Finance Sourcing in a Supply Chain

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Abstract

We examine the relative merits of bank versus trade credit in a supply chain consisting of a manufacturer and a capital-constrained retailer. We show that trade credit is more effective than bank credit in mitigating double marginalization in the supply chain when marginal production cost is relatively low, and that bank credit becomes more effective otherwise.

1. Introduction

Trade credit refers to the credit a seller lends to its buyer for the purchase of goods. The prevalence of trade credit has been well documented for both developed economies such as the US and the other G7 countries (e.g., [10], [11]) and economies with less developed financial markets or weak bank-firm relationships ([1], [2]). The extant literature has given several theories over the use of trade credit. In the current article, we provide a new explanation of trade credit that it may help alleviate double marginalization in a supply chain over bank credit.

Much research attention has been paid to why the seller may extend trade credit even in the presence of specialized financial institutions such as banks. When buyers have private information about their heterogeneous payoff distribution, a lender wishes to be able to price discriminate among them, and [3] and [13] show that trade credit is a more effective screening device than bank credit. The information advantage theory argues that while the manufacturer gathers useful information about the buyer in the normal course of business, financial institutions must overcome additional barriers to obtain such information (e.g., [1], [4], [5], and [12]). In the event of buyer default, the manufacturer may have an advantage in salvaging collateral than a financial institution ([9]). When the manufacturer has private information about the quality of its product, trade credit may be used to signal product quality ([7], [8]). [6] proposes that trade credit may help control the transaction costs between trading partners. In addition, [15] provides a relationship-lending theory of trade credit. [10] provides a more detailed discussion of some of these perspectives.

We develop a model of a supply chain consisting of one manufacturer and one retailer. While the manufacturer has sufficient capital endowment, the retailer is endowed with zero working capital. Demand for the product in the end market is stochastic and does not realize until the end of the selling season. We assume that the product is a commodity and that its prior demand distribution is common knowledge. In addition to the product market, there also exists a market of specialized financial intermediaries such as banks. All players (the manufacturer, retailer and banks) are risk neutral and none of them receives further signals about demand prior to its realization. We thus examine a scenario where no player possesses any informational advantage over the others. Following the convention in the literature, we assume the bank market to be competitive so that a lending bank makes zero expected profits. The retailer can always borrow
from the bank market to finance its operations. In addition, the manufacturer may find it optimal to offer trade credit to the retailer. When trade credit is available, the retailer chooses between bank and trade credit to fund its inventory purchase.

At the beginning of the selling season, the manufacturer announces a wholesale price and acceptable payment schemes. Specifically, the manufacturer declares whether payment must be in cash or can be postponed until the end of the season. In the event that the manufacturer accepts postponed payment, it is essentially extending trade credit to the retailer. If the manufacturer accepts only up-front cash payment, then the retailer decides on an inventory level and borrows bank credit to finance the purchase. If delayed payment is also allowed, then the retailer decides on an inventory level and chooses between bank or trade credit to finance the purchase. We assume the product is a perishable so that at the end of season any leftover units have zero salvage value. This means that unsold inventory cannot serve as collateral on loan application. Clearly, the retailer's revenue is always stochastic, regardless of its source of financing. The revenue of the manufacturer is also stochastic when it extends and the retailer borrows trade credit. At the end of the season, the retailer's revenue realizes and it repays its creditor the smaller of its revenue and the loan (including interest). Note that by focusing on a supply chain with a single retailer, our model naturally excludes any price-discrimination effect of trade credit.

In a supply chain context such as ours, it is well known that the total profits of the manufacturer and the retailer are below those in a centralized supply chain, where the same firm produces and sells directly to its end customers. Such a phenomenon is commonly referred to as double marginalization ([14]). Our central result is that both bank and trade credit have their comparative merits in ameliorating double marginalization in the supply chain. When marginal production cost is sufficiently low (relative to market demand parameters), trade credit yields higher total profits in the supply chain than bank credit. In this case, trade credit helps mitigate double marginalization and then allows the manufacturer to make more profits. When marginal production cost is relatively high, bank financing yields higher total profits in the supply chain than trade financing. In this case, bank credit better alleviates double marginalization and leads to higher profits for the manufacturer. The manufacturer thus will not issue trade credit.

