A Stochastic Model for Implementing Postponement Strategies in Distribution Networks

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Abstract

An essential aspect of modern logistic concepts is the question how to deal with demand uncertainties. One concept that has been proposed but not yet fully explored in this context is postponement. In this concept the finalization of a product is procrastinated, i.e., the final products are not completed in single factories but in facilities in a distribution network that are located on the itinerary from factories to customers.

Up to now, mainly global evaluations of pros and cons of postponement strategies are available. In order to allow for decision making for specific postponement implementations including its various effects in distribution networks, a double-stage stochastic mixed integer linear programming (MIP) model is presented in this paper. This model supports managers to find out appropriate implementations of postponement strategies to manage demand uncertainties.

By solving a set of problem instances it becomes obvious that the presented model formulation can be used to obtain solutions with (commercially) available mathematical programming software.

1. Introduction

Different challenges like, e.g., a higher diversity of variants, shorter product life cycles or gradually less predictable customer demands have led to considerations of transferring manufacturing steps of a product towards the end of a supply chain. This approach is called postponement. Following this concept, a product is not necessarily finished at the production site. Instead different manufacturing steps, like the assembly of some components or the packaging may be executed while the product is shipped to the customer. In this way it is possible to carry out delayed adjustments concerning the customer demand by transferring the product as a standard version from the production site into the distribution network where it is client-specifically individualized at a later stage.

This approach is suggested for various types of industries like, e.g., manufacturers of computers and computer components as well as the apparel industry, the food industry, or retail.

This paper examines in which way certain production steps – especially final assembly and packaging – can be relocated from the production sites to certain stages within the supply chain. Therefore, a distribution network which consists of production sites, central warehouses, regional warehouses and sales regions is examined.

By means of a mixed integer linear programming (MIP) model a cost-minimum solution is determined. This solution specifies the number and location of different types of potential facilities to be selected from a set of possible candidates within the examined distribution network in order to satisfy the given demands. Furthermore, solutions also indicate where postponement strategies should be implemented. One characteristic of the introduced MIP-model is the assumption of stochastic demands which leads to the formulation of a multi-stage stochastic MIP-model.

The solution of some test instances demonstrates the applicability of the presented model and, as anticipated, different levels of uncertainty result in different solutions. Furthermore, the impact of different ratios of fixed and variable costs on the solutions is considered.

The presented model enables decision-makers to free themselves from global evaluations of different types of postponement strategies and renders decision-making for specific problems possible despite the contradictory effects of implementing postponement strategies.
The remainder of the paper is organized as follows. In Section 2, a brief literature review on postponement as well as the design of distribution networks is given. In Section 3, a specific distribution network incorporating postponement approaches is depicted in detail. Section 4 contains a mathematical formulation of the considered problem. Furthermore, the method of including uncertainty in the model is explained. Section 5 goes into detail about the model parameters and the test instances. Section 6 contains an analysis of the solutions. A summary as well as an outlook to further research are given in Section 7.

2. Literature review

Postponement is widely regarded as an approach that may result in superior supply chains (see, e.g., Cooper [13] or Jones and Riley [22]), and it has been recognized as a growing trend in manufacturing and distribution (see Skipworth and Harrison [37]). According to Yang and Burns [43], much has been written in the literature on the benefits of postponement, yet little is known about its implementation.

Extensive investigations of benefits of postponement as well as postponement strategies have been carried out in the context of marketing and logistics as well as supply chain management. Two early papers in this context are due to Alderson [2] and Bucklin [9]. Alderson appears to be the first who coined this term, and an early extension of the concept is presented by Bucklin. Alderson regards postponement from the marketing point of view as promising for reactions towards demand uncertainties in order to reduce costs. Bucklin broadens this concept to the distribution channel and raises the question of where, when and how inventories can be stocked to trigger cost reductions.

