A Multi-item Fuzzy Economic Production Quantity Problem with limited storage space

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Abstract

This paper extends a fruitful fuzzy EOQ (Economic Order Quantity) track of research with a multi-item model that takes storage limitation into consideration as well as a finite production rate (i.e., the products are produced to stock and not purchased). The order cycle time and the demand are allowed to be triangular fuzzy numbers. In the process industry, this is a situation that often occurs, since the production equipment typically manufactures a large amount of only one product at a time. The model is developed and solved with Lagrange multipliers. Two illustrative examples are also given.

1. Introduction

Many research contributions have been presented to the EOQ-theory (Economic Order Quantity) after the first model by [1]. His contribution was the basic model, where the order size needed to be determined given holding costs, order setup costs and annual demand. After this basic model, developments in different directions have been presented. For instance, instead of purchasing the products to stock, they are produced to stock. Another extension is to solve the optimal order quantity for several products using the same production equipment. This is typically the case in the process industry. In addition, the storage space is limited, sometimes to the extent of only having storage capacity for one week’s production (this was the case in a Nordic plywood production facility that we collaborated with).

Needless to say, the process industry environment is such that it is inevitably necessary to produce to stock, even if a lot of effort has been done to reduce this inventory. Therefore, we need extensions to the EOQ-models that take these uncertainties into consideration in an appropriate manner. In the process industry, many uncertainties are found and they are sometimes inherently fuzzy, c.f. [2], [3] and [4]. A lot of research contributions are found in this line of research. For instance [2], solved an EOQ model with backorders and infinite replenishment lead time with fuzzy lead times. The uncertainties stem from different sources, however, and in many situations not only demand is fuzzy but also the cycle time. In the Nordic process industry context, the uncertainties in cycle time comes from different sources; for instance, sometimes the raw material needed is not available at the planned time of production and therefore production planning increases the amount of products produced at the time on a particular machine. This rationale comes from the fact that it is very costly to let the capital intensive production units to be idle. In addition, the setup times when changing products may be significant. Also other factors contribute to the situation that the overall cycle time (as well as the demand) is often inherently fuzzy.

It is not uncommon that the EOQ-models with their extensions are solved analytically through the solution of the derivatives (as also done originally by Harris, [1]). There are other analytical methods as well, for instance [5] that proved the EOQ-case with backorders without using derivatives. If no analytical solutions are to be found, several methods can be used, for instance genetic algorithms as in [6]. If the uncertainties in the EOQ-models can be modeled stochastically (as done in [7]), the track of probabilistic models should be conducted, but this is not always possible in the process industry. For the uncertainties relevant to this paper, it is better to use fuzzy numbers instead of probabilistic approaches ([8] and [9]). In the line of research of fuzzy EOQ-models, there are contributions for instance like [10], who worked out fuzzy modifications of the model of [11], which took the defective rate of the goods into account. [12] and [13] solved an EOQ-model with lead times as decision
variables in addition to the order quantities. [14] introduced an EOQ-model, without backorders, but for two replaceable merchandizes. [15] used the signed distance method for a fuzzy demand EOQ-model without backorders. These are only a few examples from current literature.

The approach used in this paper is novel since there are no papers (to our knowledge) that focus on the multi-item EPQ (Economic Production Quantity) model, with limited storage and fuzzy demand and cycle times. Our solution methodology is similar to [4] and [16], where the fuzzy model is defuzzified using the signed distance method [17] and then the solution is found through the derivatives. This paper extends the results in a recent publication by [18] with the limited storage capacity restriction. However, the convexity proof of the model presented [18] is similar to the one needed in this paper. Therefore it is assumed that the defuzzified model in this paper is convex and thus, solvable through the derivatives. However, this model extends the model by [18] by the storage space restriction, which is taken into consideration using Lagrange multipliers.

The paper has the following outline: First we will show the motivation for conducting this research within a supply chain context. Then we will focus on developing the fuzzy model from the basic crisp model in chapter 3. After this, some illustrative examples are given and finally some concluding remarks will be made.