The rationale behind our above result is as follows. When the retailer finances with bank credit, it is the lending bank that bears the retailer's default risk in the event of low market demand. Because the lending bank makes zero expected profits due to Bertrand competition, the capital-constrained retailer financing via bank credit behaves just like a self-sufficient retailer facing no capital constraint. In particular, the capital-constrained retailer will stock the same inventory as a retailer endowed with sufficient capital. In this sense, the retailer's limited liability does not play any active role under bank credit. In contrast, when the retailer finances with trade credit, the manufacturer solely bears the retailer's default risk. Because the manufacturer shares the retailer's risk in the event of low market demand, limited liability is now effective and prompts the retailer to stock a higher inventory than it does under bank credit. When marginal production cost is low enough, such a higher inventory mitigates double marginalization, making the manufacturer strictly better off and the retailer equally well off relative to bank financing. Trade credit is thus the unique financing equilibrium. On the other hand, when marginal production cost is sufficiently high, the higher inventory under trade credit aggravates double marginalization, and bank credit becomes the unique financing equilibrium.

The remainder of the article is organized as
follows. Section 2 develops a model of a supply chain with a discrete demand distribution. Section 3 presents the centralized supply chain as a benchmark, derives the market equilibrium under bank and trade credit, respectively, and then characterizes the financing equilibrium in the supply chain. As section 4 shows, the basic insight from the discrete model continues to hold in a model of continuous demand. Section 5 concludes.

2. Model

We examine a product market comprising one manufacturer and one retailer. The manufacturer does not sell directly to the end consumers. Instead, distribution is purveyed by the retailer. Such a manufacturer-retailer relationship is commonly known as a supply chain. While the manufacturer is endowed with sufficient working capital, the retailer faces capital constraints. The retailer's capital endowment is normalized to zero without loss of generality. In addition to the product market, there also exists a market of financial intermediaries (banks, say). Clearly, for the product market to be viable the retailer has to borrow either bank or trade credit. Following the convention in the trade credit literature (e.g., [3], [4]), we assume that the bank market is competitive and that banks have access to unlimited funds at a risk-free interest rate $r_f$, which is normalized to zero (without loss of generality). A zero risk-free interest rate also confers the advantage of allowing us to ignore discounting. All players in the model (i.e., the retailer, the manufacturer and the banks) are risk neutral and maximize their respective expected profits.

Product demand in the retail market, $D$, is stochastic and not realized until the end of the selling season. To ease exposition, the demand distribution is assumed to be binary: $D = H$ with probability $\alpha$ ($0 < \alpha < 1$) and $D = L$ (where $0 < L < H$) with the remaining probability, $1 - \alpha$. Retail price is assumed to be fixed at 1. Neither member of the chain has fixed operating costs. The marginal production cost is constant at $c$, with $0 < c < 1$. Without loss of generality, we assume that the retailer incurs no variable costs besides the wholesale price. The product is perishable so that any unsold units have zero salvage value by the end of the season. Therefore, the retailer can not use leftover inventory as collateral on its loan.

The retailer has limited liability. If its revenue exceeds its loan, the retailer repays its loan fully. Otherwise, it repays its entire revenue and defaults on the remainder portion of the loan. We assume that borrowing is exclusive in that the retailer may borrow from only one creditor.