A seminal work in postponement classification is the fundamental classification of Zinn and Bowersox [45]. They introduce labeling, packaging, assembly and manufacturing postponement which are based on the type of the postponed manufacturing operation and can be characterized as form postponement. Time postponement occurs during transportation and means that the forward movement of inventories is delayed. Labeling postponement means that products which are technically identical but that are, nonetheless, launched under different brand names leave the production site without labels. The labeling is done at a later stage in the supply chain, e.g., if respective customer orders have been received specifying which products are demanded in which quantity. During packaging postponement the goods are at first bulk shipped to the warehouses and then packed customer-specifically. Assembly postponement means that a standard product is sold in different variants. The different variants are only differentiated in a single detail, e.g., the color of component. The standard product is not differentiated ex works, but only later within the supply chain when the diversified part is installed. Manufacturing postponement can be distinguished from assembly postponement in its extent, as more than one installation is relocated from the production site to the warehouses. While assembly postponement contains only the execution of one production step on the goods, the extent of production steps is larger for manufacturing postponement. Additionally, different components of the products can be received from different sites and combined at the warehouses. Time postponement does not mean that goods are transported to the warehouses based on a forecast, but that products are shipped to customers only following order receipt which results in central inventories.

Besides the frameworks and classification mentioned above, many further postponement strategies as well as conceptual investigations of benefits of postponement are presented by Cooper [13], Feitzinger and Lee [16], Pagh and Cooper [31], van Hoek [40, 41] and Yang and Burns [43] as well as Mikkola and Skjøtt-Larsen [28]. Whereas these studies are primarily qualitative, diverse papers focus on quantifying the benefits and criteria of various postponement strategies. See, e.g., Appelqvist and Gubi [5], Ernst and Kamrad [14], Garg and Tang [18], Billington and Carter [26], Ma et al. [27], Skipworth and Harrison [37], Swamianthan and Tayur [38] or Yeh and Yang [44].

Models and methods for distribution planning are available in operations research since its early days, in particular for warehouse location, but also for more comprehensive design problems regarding multiple products, limited capacities, single resource constraints or nonlinear transportation costs (see, e.g., the reviews of Aikens [1], Klose and Drexl [25] or Owen and Daskin [30]). Geoffrion and Graves [19] were among the first to investigate the use of intermediate distribution. They present a model to solve the problem of designing a distribution system with an optimal location of intermediate distribution facilities between factories and customers. A MIP formulation with the objective of maximizing the total after-tax profit for manufacturing facilities and distribution centers is presented by Cohen and Lee [11]. The model determines the optimal deployment of resources associated with a particular policy option. The considered product structure in the model encompasses three levels (major components, subassemblies, finished products). Extending the model of Cohen and Lee, Cohen and Moon [12] investigate effects of various parameters on supply chain costs and determine which manufacturing facilities and distribution centers should be established.
Fleischmann [17] developed a multi-commodity three-stage network flow model with arbitrary nonlinear transport and warehouse costs which may include fixed costs. In contrast to common models, location decisions are not determined by integer programming or add/drop procedures, but result from the solution of a network flow problem. A MIP model presented by Pooley [33] allows for deciding where to locate factories and depots, allocating the production as well as how to serve customers. A MIP global supply chain model presented by Arntzen et al. [6] determines the number and location of distribution centers, customer-distribution center assignments, the number of tiers, and the product-factory assignment. Camm et al. [10] developed an integer model for finding the location of distribution centers and to assign those selected to customer zones. Amiri [4] presents a model that takes into consideration different capacity levels. A tri-echelon multi-commodity system incorporating production, transportation and distribution planning is considered by Pirikul and Jayaraman [32]. In a succeeding work, Jayaraman and Pirikul [21] present a model that determines the location of a number of production plants and distribution centers. Further models of distribution networks with several layers are also presented in, e.g., Ambrosino and Scutellà [3], Klincewicz [24], or Tsiaakis et al. [39]. A distribution network model taking into account mode selection, lead times as well as capacitated vehicle distribution centers is proposed in Eskirgun et al. [15]. Kaclesis et al. [23] developed a generic strategic planning and design model for global supply chains which captures essential elements of many industrial environments. Further papers concerning distribution networks can be found in comprehensive reviews of, e.g., Bilgen and Ozkarahan [7], Goetschalckx et al. [20] or Vidal and Goetschalckx [42].

