EXPLORING CONTRACTS WITH OPTIONS IN LOYALTY REWARD PROGRAMS SUPPLY CHAIN

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Abstract
This paper explores analytically the issue of whether an option contract mechanism is a viable alternative to help hedge against demand uncertainties for rewards in a Loyalty Reward Programs (LRP) enterprise-led supply chain. We introduce an analysis framework based on the study of the problem of planning the supply of rewards (and points) given demand uncertainties and considering option contracts featuring two parameters namely the option price and the option exercise. A two-stage stochastic linear programming with simple recourse model is developed to formulate this problem and solved using a solution procedure based on the sampling average approximation scheme. We benchmark our analysis with a wholesale price contract, a commonly used mechanism in buyer-supplier supply chains. Our preliminary numerical experiments show that option contracts can be considered as an attractive means to mitigate higher level of redemption demand variability as they tend to yield a higher LRP enterprise profitability and lower increases in both the liability level and budget usages.

1. Introduction

Supply chain contracts are widely recognized as a necessary means for governing buyer-supplier relationships in the supply chain, coordinating the decisions of the supply chain partners, sharing the risks arising from various sources of uncertainty, or for facilitating long-term partnerships ([1]).

A variety of within and across industries supply contracts have been investigated in the academic literature and contributions have been reported on a number of issues related to their critical role, selection, design, evaluation, and implementation (for survey papers see for example, [1], [2]). Wholesale-price contracts, two-part tariff linear price contracts, two-part tariff non-linear price contracts are examples of supply chain contracts that are defined on the basis of pricing policies ([3]). Quantity-flexibility contracts, back-up agreement contracts, revenue-sharing contracts, option contracts, buyback or return contracts, quantity discount, shared-saving contracts and their many variations are other examples of supply chain contracts defined on the basis of incentives or purposes ([2], [4]-[7]). Most existing studies on supply chain contracts have focused on supply chains that deal with tangible products, i.e. good supply chain products. The analysis in most studies primarily assumes a supply chain involving a single retailer and a single supplier.

Loyalty reward programs (LRPs) are marketing programs aimed at rewarding customers for continuing to use a product or service. They have been recognized as an innovative business practice to build and nurture long-term relationship with customers, to better allocate marketing budgets, and to derive as much as revenue as possible from customers over their lifetime (see [8] for a brief introduction to LRPs). A traditional supply chain (TSC) and a loyalty reward programs supply chain (RSC) share many common characteristics. Generally speaking, a TSC consists of multiple independent business entities such as vendors, manufacturers, distributors, retailers, etc. Similarly, an RSC typically consists also of business entities including an LRP host (i.e., the firm that owns the program), LRP accumulation partners (i.e., firms that join the program to provide members with accumulation options), LRP redemption partners (i.e., firms that join the program to provide members with redemption options). The revenue flow in both TSCs and RSCs is created by end consumers and shared...
among all business entities in the system. However, important differences exist between TSCs and RSCs. In the TSC, products provided by the business entities are in the same good/service categories. In an RSC, rewards (tangible) in different good/service categories are provided by redemption partners and points are a special type of product (intangible) provided by the LRP host and distributed by accumulation partners. End consumers’ demand relates to products only in the TSC, while in the RSC, end consumers’ demand relates to both points and rewards. With regard to the system structure, an TSC is mainly sequential-based. Following the production flow, the business entities involved are operated sequentially. Downstream entities play a key role in the interaction with end consumers. In contrast, the structure of RSC is parallel-based. The LRP host firm is at the center of the system. The redemption and accumulation partners are LRP host’s multiple channel partners and they operate independently. All these entities in the RSC interact with end consumers (i.e., members who join the LRP) directly. As such, the structure of RSCs is very similar to decentralized multi-channel supply chains. Another key difference between an TSC and an RSC is that costs and revenues are generated in a different time order. In the TSC, costs associated with production occur first, and then revenues are generated through selling products. In the RSC, revenues are generated through selling points first (this applies only to a LRP operating as a profit center), and then costs associated with rewards occur later when LRP members redeem their points for rewards.