The sequence of events is as follows. At the start of the season, the manufacturer announces a per-unit wholesale price $w$ and specifies whether full payment must be made upon purchase or payment can also be made when retail revenue materializes at the end of the season. Observing the wholesale price and the permitted payment choice(s), the retailer chooses a financing scheme and a corresponding inventory level. If the manufacturer does not extend trade credit and accepts only cash payment, the retailer has to borrow from the bank market to finance her inventory purchase. If the manufacturer accepts both instant (cash) payment and delayed payment, then the retailer chooses between bank and trade credit.

Let $Q$ denote the retailer's inventory level and $r$ the bank's interest rate to the retailer. If the retailer chooses bank credit, it borrows $wQ$ from a bank and pays this amount to the manufacturer for her inventory purchase. At the end of the season, it repays the lending bank the smaller of her revenue $(\min\{D, Q\})$ and the principal and interest $(wQ + QrQ)$ of her loan. If the retailer chooses trade credit, it receives $Q$ units of product at the start of the season and pays the manufacturer the smaller of her revenue
Clearly, the retailer will choose inventory \( Q \) such that \( L \leq Q \leq H \) provided \( w(1+r) \leq 1 \) under bank credit or \( w \leq 1 \) under trade credit.

Finally, we make the following tie-breaking rules. If the retailer is indifferent between two inventory levels, we assume that it chooses the higher inventory. If trade credit is available and the retailer is indifferent between bank and trade credit, we assume that it uses trade credit.

3. Analysis

3.1. The centralized supply chain as a benchmark

In the centralized supply chain, one firm produces and also sells its products directly to the end customers. Below we separately consider centralized supply chains with and without capital constraints; the two cases yield essentially identical outcomes. Let \( Q_c \) denote the production quantity in the centralized chain.

**Case A: The case without capital constraint.**

Suppose that the firm in the centralized supply chain has sufficient capital endowment and thus does not resort to external financing. Since demand satisfies \( L \leq D \leq H \), the firm chooses production level \( Q_c \) between \( L \) and \( H \) to maximize its expected profits:

\[
\pi_c = \alpha Q_c + (1-\alpha)L - cQ_c. \tag{1}
\]

It is easy to see that the optimal production volume is

\[
Q_c = \begin{cases} 
H & \text{if } c \leq \alpha \leq 1, \\
L & \text{if } 0 < \alpha < c 
\end{cases} \tag{2}
\]

and that the optimal firm profits are

\[
\pi_c = \begin{cases} 
(\alpha-c)H + (1-\alpha)L & \text{if } c \leq \alpha \leq 1 \\
(1-c)L & \text{if } 0 < \alpha < c 
\end{cases} \tag{3}
\]

**Case B: The case with capital constraint.**

Now, suppose that the firm is endowed with zero working capital. Let \( r_c \) denote the banks' interest rate. To operate the firm has to borrow bank credit \( cQ_c \) at time zero. At the end of the season, the firm repays the lending bank the smaller of its revenue and its loan \((c(1+r)Q_c)\). The firm chooses \( Q_c \) to maximize its expected profits

\[
\pi_c = E[min\{D,Q_c\} - c(1+r)\min\{D,Q_c\}], \tag{4}
\]

subject to the zero-profit condition in the bank market:

\[
cQ_c = E\min\{c(1+r)Q_c, \min\{D,Q_c\}\} \tag{5}
\]

Here \( x^+ \) equals \( x \) when \( x > 0 \) and equals 0 otherwise. Expanding and substituting the latter equation into the former convert the firm's problem into one without any capital constraint:

\[
\pi_c = \alpha Q_c + (1-\alpha)L - cQ_c. \tag{6}
\]

The optimal production and profit levels are thus the same as in the centralized supply chain without capital constraint. Our intuition suggests that due to limited liability, the capital-constrained firm will order more than it would when it were endowed with sufficient capital. However, the firm still acts as if it faced no capital constraint. This is because the lending bank makes zero expected profits due to Bertrand competition and the capital-constrained firm and the lending bank jointly function as a self-sufficient firm.