2. Why is inventory optimization so important in Supply Chain Management?

Two popular paradigms in supply chain management (SCM) are the so called lean- and agile operations. These have been around for decades and are quite often simultaneously used as supply chain strategies (as also shown in our example below). More recently, increased awareness of sustainability and “green” thinking have also been extended to the supply chain, particularly in the form of requirements for a reduced carbon footprint through more efficient transportation, but also through for example less scrapping and overproduction.

When discussing lean thinking, there is one prevailing “mantra” that is widely used to describe the essence of lean thinking, namely that of eliminating so called waste [19]. Toyota is often credited as the company pioneering lean thinking and according to Liker [20], the company originally devised seven types of waste. For our purposes, three of these types are critical; namely that of elimination of excess inventory (in all different stages of production), overproduction, and waiting (this originally refers to workers being idle but can also be extended to idle production machinery in our process industry context). As previously noted, process industry is capital intensive and thus a balance between potential overproduction (and resulting excess inventory) needs to be balanced with capacity utilization, or in other words, “waiting”. Having idle machines that do not produce anything is costly due to the capital tied in machinery, and in certain instances it then makes sense to produce to stock. Naylor [21] describes leaness as a strategy that strives for ensuring a “level schedule”, that is to say no unnecessary fluctuation in production volumes. In addition to this, cost optimization is also defined as a key target (among other things). Both of these interlinked features fit well into our process industry environment. Thus it can be said that leanness is one of the key goals for a typical process industry supply chain. When considering the topics of elimination of waste (in particular the three types outlined here), a level schedule, and cost optimization, the model found in this article provides one way to better work out a solution to these challenges.

Whereas leanness focuses on the elimination of waste and a level schedule, Naylor [21] defines agility as the ability to “exploit profitable opportunities in a volatile marketplace”. In other words, agility is about responsiveness to changed demands but it is to be noted it also refers to the ability to react to other changed conditions (for example actions to cope with a lack of supply of a particular raw material). Many definitions of agility rely on the term “flexibility” (for example [22] and [23]) and thus attempt to describe the fact that adaptability and adjustability are key features required to cope with changing conditions. Different types of flexibility in all areas of the company is inherent for an agile supply chain [24]. As previously noted, in the process industry demand is rarely constant and fully predictable, while raw material supply can at times also be uncertain. This creates a need for agility. Naim and Gosling [25] distinguish between lean and agile by looking at how uncertainty is managed. Lean tries to, as far as possible, eliminate variation. This is a good cause. Agility on the other is not trying to eliminate variation but, instead, it is a strategy to cope with variation. Numerous ways have been developed to support agility. For our purposes however, inventory management can be seen as one key enabler to create flexibility and manage uncertainty. While inventory can in certain cases reduce your responsiveness (for example if old products need to be “flushed out” of the supply chain before new ones can be taken in), stocks are generally perceived as a “necessary evil” that enables an appropriate response to changed conditions [26]. The key however is in establishing appropriate
buffers that take into account for example demand variability and other parameters that need to be considered (in our case for example the storage capacity). Inventory optimization thus becomes important also in the agile supply chain, and the model outlined in this paper provides one way of coping with the problem of determining optimal buffers.

Green thinking often refers to environmentally sound practices that help conserve resources. A commonly used term is sustainability, which stresses the need to engage in practices that allow us to leave the planet in a state where also future generations can live and prosper. Carter and Rogers [27] define sustainability as “an integration of social, environmental and economic responsibilities”. This is commonly also referred to as the “triple bottom line”, originally coined by Elkington in 1998. This definition contains an important point as it also stresses the economic responsibility of the corporation. This gives us further weight to the need for aligning green concepts with previously discussed lean and agile practices. Intuitively, lean and green should go well together as lean strives to eliminate “waste”, including unnecessary movement and inventory. As such, in a study by King and Lennox, the statistical relationship between lean operations and environmental performance was conducted [28]. This study looked at statistical data from more than 17,000 US chemical companies in an effort to understand linkages between lean and green. One focus was on the adoption of the appropriate ISO standards (and correlation between these), but also waste generation (here referred to in the “classical”, not broad, sense as in lean thinking) and inventory data was looked at. The study finds that there is a relationship between the adoption of lean practices and sustainability. Lean is indeed green. This is further supported in a study by Mollenkopf et al. [29] where they conclude that waste elimination is in fact a key area of complementary targets between sustainability and lean operations. While certain instances of contradictory elements in lean operations and sustainability have been reported, waste elimination (both in a broad sense as well as specifically referring to pollution) and inventory management is as such an important part of ensuring also environmentally friendly operations. Again, the model outlined below serves as one way to improve leanness, and thus also sustainability.