This paper aims at exploring the issue of whether an option contract mechanism is a viable alternative to coping with demand uncertainties for rewards in a LRP supply chain. We benchmark our analysis with a wholesale price contract, a commonly used mechanism in manufacturer-retailer supply chains. It has been shown in the literature that option contracts seem to generalize several widely practiced ‘specialised-case’ contracting schemes such as quantity flexibility contracts, backup agreements contracts, pay-to-delay capacity reservation, and buyback/return contracts ([5], [9]). Unlike most of the existing work on supply chain contracts in TSCs, we consider a supply chain composed of multiple commercial partners. We introduce a framework based on the study of the problem of planning the supply of rewards (and points), one of the critical operational issues faced by LRP enterprises in order to achieve business goals such as meeting customer demand, improving customer satisfaction, lowering operational costs or generating higher profits, while taking into account both internal dynamics and external uncertainties (e.g., changes in the management or resource constraints, demand uncertainties).

The paper is organized as follows. We present a brief literature review on supply chain contracts with options in Section 2. A description of the rewards-supply planning problem under option contracts, the corresponding mathematical model, and the solution methodology are included in Section 3. We report our computational experiments and preliminary findings in Section 4. Finally, conclusions are presented in Section 5.

2. Literature Review

Option contracts used as a risk management mechanism in the financial area, have attracted enormous attention in the academic literature on supply chains in the recent years. In this context, an option contract refers to a contract in which a supplier (e.g. manufacturer) allows a buyer (e.g., retailer) to purchase up to a given quantity of a product during a specified time interval at specified pricing ([10]). In this type of contract, capacity is commonly regarded as an option to be exercised in the future to produce needed goods. Hence, the capacity reservation fee (i.e. option price) and the execution fee (i.e. exercise price) are the two key parameters in these contracts. A typical option setting follows a two-phase structure. In the first phase, an option contract specifying a reservation fee and an execution fee is offered by a supplier. Based on this information, the buyer chooses an optional ordering quantity which is matched by the supplier’s capacity. While the reservation fee is immediately payable, the exercise fee is due when the option is exercised (after demand uncertainty is resolved). In the second phase, the buyer decides on the exercise amount and pays the exercise fee after observing the demand.

In Operations Management literature, the authors in [11] are reported to be the first to propose a two-period model where option contracts are considered given a stochastic demand. In their model, assuming that option prices are provided, the buyer has the choice of carrying inventory from the first period to the second and can utilize first-period demand to update the forecast for the second period’s demand and then exercise the options based on the updated information at the beginning of the second period. In [5] the authors also analyzed a two-period model and discussed the sufficient conditions that are required to achieve channel coordination when a return incentive is considered in the contract and supplier is limited to
using a linear pricing policy in the contract design. Other works with option contracts considering a supply chain with a single seller and a single buyer include, among others, [9], [12]-[16]. The case in which the buyer’s reservation fee (option price) is deductible from the purchasing price (exercise price) has been investigated in [12]. Assuming that the demand distribution is influenced by retailer’s pricing decisions, it has been shown in [13] that the introduction of option contracts causes the wholesale price to increase and the volatility of the retail price to decrease. The authors derived the conditions under which manufacturer and retailer will be better off respectively with option contracts. In [14] the authors considered the case where both the supplier and the buyer have access to option contracts and to spot market to sell or purchase no-storage goods (e.g. electricity), assuming that the buyer’s future demand, the seller’s future marginal costs, and the future spot price are uncertain. A model has been developed in [9] to address the optimal decision order of the buyer, specifying both the committed order quantity and the number of options contracts, but also the optimal pricing decision of the supplier, specifying both the option and the exercise prices. It is one of the few models where the pricing of options are considered as decision variables. In [15] the authors included in their work the supply chain members’ risk preferences and negotiating powers. They analyzed how these factors impact the profit allocation between the retailer and the manufacturer. Assuming a retailer-led supply chain in which the retailer holds the decision right of pricing and takes the initiative to coordinate the manufacturer’s production quantity. The authors in [16] showed that to achieve a successful coordination through option contracts the firm commitment must be no greater than the optimal production quantity in a centralized system. In addition they found that one needs to maintain a negative correlation between exercise price and option price.