3.2. The retailer finances with bank credit

In this subsection, we suppose that the manufacturer does not offer trade credit and accepts only cash payment. We analyze the market equilibrium when the retailer finances with bank credit. At the start of the season, the manufacturer announces a wholesale cash price \( w \) and the retailer then chooses an inventory level \( Q \). The retailer approaches the banks and requests a loan of \( wQ \) dollars to fund its inventory purchase. The banks simultaneously announce interest rate \( r \). At the end of the season, the retailer collects revenue \( \min\{D,Q\} \) and repays the lending bank \( \min\{w(1+r)Q, \min\{D,Q\}\} \).

Because there is no discounting in our model,
the retailer's problem is to maximize its expected profits (for \( L \leq Q \leq H \))
\[
\pi^H_b(Q) = E(\min\{D, Q\} - w(1+r)Q),
\]
subject to the banks' zero-profit condition
\[
wQ = E \min\{w(1+r)Q, \min\{D, Q\}\}
\]
(8)
The LHS of (8) is the lending bank's costs of fund and the RHS its expected revenue. When \( D < w(1+r)Q \), the retailer's revenue is not sufficient to cover its loan (including the principal and interest). In such a case the retailer only repays its entire revenue and the lending bank absorbs the retailer's default risk.

Since interest rate \( r \) measures the retailer's cost of financing via bank credit, a necessary condition to enable decentralized distribution is that the discounted retail price exceeds the wholesale cash price, i.e., \( \alpha > c + (1-c)\frac{L}{H} \), \( w^*_b = \alpha \) and \( Q^*_b = H \), and the banks' zero-profit condition becomes \( \alpha H = (1+r)H + (1-\alpha)L \), which leads to \( r = \frac{(1-\alpha)(\alpha H - L)}{\alpha^2 H} \). In this case, the retailer borrows bank credit \( \alpha H \) and will default when realized demand is low. Here a positive break-even interest rate \( r^* \) compensates the lending bank for the retailer's default risk. When \( \alpha < c + (1-c)\frac{L}{H} \), \( w^*_b = 1 \) and \( Q^*_b = L \). At time zero the retailer takes a bank loan of \( L \) dollars. At the end of the season, the retailer collects sales revenue \( L \) dollars with certainty. In this case, the retailer never defaults and the corresponding interest rate is thus zero. Proposition 1 summarizes the equilibrium when the retailer uses bank credit.

**Proposition 1.** When the retailer finances with bank credit, the market equilibrium is: (1). When \( \alpha > c + (1-c)\frac{L}{H} \), \( w^*_b = \alpha \), \( Q^*_b = H \), \( r = \frac{(1-\alpha)(\alpha H - L)}{\alpha^2 H} \), \( \pi^H_b = (\alpha - c)H \) and \( \pi^w_b = (1-\alpha)L \). (2). When \( \alpha < c + (1-c)\frac{L}{H} \), \( w^*_b = 1 \), \( Q^*_b = L \), \( r = 0 \), \( \pi^H_b = (1-\alpha)L \) and \( \pi^w_b = 0 \).

Under bank financing, the capital-constrained retailer together with a competitive bank market amounts to a retailer with sufficient capital endowment. Therefore, the incentives of both the manufacturer and the retailer and hence the supply chain equilibrium remain the same as those when the retailer faces no capital constraint. When demand for the product is high ( \( \alpha > c + (1-c)\frac{L}{H} \) ), the manufacturer sets a lower wholesale cash price (at
\( \alpha \) to encourage the retailer to carry more inventory \((H)\). Otherwise, it sets a higher wholesale cash price \((1)\) to induce the retailer to order \(L\). Comparing this market equilibrium with the centralized case above, we see that when \(c < \alpha < c + (1-c)L/H\), the decentralized supply chain stocks a lower inventory \((Q_b = L < Q_c = H)\), and the total supply chain profits \((1 - \alpha)L\) are strictly lower than those of the centralized chain \((\alpha - c)H + (1 - \alpha)L\). This reflects the efficiency loss caused by double marginalization.

