies and is much less costly than building additional fixed generation to meet fixed demand.

Implementing such contracts in practice requires communication and operations infrastructure that can execute and validate these transactions. Consumers should be able to defer or anticipate planned consumption and the producer needs to inform the consumer of the availability of power at the desired quantities. Also, the quantity plans for the different bundles of power at the consumer site need to be computed from existing consumption data for that particular consumer. For example, multiple sensors can inform which electricity uses can be bundled together and deferred or anticipated. Devices at the consumer site can be preprogrammed to respond to requests to consume or defer consumption. Such infrastructure only exists partially today but can be foreseen to be easily deployed. For example, many of the sensors and programming capabilities are already being made available for appliances since they provide diagnostic information for the manufacturers and some features to consumers. A direction for future research we have started to explore is how to estimate a consumer’s requirement for different levels of reliability from detailed consumption data available for him.

VI. CONCLUSION

Wind producers in California receive an implicit subsidy in the form of reserves that compensate for wind power intermittency and unpredictability. These reserves are purchased by CAISO and ultimately paid for by electricity consumers. A stylized model calculation suggests that the cost of these reserves is between 21 percent of total wind revenue if wind is perfectly predictable and 39 percent if it cannot be predicted at all. This cost seems large. The optimum reserves will also spill (waste) some wind power.

In the alternative market proposed here wind power producers sell electricity with different levels of reliability: \( \rho \)-reliability service will deliver electricity with probability \( \rho \) and not deliver with probability \( (1 - \rho) \). This random wind power market requires no reserves. The real-time or spot market also needs no reserves. Since variable-reliability service is sold in a forward market (period 1 for delivery in period 2), its price is determined in advance, whereas the
real-time price is determined in the sport market at the time of sale.

An important difference between the three markets (firm-only, variable-reliability and real-time) is the way in which the risk of uncertainty and variability of wind power is shared. In the firm-only market, the risk is absorbed by reserves, whose cost is socialized. In the variable-reliability market, the risk takes the form of quantity risk borne by customers who purchase less reliable electricity, but this risk is compensated by lower prices. Since prices are determined in a forward market, consumers face no (or reduced) price risk, and producers face no (or reduced) profit risk. In the real-time market, the risk is borne by consumers who face both price and quantity risk, and the wind producer who faces profit risk. Spot prices will be more volatile than variable-reliability prices.

One potential advantage of variable-reliability service is that it opens up profitable opportunities for technologies and services that assist customers in reducing quantity risk, by making use of deferrable demand, storage, and coordination with local renewable sources. Of course, some of these technologies may also be used to reduce the risk in real-time markets.

REFERENCES