Not all supply chains necessarily need to achieve leanness, agility and sustainability at the same time but all three are becoming more and more important in today’s world. In our process industry context, leanness is paramount from a purely financial point of view, but as uncertainty also needs to be managed, a certain degree of agility is required. As for sustainability, consumers demand it more and more. In addition to public pressure, the cost of energy and material also needs to be considered [30]. This also serves to create a link between lean and sustainable operations.

Although there are multiple strategies and ways of achieving the previously outlined supply chain paradigms (that are not covered as a part of this article), inventory optimization is one way of supporting the implementation of lean, agile and sustainable supply chains. The model outlined here is particularly suitable in a process industry context but can also be applied elsewhere.

3. The Model

In order to put this model into the research perspective of other fuzzy EOQ-models, also using the signed distance measure for the defuzzification step, a brief overview of this specific topic is given before the actual model. Typically, the above mentioned approach is used for solving the optimization problems analytically. For instance [2] solved the EOQ-problem with backorders, where the backorder were allowed to be symmetrical triangular fuzzy numbers. This result was extended to cover asymmetrical fuzzy numbers and fuzzy demand parameters as well in [31]. In [16] again, the focus was on a situation, where the production rate is finite and the cycle time is fuzzy. These results were extended to multiple machines in [18], still providing the analytical solution for the optimization problem. Now in this paper, the focus is set on the situation when the production rate is finite; there are several machines with a joint inventory space limitation. This situation is very realistic in the paper industry today. So in a way the research direction is towards relevance from a field of research that has sometimes been criticized for being too theoretical.

The parameters and variables (can be assumed strictly greater than zero) in the classical multi-item EPQ model with shared cycle time are the following (where the index \( i \in I = \{1, 2, \ldots, |I|\} \) is denoting the products):

- \( y_i \) is the production batch size (variable)
- \( K_i \) is the fixed cost per production batch (parameter)
- \( D_i \) is the annual demand of the product (parameter)
- \( h_i \) is the unit holding cost per year (parameter)
- \( T \) is the cycle time (variable)
- \( P_i \) is the annual production rate (parameter)
- \( a_i \) is the storage area requirement per inventory unit (parameter)
- \( A \) is the maximum available storage area (parameter)
The total cost function (TCU), including production setup costs and inventory holding costs for all products, and the additional constraint limiting the storage size are given by

\[
TCU(y_1, \ldots, y_n) = \sum_{i=1}^{n} \frac{K_iD_i}{y_i} + \sum_{i=1}^{n} \frac{h_iy_i}{2} \rho_i
\]

\[s.t. \quad \sum_{i=1}^{n} a_iy_i \leq A \]  

(1)

where \( \rho_i = 1 - \frac{D_i}{P_i} \). Here we assume that \( \frac{D_i}{P_i} \) is strictly less than 1. The production batch size \( y_i \) can also be replaced with the cycle time \( T \) according the formula \( y_i = TD_i \). The insertion of this formula into (1) yields the total cost function to minimize

\[
TCU(T) = \sum_{i=1}^{n} \frac{K_i}{T} + \sum_{i=1}^{n} \frac{T h_i D_i}{2} - \sum_{i=1}^{n} \frac{T h_i D_i^2}{2P_i}
\]

\[s.t. \quad T \sum_{i=1}^{n} a_i D_i \leq A \]  

(2)

Eq (2) is one version of the crisp (classical) multi-item EOQ-model with shared production capacity, cycle time and storage limitation. This problem can be solved using the derivatives, since all the terms in Eq. (2) are convex.