3.3. The retailer finances with trade credit

We now examine the market equilibrium when the manufacturer extends and the retailer borrows trade credit. At the start of the season, the manufacturer announces a wholesale price \(w\) and accepts both instant and delayed payment. Suppose the retailer orders \(Q\) units on trade credit. When demand \(D\) realizes at the end of the season, the retailer repays \(\min{\min\{D, Q\}, wQ}\) to the manufacturer. The retailer thus fully repays its loan \((wQ)\) when its realized revenue exceeds \(wQ\) and repays its entire revenue otherwise. Therefore, under trade credit it is the manufacturer that bears the retailer's default risk.

For a given wholesale price \(w \leq 1\), the retailer chooses \(Q\) to maximize its expected profits

\[
\pi_r^T(Q) = E[\min\{D, Q\} - wQ],
\]

which may be rewritten as

\[
\pi_r^T(Q) = \begin{cases} 
\alpha Q + (1 - \alpha)L - wQ & \text{if } L \geq wQ \\
\alpha(1-w)Q & \text{if } L < wQ
\end{cases}.
\]

We can readily verify that the retailer's optimal inventory decision splits into several cases: (1) When \(w \leq \alpha\) and \(w \leq L/H\), \(Q = H\) and \(\pi_r^T = (\alpha - w)H + (1 - \alpha)L\); (2) When \(w > L/H > \alpha\), the retailer's optimal inventory is \(Q = L\) and its profits are \(\pi_r^T = (1 - w)L\); (3) When \(w > \alpha \geq L/H\), \(Q = H\) and \(\pi_r^T = \alpha(1 - w)H\); (4) When \(L/H \geq w > \alpha\), \(Q = L\) and \(\pi_r^T = (1 - w)L\); and (5) When \(L/H \leq w \leq \alpha\), \(Q = H\) and \(\pi_r^T = \alpha(1 - w)H\).

The manufacturer incurs production costs of \(cQ\) at time zero. At the end of the season, the manufacturer's revenue equals the lower of the retailer's realized revenue \((\min\{D, Q\})\) and the loan \((wQ)\). Its objective function is

\[
\pi_m^T(w) = E[\min\{\min\{D, Q\}, wQ\} - cQ].
\]

Proposition 2 below summarizes the supply chain equilibrium when the retailer finances with trade credit.

**Proposition 2.** When the manufacturer extends and the retailer uses trade credit, the supply chain equilibrium is \(w_r = 1\) and \(Q_r = H\), with expected profits \(\pi_r^T = (\alpha - c)H + (1 - \alpha)L\) and \(\pi_m^T = 0\).

Proof: See the appendix.

When the retailer finances with trade credit, the supply chain also suffers from double marginalization. The retailer always stocks inventory \(H\), which exceeds the inventory in the centralized chain when \(\alpha < c\). The total profits in the supply chain, \((\alpha - c)H + (1 - \alpha)L\), are also lower than those of the centralized chain when \(\alpha < c\).

However, trade credit has its comparative advantage over bank credit under some circumstances, as we shall see in the next subsection.

3.4. Equilibrium financing

We have seen above that neither bank nor trade credit alone eliminates double marginalization under all circumstances. As the next Proposition shows, however, the combined use of these two financing schemes removes double marginalization.

**Proposition 3.** (1). When \(0 < \alpha < c\), the manufacturer does not offer trade credit and the retailer borrows bank credit. In equilibrium, \(w_b = 1\), \(Q_b = L\), \(r = 0\), \(\pi_b^M = (1 - c)L\) and \(\pi_b^S = 0\).