None of the papers discussed above explicitly deals with the implementation of postponement strategies in the context of planning a distribution network, and there are only few papers, e.g., the papers of Arntzen et al. [6], Cohen and Lee [11] or Cohen and Moon [12] that rudimentarily combine aspects of postponement and distribution network design. Two papers that comprehensively take into account the aspects of postponement and distribution network design are presented by Schwartz and Voß [35, 36]. In these two papers deterministic demands are examined. However, while Alderson [2] initially devised the idea of postponement as a reaction to demand uncertainties, respective models considering stochastic demands still need to be developed. Such a model is presented in the subsequent sections.

3. A distribution network with postponement and stochastic demand

In this paper a distribution network with several facilities at different tiers of the network is considered. The network structure is based on the structure presented by Schwartz and Voß [35]. However, in order to broaden the existing approach, here stochastic in lieu of deterministic demands are assumed. Different products are delivered from factories to satisfy the stochastic demands of several customer zones. Figure 1 shows an exemplary distribution network with factories, central warehouses, regional warehouses and customer zones. The arrows represent potential flows of the products from the factories up to the customer zones. Typically, the goods flow from factories to central warehouses, from central warehouses to regional warehouses and then to the customer zones. The locations of central and regional warehouses are unknown and have to be selected from a set of candidate locations. Furthermore, there is also the alternative to ship goods directly from factories to regional warehouses, from central warehouses to customer zones as well as from factories up to customer zones.

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from factories to customers. Here this assumption is relaxed such that activities to finish a product could be performed later, or rather postponed along the route from the factories up to the customers. Potential locations for postponed activities are both central and regional warehouses.

In the considered distribution network planning problem two types of postponement strategies are to be established: Packaging postponement as well as assembly postponement (see, e.g., Cooper [13]). That is, products may eventually be specified according to either the final assembly step or even the packaging. Especially for regions with large varieties in customer demands this may allow additional degrees of freedom. Thus, Yang and Burns [43] postulate that in an environment with extreme demand uncertainty manufacturers may derive significant economic benefit from locating production geographically closer to customers.

Regarding the implementation of the corresponding processes to assemble unfinished goods respectively to pack finished goods, the required resources can be installed on every tier of the distribution network, namely in the factories, in the central warehouses, or in the regional warehouses. Feasible routings through the distribution network are depicted in Figure 2.

Postponement may result in considerable cost tradeoffs. Regarding the considered assembly and packaging postponement strategy, processing costs for assembly and packaging, respectively, increase and the transportation costs decrease. Increased per unit assembly costs as well as increased per unit packaging costs result from reduced economies of scale in the concerned warehouses in contrast to assembly and packaging in a central plant. Regarding unfinished products that have yet to be assembled, the reduction of transportation costs is typically due to a better density ratio of unassembled products compared to assembled products. In the context of packaging products, a reduction of transportation costs results from bulk shipping finished, but not packed products from factories to central or regional warehouses or from central warehouses to regional warehouses.

Taking into account stochastic demands may be more cost-intensive as capacities have to be installed that are only used if accidentally higher demands occur. Facilities installed at the production site usually have higher fixed costs and lower costs per unit as decentralized facilities in central or regional warehouses. These are basically installed to handle smaller lot sizes and are characterized by lower fixed costs, but higher variable processing costs. While following postponement strategies it might happen that the variable costs for the final product increase. This process is, nevertheless, accompanied by visibly reduced fixed costs for the high-capacity facilities which are not required at the production sites if postponement strategies are implemented.