First, let us assume that the cycle time is uncertain but it is possible to describe it with a triangular fuzzy number (symmetric).

**Definition 1.** Consider the fuzzy set \( \tilde{A} = (a, b, c) \) where \( a < b < c \) and defined on \( R \), which is called a triangular fuzzy number, if the membership function of \( \tilde{A} \) is given by

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & \text{otherwise.}
\end{cases}
\]

The fuzzy cycle time \( \tilde{T} \) will then be

\[
\tilde{T} = (T - \Delta, T, T + \Delta)
\]

Annual demand is also uncertain, but assumed to be captured by a (symmetric) fuzzy number \( \tilde{D}_t = (D_i - \Delta_i, D_i, D_i + \Delta_i) \).

In order to find non-fuzzy values for the model, we need to use some distance measures, and as in [10] we will use the signed distance [17]. Before the definition of this distance, we need to introduce the concept of \( \alpha \)-cut of a fuzzy set.

**Definition 2.** Let \( \tilde{B} \) be a fuzzy set on \( R \) and \( 0 \leq \alpha \leq 1 \). The \( \alpha \)-cut of \( \tilde{B} \) is the set of all the points \( x \) such that \( \mu_{\tilde{B}}(x) \geq \alpha \), i.e.

\[
\tilde{B}(\alpha) = \{x | \mu_{\tilde{B}}(x) \geq \alpha \}.
\]

Let \( \Omega \) be the family of all fuzzy sets \( \tilde{B} \) defined on \( R \) for which the \( \alpha \)-cut \( \tilde{B}(\alpha) \) is [\( \tilde{B}_l(\alpha), \tilde{B}_u(\alpha) \)] exists for every \( 0 \leq \alpha \leq 1 \), and both \( \tilde{B}_l(\alpha) \) and \( \tilde{B}_u(\alpha) \) are continuous functions on \( \alpha \in [0,1] \).

**Definition 3.** For \( \tilde{B} \in \Omega \) define the signed distance of \( \tilde{B} \) to \( \tilde{0} \) as

\[
d(\tilde{B}, \tilde{0}) = \frac{1}{2} \int_0^1 [\tilde{B}_l(\alpha) + \tilde{B}_u(\alpha)] d\alpha
\]

The Total Annual Cost in the fuzzy sense will be

\[
TCU(\tilde{T}) = \sum_{i=1}^{n} \frac{K_i}{\tilde{T}} + \sum_{i=1}^{n} \frac{\tilde{T} h_i \tilde{D}_i}{2} - \sum_{i=1}^{n} \frac{\tilde{T} h_i \tilde{D}_i^2}{2P_i}
\]

\[s.t. \quad \tilde{T} \sum_{i=1}^{n} a_i \tilde{D}_i \leq A \]  

(3)

The signed distance of TCU and \( \tilde{0} \) is given by
\begin{equation}
\begin{aligned}
d(TCU(\bar{T}), \bar{0}) &= \sum_{i=1}^{n} K_i d(1/\bar{T}, \bar{0}) \\
&\quad + \sum_{i=1}^{n} \frac{h_i}{2} d(\bar{T} \bar{D}_i, \bar{0}) \\
&\quad - \sum_{i=1}^{n} \frac{h_i}{2P_i} d(\bar{T} \bar{D}_i^2, \bar{0})
\end{aligned}
\end{equation}

The constraint takes the following form

\begin{equation}
\sum_{i=1}^{n} a_i d(\bar{T} \bar{D}_i, \bar{0}) \leq A
\end{equation}