When \(c \leq \alpha < 1\), the manufacturer offers and the retailer borrows trade credit. In equilibrium, \(w_r = 1\),
The proof is straightforward by comparing the expected profits of each firm under the two financing schemes (see Propositions 1 and 2) and is omitted. Proposition 2 shows that when the manufacturer extends and the retailer borrows trade credit, the former charges wholesale price 1 and the latter stocks $H$ units. The size of the loan is thus $H$. When the probability of high demand is low ($\alpha < \alpha_0$), the retailer is more likely to default, and the manufacturer prefers not to offer trade credit. In this case, the retailer borrows from the bank system and stocks a lower inventory ($L$), and the lending bank absorbs the retailer's default risk. When the probability of high demand is relatively high ($\alpha \geq \alpha_0$), the retailer's risk of default is low enough so that the manufacturer is better off by extending trade credit and thus bearing the retailer's default risk. In this case, the retailer stocks a higher inventory, $H$, the inventory in a centralized supply chain. Trade credit improves the efficiency of the chain by eliminating double marginalization, which in turn allows the manufacturer to extract greater surplus.

4. Robustness check: a continuous demand distribution

In a model of discrete demand distribution, we have shown that the manufacturer offers trade credit when marginal production cost is low (relative to demand parameter $\alpha$) and does not otherwise. We now examine whether a similar result continues to hold when product demand is continuously distributed. For simplicity, we focus on the Uniform distribution over $[0,1]$.

First, suppose that the manufacturer does not extend trade credit and the capital constrained retailer finances with bank credit. For a given wholesale (cash) price $w$, the retailer's objective becomes

$$
\pi^B_w(Q) = \int_0^Q (x - w(1 + r)Q)dx + \int_Q^1 (Q - w(1 + r)Q)dx
$$

subject to the banks' zero-profit condition

$$
wQ = \int_0^{(1+r)Q} xdx + \int_{(1+r)Q}^1 w(1 + r)Qdx.
$$

Substituting the latter equation into the former converts the retailer's inventory decision into one without capital constraint:

$$
\pi^B_w(Q) = \int_0^Q xdx + \int_Q^1 Qdx - wQ.
$$

We can readily check that the retailer's optimal inventory level is $Q_a = 1 - w$. To maximize its profits $(w - c)(1 - w)$, the manufacturer will choose wholesale cash price $w_a = (1 + c)/2$. The equilibrium inventory level is thus $Q_b = (1 - c)/2$. The bank's zero-profit condition reduces to $1 = (1 + r)[1 - (1/2)w_a(1-w_a)]$, from which we obtain the bank's interest rate $r = 4(1 - \sqrt{(1+c^2)/2})/(1-c^2) - 1$. The equilibrium profits of the manufacturer and the retailer are $\pi^M_w = (1-c)^2/4$ and $\pi^R_w = (1-c)^2/8$, respectively.

Next, consider the case where the manufacturer extends and the retailer borrows trade credit. For a given wholesale price $w$, the retailer's profit function is

$$
\pi^B_T(Q) = \int_0^Q (x - wQ)dx + \int_Q^1 (Q - wQ)dx.
$$

It follows that the retailer's optimal inventory is $Q_T = 1/(1 + w)$.

Under trade credit the manufacturer's profit function is
\[
\pi^M_T (w) = \int_0^{Q_T} x dx + \int_{Q_T}^w wQ_T dx - cQ_T \\
= \frac{w-c}{1+w} \frac{w^2}{2(1+w)^2}.
\]

Since \( \frac{d\pi^M_T}{dw} = \frac{1+c(1+w)}{(1+w)^3} > 0 \), the optimal delayed wholesale price is \( w_r = 1 \). The equilibrium inventory is thus \( Q_r = \frac{1}{2} \). Under trade financing, the firms' equilibrium profits are \( \pi^M_T = \frac{3-4c}{8} \) and \( \pi^B_T = 0 \). When the wholesale price is \( w = 1 \), the retailer makes zero profits by financing via bank credit and thus has no incentive to opt for bank credit.

Comparing each firm's profits under the two financing schemes readily yields the next result.