In the case of a supply chain with multiple suppliers and/or multiple buyers, existing works have focused on the strategic interactions between the supplier(s) and the buyer(s) or in the competitive behavior of independent players with option contracts. In [17] the authors examined the situation where several suppliers compete to provide capacity to a single buyer. They investigated the optimal portfolio of contracting and spot market transactions for the buyer and the suppliers, and determined the market equilibrium pricing strategies. In [10] the authors provided a two-stage mixed integer stochastic programming model and a solution procedure to assess the impacts of uncertainty in the availability of parts from suppliers. Their model aims to select which parts option contracts should apply to and with which suppliers. The authors in [18] analyzed the suppliers’ pricing strategy when they are competing through option price and flexibility. They found that in the market equilibrium, a variety of suppliers coexist, and these suppliers offer different prices. However, their equilibrium solution cannot achieve supply chain coordination. In [19] the authors considered a competitive supply chain in a multi-period setting as well. In their model, they treated “spot market” as a dummy supplier with stochastic costs and unlimited capacity. Unlike in [18], the authors in [19] showed that the equilibrium policies still coordinate a supply chain.

Our work focuses on a supply chain with multiple partners (redemption partners and accumulation partners). We consider a supply chain that offers non-storable goods in an industry that has not been considered yet in the existing literature. Rather than examining strategic interactions between the suppliers and the buyers or their competitive behavior, we focus on LRP host firm’s ordering decisions to better allocate the limited resources available (e.g. budget for rewards) while taking into account a number of internal dynamics (e.g., control of liability level, demand uncertainties for rewards) and, in a simultaneous manner, supply chain partners offerings (reservation fee, exercise fee, and limits in the supply of rewards). As the stochastic nature of the demand for rewards is one of the key concerns in LRP management, we explore how the option contracts can help to hedge against this uncertainty.

3. Modeling Framework

Our analysis framework is in the context of planning the supply of rewards (and points), one of the critical operational issues faced by LRP enterprises in order to achieve business goals such as meeting customer demand, improving customer satisfaction, lowering operational costs or generating higher profits, while taking into account both internal dynamics and external uncertainties (e.g., changes in management or resource constraints, demand uncertainties). This problem consists of determining, given the ordering quantities of points from LRP accumulation partners, \( A_i \)’s, (i.e., \( q_{i}^p \)), the LRP host’s optimal ordering quantity decisions of rewards from LRP partners \( R_j \)’s (i.e., \( q_{j}^p \)). The problem consists of maximizing the LRP host profitability (as measured by the value creation), subject to LRP
partners Rj’s capacities on offering rewards, the LRP host’s overall budget for purchasing rewards, and the LRP host’s control on points-liability (i.e., the value of future redemption obligations). To address this problem a model based on a single-period constrained newsvendor is developed. We assume that the relationships between LRP accumulation partners and the LRP host firm are governed by wholesale-price contracts. Under this contract setting, the LRP host firm guarantees each LRP accumulation partner Ai a wholesale unit price of points, wiA. The LRP accumulation partner Ai decides on the quantity of points to order (qAi) during the planning horizon at the given wholesale unit price, wiA. The LRP members’ accumulation demand (in points) towards the LRP redemption partner Rj (i.e., Dij) is not known with certainty, but follows a known probability distribution. At the end of the planning horizon, if the LRP members’ redemption demand towards partner Rj is higher than the LRP host ordering quantity (qAj + qij), the excess demand is assumed to be lost and the under-stocking cost is vji per unit of points. Alternatively, the excess ordering quantity is sold at sji, the over-stocking unit sale price. Let pji defines the per point unit value of rewards offered by partner Rj. Hence, the LRP Host’s profitability (i.e., value creation) function at the redemption side can be defined as follows (BP-R):

\[
\pi_{HR}(q_{ji}^A, q_{ij}^A, D_{ij}^A) = \sum_{i,j} \left( p_{ji} \times \min\{q_{ji}^A + q_{ij}^A, D_{ij}^A\} - w_{ij}^A \times q_{ji}^A - w_{ij}^A \times \left[D_{ij}^A - q_{ij}^A\right] \right)
\]

The first term in relation (2) indicates the value of rewards offered by each partner Rj. The second term indicates the LRP host’s purchasing cost of rewards. The third and fourth terms indicate the LRP host’s costs for purchasing and exercising options. The fifth term refers to the under-stocking cost of rewards. Finally, the sixth term refers to the salvage value of over-stocking rewards.