If we calculate the signed distances, we obtain that

\begin{equation}
\begin{aligned}
d(1/\bar{T}, \bar{0}) &= \frac{1}{2} \int_{0}^{1} [(1/\bar{T})_i(\alpha) \\
&\quad + (1/\bar{T})_u(\alpha)]d\alpha \\
&= \frac{1}{2} \int_{0}^{1} \left[ \frac{1}{T + \Delta - \Delta \alpha} \\
&\quad + \frac{1}{T - \Delta + \Delta \alpha} \right] d\alpha \\
&= \frac{1}{2\Delta} [\ln(T + \Delta) - \ln(T - \Delta)]
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
d(\bar{T} \bar{D}_i, \bar{0}) &= \frac{1}{2} \int_{0}^{1} [(\bar{T} \bar{D}_i)_i(\alpha) \\
&\quad + (\bar{T} \bar{D}_i)_u(\alpha)]d\alpha \\
&= \frac{1}{2} \int_{0}^{1} \left[ (D_i - \Delta_i + \Delta_i \alpha)(T - \Delta \\
&\quad + \Delta \alpha) \\
&\quad + (D_i + \Delta_i - \Delta_i \alpha)(T + \Delta \\
&\quad - \Delta \alpha) \right] d\alpha = D_i T + \frac{\Delta \Delta_i}{3}
\end{aligned}
\end{equation}

The defuzzified total cost function and constraint

\begin{equation}
TCU(\bar{T}) = \sum_{i=1}^{n} \frac{K_i}{2\Delta} [\ln(T + \Delta) - \ln(T - \Delta)] \\
+ \sum_{i=1}^{n} \frac{h_i (D_i T + \frac{\Delta \Delta_i}{3})}{2} \\
- \sum_{i=1}^{n} \frac{h_i (D_i^2 T + \frac{2\Delta \Delta_i D_i + \frac{\Delta^2 \Delta_i}{3}}{2P_i})}{3}
\end{equation}

s.t. \sum_{i=1}^{n} a_i (D_i T + \frac{\Delta \Delta_i}{3}) \leq A

If we consider the cost function, since it is convex in \( T \), we can find the optimal value if we solve the equation \( \frac{dT_{CU}}{dT} = 0 \). The solution is the following:

\begin{equation}
T^* = \sqrt{\frac{\Delta^2 + \frac{2 \sum_{i=1}^{n} K_i}{\sum_{i=1}^{n} D_i h_i - \sum_{i=1}^{n} h_i (D_i^2 + \frac{\Delta^2 \Delta_i}{3})}}{2}}
\end{equation}

(4)

If this value satisfies the storage constraint, we have found the optimal solution for the problem; otherwise we can employ the Lagrangian relaxation method. For
To find the optimal solution in this case, we need to introduce the constraint into the objective function with a non-negative multiplier:

\[ L(TCU, \lambda) = \sum_{i=1}^{n} \frac{K_i}{2\Delta} \left[ \ln(T + \Delta) - \ln(T - \Delta) \right] + \sum_{i=1}^{n} \frac{h_i(D_i T + \frac{\Delta \Delta_i}{3})}{2} - \sum_{i=1}^{n} \frac{h_i(D_i^2 T + \frac{2\Delta \Delta_i}{3})}{2P_i} + \lambda \left( \sum_{i=1}^{n} a_i(D_i T + \frac{\Delta \Delta_i}{3}) - A \right) \]

For any non-negative \( \lambda \) the solution of this new problem will be a lower bound for the original problem. From the general theory we know that if \( x^* \) is an optimal solution for the relaxation problem, the solution of this new problem will be a lower bound for the original problem if \( \sum_{i=1}^{n} a_i(D_i T + \frac{\Delta \Delta_i}{3}) \leq A \) and \( \lambda \left( \sum_{i=1}^{n} a_i(D_i T + \Delta \Delta_i/3) - A \right) = 0 \).