**Proposition 4.** (1) When \( \sqrt{2}/2 < c < 1 \), the manufacturer does not offer trade credit and the retailer uses bank credit. In equilibrium, \( w_B = (1+c)/2 \), \( Q_B = (1-c)/2 \), \( r = 4(1-\sqrt{(1+c^2)/2})/(1-c^2)-1 \), \( \pi^M_B = (1-c)^2/4 \) and \( \pi^B_B = (1-c)^2/8 \). (2) When \( c \leq \sqrt{2}/2 \), the manufacturer offers and the retailer uses trade credit. In equilibrium, \( w_T = 1 \), \( Q_T = \frac{1}{2} \), \( \pi^M_T = \frac{3-4c}{8} \) and \( \pi^B_T = 0 \).

Therefore, trade credit is the unique financing equilibrium when marginal production cost is relatively low \( (c \leq \sqrt{2}/2) \). The reason is still that in such a case trade credit effectively mitigates double marginalization compared with the use of bank credit. The improved overall efficiency in the supply chain makes the manufacturer better off while not making the retailer worse off.

**5. Conclusion**

The extant economic literature has provided several plausible theories of trade credit. The current article adds to this stream of literature by pointing out its role in containing double marginalization in a supply chain. When the capital-constrained retailer finances through a competitive, risk-neutral bank market, it behaves as if it had sufficient capital endowment, because the lending bank makes zero expected profits. The effect of double marginalization thus remains the same as when the retailer faces no capital constraint. In contrast, trade credit effectively mitigates double marginalization when the marginal production cost of the manufacturer is sufficiently low (relative to market demand parameters). Consequently, bank and trade credit are financing equilibrium under mutually exclusive sets of conditions.

**6. Appendix**

**Proof of Proposition 2.** We compare the manufacturer's expected profits under the following five possible cases.

(1) When \( w \leq \min\{\alpha, L/H\} \), the retailer chooses \( Q = H \) and the manufacturer's profits reduce to \( \pi^M_T (w) = (w-c)H \). In this case the optimal wholesale price is \( w = \min\{\alpha, L/H\} \), with manufacturer profits \( \pi^M_T (w) = (\min\{\alpha, L/H\}-c)H \).

(2) When \( w > L/H > \alpha \), the retailer chooses \( Q = L \) and the manufacturer's profits are \( \pi^M_T (w) = (w-c)L \). The optimal wholesale price is \( w = 1 \), with manufacturer profits \( \pi^M_T (w) = (1-c)L \).

(3) When \( w > \alpha \geq L/H \), the retailer chooses \( Q = H \) and the manufacturer's profits are \( \pi^M_T (w) = (aw-c)H + (1-\alpha)L \). The optimal wholesale price is \( w = 1 \), with manufacturer profits \( \pi^M_T (w) = (\alpha-c)H + (1-\alpha)L \).

(4) When \( L/H \geq w > \alpha \), the retailer
chooses $Q = L$ and the manufacturer's profits are $\pi^M_r(w) = (w-c)L$. The optimal wholesale price is $w = L/H$, with manufacturer profits $\pi^M_r(w) = (L/H-c)L$.

(5) When $\alpha \geq w \geq L/H$, the retailer chooses $Q = H$ and the manufacturer's profits are $\pi^M_r(w) = (\alpha w-c)H + (1-\alpha)L$. The optimal wholesale price is $w = \alpha$, with manufacturer profits $\pi^M_r(w) = (\alpha^2-c)H + (1-\alpha)L$.

We note that cases (1) and (5) are strictly dominated by case (3), and that case (4) is strictly dominated by case (2). Therefore, we only need to compare cases (2) and (3). In both of these cases, the manufacturer sets a wholesale price $w = 1$ and the retailer makes zero expected profits. By our tie-breaking rule, the retailer will choose inventory $Q = H$, leading to manufacturer profits $\pi^M_r(w) = (\alpha-c)H + (1-\alpha)L$. Q.E.D.

7. References