In our modeling problem, we make the following additional assumptions: (a) LRP redemption partners have capacity limitations on offering rewards; (b) LRP host firm has no capacity limitation on issuing points; (c) LRP members accumulation and redemption demands are not known with certainty, but have known probability distributions and both demands are price-independent; (d) one universal static redemption scheme is adopted by all LRP redemption partners; (e) one universal static accumulation scheme is adopted by all LRP accumulation partners; (f) LRP members demands for points will always be met; and (g) at the beginning of the period, an LRP partner offers the LRP host firm a single-level option contract with parameters (wji, eji). Assumptions (b) and (f) relate to the unique features of points. As points are a kind of information symbol for recording and accounting
LRP members’ purchase effort in the LRP system, the LRP host does not have any “production” related costs and “resource” related capacity limitations on offering points. As such, to a host, there is no capacity limitation on issuing points. Furthermore, unlike tangible products, production and movement of points are not limited by time and physical space. Points are never “stock-out” in the sense that there is no time lag between the production of points and meeting customers’ accumulation demand on points. Therefore, members’ accumulation demands will always be met. In other words, the LRP host allows the LRP partner always to “back-order” points that are over its initial ordering quantity. Now let’s consider the additional notation shown in Figure 1. Combining (BP-A) and (BP-R), the problem of planning the supply of rewards can be formulated as follows (hereafter Problem BP):

\[
\Pi_h^l(q^R_j, m^R_j, q^R_j; D^R_j, D^A_j) = \\
\frac{\sum_{j=1}^J (w^R_j \times q^R_j) - \sum_{j=1}^J \left[ w^R_j \times q^R_j + \left( w^R_j m^R_j + e^R_j q^R_j \right) \right] + E \left[ \sum_{j=1}^J D^A_j - q^R_j \right] + E \left[ \sum_{j=1}^J q^R_j \times \min \left\{ q^R_j + q^R_j, D^A_j \right\} \right] + E \left[ \sum_{j=1}^J q^R_j \times \left( D^R_j - \left( q^R_j + q^R_j \right) \right) \right]}{\sum_{j=1}^J (w^R_j q^R_j + w^R_j m^R_j + e^R_j q^R_j) \leq W^R_j} \leq L_{UB}, \text{ where} \\
l = l_0 + \sum_{j=1}^J \left( q^R_j + \left[ D^R_j - q^R_j \right] \right) - \sum_{j=1}^J (q^R_j + q^R_j) \leq L_{UB} \leq (1+\delta) \sum_{j=1}^J (q^R_j + \left[ D^R_j - q^R_j \right] \right) (4)
\]

Redemption partners’ capacity limitations on offering rewards:

\[
q^R_j \leq Q^R_j (5)
\]

LRP host’s budget constraint on purchasing rewards:

\[
\sum_{j=1}^J (w^R_j q^R_j + w^R_j m^R_j + e^R_j q^R_j) \leq W^R_j (6)
\]

Upper bound of option quantity that host is allowed to purchase:

\[
m^R_j \leq M^R_j, \text{ for } j = 1, \ldots, J (7)
\]

Upper bound of option quantity that host is allowed to exercise:

\[
q^R_j \leq m^R_j, \text{ for } j = 1, \ldots, J (8)
\]

Non-negativity constraints:

\[
q^R_j, m^R_j, q^R_j \geq 0, \text{ for } j = 1, \ldots, J (10)
\]

In constraints (4), \( \sum_{j=1}^J \left( q^R_j + \left[ D^R_j - q^R_j \right] \right) \) is the overall amount of points accumulated during the planning horizon. Therefore, the overall liability at the end of the planning horizon \( (l) \) is equal to the initial liability \( (l_0) \) at the beginning of the planning horizon plus the overall amount of points collected by members during the planning horizon, and then minus the overall amount of points redeemed by members for rewards during the same planning horizon. An upper bound, \( L_{UB} \), is introduced as a liability control parameter. If the LRP host wishes to reduce the liability by a minimum fraction of \( \delta_{\text{min}} \), then \( L_{UB} \) would be equal to \((1-\delta_{\text{min}})\). If the LRP host plans to keep the liability at the same level as before, then \( L_{UB} \) would be equal to 1. When \( L_{UB} = (1+\delta_{\text{max}}) \), then the LRP host allows the liability to be at a higher level than that in the previous planning horizon, but within a certain range. The actual liability is computed as follows:

\[
l' = l_0 + \sum_{j=1}^J \left( q^R_j + \left[ D^R_j - q^R_j \right] \right) - \sum_{j=1}^J (q^R_j + q^R_j) \leq L_{UB} \leq (1+\delta_{\text{max}}) \sum_{j=1}^J \left( q^R_j + \left[ D^R_j - q^R_j \right] \right) (11)
\]