To find the optimal solution in this case, we need to find the solution for the system of equations:

\[ \frac{dL(TCU, \lambda)}{d\lambda} = 0, \quad \frac{dL(TCU, \lambda)}{dT} = 0 \]

\[ \frac{\partial L}{\partial T} = \sum_{i=1}^{n} \frac{K_i}{2\Delta} \left[ \frac{1}{T + \Delta} - \frac{1}{T - \Delta} \right] + \sum_{i=1}^{n} \frac{h_i D_i}{2} - \sum_{i=1}^{n} \frac{h_i(D_i^2 T + \frac{2\Delta \Delta_i}{3})}{2P_i} + \lambda \sum_{i=1}^{n} a_i D_i \]

\[ \frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} a_i(D_i T + \frac{\Delta \Delta_i}{3}) - A \]

Using the first derivative we obtain the following equation:

\[ \sum_{i=1}^{n} \frac{K_i}{2\Delta} \left[ \frac{1}{T + \Delta} - \frac{1}{T - \Delta} \right] + \sum_{i=1}^{n} \frac{h_i D_i}{2} - \sum_{i=1}^{n} \frac{h_i(D_i^2 T + \frac{2\Delta \Delta_i}{3})}{2P_i} + \lambda \sum_{i=1}^{n} a_i D_i \]

\[ = 0 \]

\[ T = \sqrt{\frac{2\Sigma_{i=1}^{n} K_i}{\Sigma_{i=1}^{n} D_i h_i - \Sigma_{i=1}^{n} h_i(D_i^2 T + \frac{2\Delta \Delta_i}{3}) + \lambda \Sigma_{i=1}^{n} a_i D_i} + \frac{2 \Sigma_{i=1}^{n} k_i}{\Sigma_{i=1}^{n} D_i h_i} + \frac{2 \Sigma_{i=1}^{n} K_i}{\Sigma_{i=1}^{n} a_i D_i} - \Delta^2 (\Sigma_{i=1}^{n} a_i D_i)^2} \]

Using the derivative with respect to \( \lambda \), we can calculate the optimal value:

\[ \lambda^* = \frac{\sum_{i=1}^{n} h_i(D_i^2 T + \frac{2\Delta \Delta_i}{3})}{\sum_{i=1}^{n} a_i D_i} \]

To summarize the results, the strategy to find the optimal solution is the following:

1. If \( T^* \) in (4) satisfies the constraint, this is the optimal solution for the problem.
2. If \( \sum_{i=1}^{n} a_i(D_i T^* + \frac{\Delta \Delta_i}{3}) > A \), then the optimal value of \( T \) can be obtained from (5) with the \( \lambda \) in (6).

4. Examples

In the first example, a very small problem with only three products is solved to optimality. The data for the example is found in table 1. This example is used to show that the optimal cycle time is often quite short, and thus the overall inventory requirement not that high. However, due to increased pressure on inventory efficiency, there are often very limited space requirements, and thus the proposed method in this paper need to be used (i.e. using the calculations of Eqs. 5-6).

In example 1, the value of \( A \) (maximum available storage area) determines if the solution is the absolute minimum of the objective function (the constraint has
no effect on the optimal solution) or we need to apply the Lagrangian multiplier. In this example the critical value of $A$ is 32.69, in other words if the available storage area is greater than this value, then the constraint has no effect on the solution for the problem, the optimal value can be obtained from (4), which is $T^* = 2.49$. If $A < 32.69$, then we need to use the results obtained in Eqs. (5) and (6): for example, if $A = 20$, then the optimal cycle time is $T^* = 1.52$.

Another example is also provided in the section. This example is theoretical but close to some actual applications that inspired this piece of research. The problem consists of 8 products, where the annual demand ($D_i$) of products are 1500 tons, 500 tons, 800 tons, 1100 tons, 400 tons, 2100 tons, 700 tons and 900 tons respectively. The production rates ($R_i$) are 3500, 3000, 3500, 3000, 3500, 3800, 3400, 3200 tons/year. There is a fixed cost incurring each time a product starts to be produced (setup costs, $K_i$): 1500, 1000, 1200, 1300, 1400, 900, 1300, and 1100 euro respectively. The holding costs are 0.25 euro per kg and annum for each product. The $\Delta$-parameter in the fuzzy case is assumed to be 0.04. and the possibility distribution for the demand is always 100 tons (i.e. $\Delta_i$). The total possible inventory space is limited to 4000 tons and the $\alpha$-parameter is assumed 1 for each product. The solution to this problem is a TCU of 1876.47 and the cycle time of 0.5 and the storage space is a limiting factor.