Constraints (5) indicate that each redemption partner has a capacity limitation on quantity of rewards offered to the LRP host. Constraints (6) indicate that the LRP host has an overall budget limit for purchasing rewards that cannot be exceeded. Constraints (7) indicate that each redemption partner has a capacity limitation on quantity of options offered to the LRP host.
Constraints (8) and (9) indicate limitations on the quantity of options that the LRP host firm can exercise from each redemption partner. Constraints (10) refer to the non-negativity constraints.

The special structure of Problem BP and the demand uncertainties involved lend to a special type of stochastic programming (SP) approach known as two-stage stochastic linear programming with recourse in which decision variables are classified into two stages according to whether they are implemented before or after an outcome of a (vector valued) random variable is observed ([20]). One common solution approach to problems with this special structure is the sample average approximation scheme (SAA). The basic idea of this scheme consists of generating an approximate solution, which is the solution of a number of instances, say M, of the SAA problems, each with N sampled scenarios. The quality of a candidate solution is then tested by bounding the optimality gap between the true objective value and the expected objective value (see equation (7)) by standard statistical procedures. A random sample \( \omega_1, \omega_2, \ldots, \omega_N \) of \( N \) realizations (scenarios) of the random vector is generated outside of the optimization routine and the expectation of second-stage objective function is approximated by the sample average function of these realizations. This is done for each instance \( m, m = 1, \ldots, M \) of the SAA problems. The term \( \hat{x}_s \) in the relation (2) is also handled in a similar manner in Problem BP-SAA. Variables in relations (21) and constraints (5) and (7) are first-stage decision variables and constraints. Variables shown in equations (22)-(23) and constraints (14)-(20) are second-stage decision variables and constraints.

Problem BP-SAA is a deterministic linear model solved for each instance \( m, m = 1, \ldots, M \), of the sampling average approximation solution procedure. Details about the implementation of this procedure can be obtained from various sources such as, [21]-[23]. The common random numbers (CRN) approach was used to construct the confidence interval for the optimality gap.
4. Preliminary Computational experiments

Our numerical experiments were carried out on a Toshiba Tecra M10 2.2 GHZ notebook. The IBM optimization subroutine library (OSL, version 3.0) was used to solve the resulting linear programming problems. Table 1 summarizes the main parameters considered in generating the test problems.

The characteristics of the problems generated randomly are presented in Table 2, where Pb# refers to problem number, NRP to number of redemption partners, NAP to number of accumulation partners, NDV to number of decision variables, and NCC to number of constraints. The size of problems in terms of the number of constraints (NC) and number of decision variables (ND) varies according to the number of redemption partners (NR) and number of accumulation partners (NA). In the implementation of the SAA solution procedure we set N=60 (number of sample demand realizations), M=30 (number of sample replications), and N'=300 (number of sample demand realizations used to assess the true objective function value). As shown in Table 2, we assume that the demand for redemption and the demand for accumulation can be represented by a normal distribution.

We report in Table 3 the quality of stochastic solutions obtained when compared to those resulting from the mean-value deterministic optimization models (obtained after replacing the demand random parameters with their mean values). The value of stochastic solutions is defined by the difference in percentage of the expected objective function values between the stochastic models and the mean-values models ([20]). AvgGDiff and StdGDiff refer to the differences in percentage between the stochastic models and the mean-values models of the mean optimality gap and the standard deviation of the mean optimality gap, respectively. These results reveal that the stochastic programming models provide better solutions than the corresponding mean-value model solutions in terms of the objective function values. These solutions also lead to significantly smaller mean gaps and smaller variability of the gaps. In addition, with the increases of demand variations, it is more obvious that the quality of the stochastic programming solutions is much better than the quality of the mean-value model solutions, increasing from around 0.5% in the case of low demand variations (DV=0.1) to more than 9% in the case of high demand variations (DV=1.8).

We report in Figures 2 to 4 our findings about how models with option contracts compare when benchmarked with models based on wholesale price contracts. These figures show for all test problems generated in this study the differences in percentages between models from both contract mechanisms (i.e., option and wholesale price contracts) in regards to (a) the mean profitability, (b) the mean liability ratio, and (c) the mean budget usage. Our findings show that models with option contracts seem to provide an increased overall profitability, in particular with high demand variations. Moreover, although the liability and the redemption budget usage seem to increase with the changes in the demand variability (from low to high demand variations), the observed increases seem to be lower in models with option contracts than those based solely on wholesale price contracts. These results seem to indicate that the adoption of option contracts worth to be pursuing in LRP supply, chains, particularly when high variations of demand for redemption awards (in points) are involved.

5. Conclusions

In this paper, we have explored analytically the issue of whether an option mechanism is a viable alternative to help hedge against demand uncertainties for rewards (and points) in a Loyalty Reward Programs (LRP) supply chain. We have introduced an analysis framework which is based on the study of the LRP host firm’s aggregate rewards-supply planning problem with option contracts in the presence of multiple commercial partners, who offer different redemption and accumulation alternatives to LRP members. The model introduced in this framework is comprehensive as it considers multiple management concerns such as the LRP host profitability defined in terms of value creation, liability control, budget and capacity limitations, and demand uncertainties. A stochastic programming solution procedure based on sampling average approximation (SAA) scheme is presented in the paper.

Considering a simple option contract with two parameters namely the option price and the option exercise and a wholesale price contract as a benchmark, our preliminary numerical experiments showed that option contracts can be considered as an attractive means to mitigate higher level of redemption demand variability as they tend to yield a higher LRP enterprise profitability and lower increases in both the liability level and budget usages. We believe that the analysis framework introduced in this work offers opportunities for further analyses and
extensions, including other types of demand distributions, more advanced settings of option contracts, other types of contracts and the application of the methodology to actual ("real") data from an existing LRP enterprise.

References


Indices:

- $R_j$: Redemption partners in the LRP system, $j=1,2,\ldots,J$
- $A_i$: Accumulation partners in the LRP system, $i=1,2,\ldots,I$

Decision variables:

- $q_{R_j}^R$: LRP host firm’s initial ordering quantity of rewards (in points) from partner $R_j$.
- $m_j^R$: Number of units of options (in points) purchased by LRP host from partner $R_j$.
- $q_j^R$: Number of units of options (in points) exercised by LRP host from partner $R_j$.

Parameters:

- $D_i^A$: Members’ accumulation demand towards partner $A_i$.
- $q_i^A$: Partner $A_i$’s ordering quantity of points.
- $w_i^A$: Wholesale price per unit of points that LRP host charges partner $A_i$.
- $w_j^A$: Back-order price per unit of points that LRP host charges partner $A_j$ when accumulation demand is over partner $A_j$’s ordering quantity.
- $p_j^R$: Per point unit value of rewards offered by partner $R_j$.
- $D_j^R$: Members’ redemption demand towards partner $R_j$’s rewards.
- $w_j^R$: Wholesale unit price of rewards (in points) that LRP partner $R_j$ charges to the LRP host firm.
- $w_i^R$: Per unit price that LRP partner $R_i$ charges to the LRP host firm to purchase options
- $c_i^R$: Per unit price that LRP partner $R_i$ charges to the LRP host firm to exercise options
- $y_j^R$: Per unit shortage penalty cost of partner $R_j$’s rewards.
- $s_j^R$: Per unit salvage value of rewards offered by partner $R_j$.
- $l_0$: Points-liability at the beginning of the targeted time period.
- $l$: Points-liability at the end of the targeted time period.
- $L_{UB}$: Upper bound of liability control limits for the targeted time period.
- $Q_j^R$: Partner $R_j$’s capacity limitation on offering rewards.
- $W^R$: LRP host firm’s budget limitation on purchasing rewards.

Table 1: Summary characteristics of the set of randomly generated problems

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Table 2: Problem generation parameters

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<td>Range of back order price factor of points (BF):</td>
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<td>Range of wholesale price factor per unit of points ordered (FA):</td>
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<td>Range for mean for redemption (DR) / accumulation (DA) demand (normal distribution):</td>
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<td>Range for the standard deviation for redemption (DR) / accumulation (DA) demand (normal distribution):</td>
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Table 3: Quality of Stochastic versus mean-value solutions

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Figure 2. Ratio of profitability between models with versus without option contracts

Figure 3. Liability ratio between models with versus without option contracts

Figure 4. Budget usage ratio between models with versus without option contracts