To analyze the sensitivity of the optimal solution with respect to the parameters in the second example, we can investigate the change in the cycle time and the total cost function for different initial values of the following parameters:

- $\Delta$, which is used in the model to represent the uncertainty connected to the cycle time;
- the $h_i$ values (the unite holding cost).

From the results of the simple sensitivity analysis (Table 2) we can make the following observations:

- the holding cost and $\Delta$ have no significant effect on the optimal cycle time; for a fixed $\Delta$, no change can be observed in the optimal $T$, and even different initial values of $\Delta$ result in a marginal change.
- If we incorporate imprecision into the Economic Production Quantity model in the form of the parameter $\Delta$, the total cost value increases as the function of $\Delta$: the more uncertain we are, the higher the total cost is. Moreover, a very slight
change in $\Delta$ results in a radical increase in the total cost function.

- The value of the holding cost has a similar effect on the total cost as $\Delta$ as we naturally expect: the increased $h_i$ results in a higher optimal cost. It is important to notice that the increment caused by the changes in $h_i$ is less significant than the effect of $\Delta$.

\[
\begin{array}{c|cccc}
  h_i & 0.02 & 0.03 & 0.04 & 0.05 \\
  \hline
  0.15 & TCU & 967.72 & 1357.96 & 1749.68 & 2143.19 \\
   & T & 0.499 & 0.499 & 0.498 & 0.498 \\
  0.2 & TCU & 1031.14 & 1421.37 & 1813.07 & 2206.57 \\
   & T & 0.499 & 0.499 & 0.498 & 0.498 \\
  0.25 & TCU & 1094.57 & 1484.78 & 1876.47 & 2269.95 \\
   & T & 0.499 & 0.499 & 0.498 & 0.498 \\
  0.3 & TCU & 1157.99 & 1548.19 & 1939.86 & 2333.33 \\
   & T & 0.499 & 0.499 & 0.498 & 0.498 \\
\end{array}
\]

Table 2. The results of the sensitivity analysis.

5. Summary

Optimization algorithms can be used to support agile and lean supply chains. Through some links between lean and sustainable supply chains, these optimization methods also serve to help in creating “green” operations. Thus, inventory optimization can have a positive impact both on responsiveness and cost as well as the environment. By using optimization techniques like the model outlined in this paper, it is possible to set an inventory strategy that ensures availability by keeping the right amount of inventory in stock while maintaining a as level schedule as possible, even with multiple products manufactured.

The model introduced in this paper is a variation of the EPQ model with many items, one manufacturing machine and limited storage space, where the cycle time and demand are fuzzy. This allows for taking expert opinion into account when modeling uncertainties in demands and cycle times, especially when new suppliers and/or products are introduced. The model provides an optimal solution that takes these uncertainties into account. This is a new contribution as an EPQ model with these uncertainties and parameters has not previously been introduced. The model is particularly suitable for solving optimization problems in a process industry context.

To better demonstrate how the model works, some examples were used to illustrate the effect of for example limited storage space. In the second example, the performance of the model was demonstrated with relevant but not actual data, that is to say data that could very well illustrate a real-world problem were used in finding an optimal solution that minimizes the total cost. Finally, a sensitivity analysis was provided to determine the effects of changes in two parameters.

Future research tracks will include increasing the presented model to several machines with shared inventory space. In this line of research, the analytical solution (most likely) not is available, so more sophisticated defuzzification methods would be needed. Finally a complete sensitivity analysis of both the cost of fuzzy cycle times and the savings such flexibility can bring about would need to be done within specific problem domains (such as the Nordic paper industry).

References