The decisions to be determined in the presented distribution network design problem represent strategic decisions and include decisions concerning the number and location of potential central and regional warehouses to be established as well as the decision where to implement assembly and packaging functions in the distribution network. As mentioned above, possible locations for assembly and packaging functions are factories, central warehouses and regional warehouses. Furthermore, decisions regarding quantities of products shipped between facilities have to be made. The objective is to minimize the combined total costs of the network for a set of scenarios of stochastic demand of several customers, taking into account both fixed infrastructure and variable operating costs.

Figure 3 represents a consolidated view of Figures 1 and 2. For reason of clarity there is only one facility displayed at each tier. Further potential facilities in alternative locations at a tier as well as the correspond-
ing flows are omitted. The displayed flows in Figure 3 comprise flows between facilities (shipping goods from one location to another) as well as flows within facilities (processing goods). In every facility displayed in Figure 3, various functions that are allowed to be implemented are shown. These functions include the function “no action” that simply refers to handling products without performing any activities like, e.g., assembly or packaging. At the second tier, the activity “no action” is represented by three nodes. This is due to the requirement to distinguish products with different completion status in a facility. The ability to distinguish products with different completion status is needed again both to determine the allowed succeeding activity in the distribution network, and to calculate the correct capacity consumption in the corresponding facility.

4. Model formulation

In this section we present a two-stage stochastic model. On the first stage of this model decisions are made about the implementation of functions in different locations, and on the second stage decisions are made about the transport amounts between the functions (see Figure 4).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Decisions in a two-stage stochastic model}
\end{figure}

Such an optimization model can be presented in a generalized manner as follows (for some general exposition of stochastic programming see, e.g., Birge and Louveaux [8]):

\begin{align*}
\min z &= c^T x + E_{\omega}[\min d(\omega)^T y(\omega)] \\
\text{Subject to} & \quad Ax = b \\
& \quad T(\omega)x + Wy(\omega) = h(\omega) \\
& \quad x \geq 0, y(\omega) \geq 0
\end{align*}

The objective function represents the minimization of costs $c^T x$ at the first stage as well as the expected costs $d^T y$ from decisions made at the second stage while meeting the first stage constraints $Ax=b$. The costs $d^T y$ depend both on $x$, the first stage decision, and on the uncertain second stage with defined probability for all considered scenarios $\omega \in \Omega$. The minimization of $d^T y$ chooses with $y(\omega)$ a different decision for every scenario $\omega$, subject to some recourse function $Tx + Wy = h$. This constraint can be characterized as a correction of the system after a random event occurs. The different scenarios are specified by parametrical variations within the objective function as well as the constraints. In the presented model, however, the different scenarios are constituted by a variation of demands which consequently corresponds to a variation of constraints.

The notation used in the mathematical formulation is as follows:

- $S$ number of tiers
- $Z$ index set of products
- $L_\omega$ index set of different functions at tier $s$
- $FCT_s$ index set of different as well as identical functions for products with different degrees of completion at tier $s$
- $LOC_s$ index set of potential locations of facilities at tier $s$
- $SL^k_s$ index set of allowed sequences of potential consecutive functions at tiers $s$ and $s+k$, $k=1,2,3$
- $\Omega$ index set of scenarios

The decision variables are:

- $y_{\omega sfi}$, if function $f$ is established in facility $i$ at tier $s$; 0 otherwise; first stage decision
- $x_{zsf\omega}$ quantity in scenario $\omega \in \Omega$ of product $z$ shipped from a facility at candidate location $i$ at tier $s$ to a facility at candidate location $j$ at tier $s+k$, performing function $f$ in facility $i$ and function $g$ in facility $j$, $k=1,2,3$; second stage decision
- $\lambda^f_{zitf\omega}$ quantity in scenario $\omega \in \Omega$ of product $z$ processed in a facility at candidate location $i$ at tier $s$ by the implemented function $f$; second stage decision

Below the parameters of the model are given:

- $c^f_{zsf}$ shipping cost per unit and per unit distance for a product $z$ processed by function $f$ at tier $s$
- $d^f_{sij}$ distances between facility $i$ at tier $s$ and facility $j$ at tier $s+k$, $k=1,2,3$
- $c^f_{zitf}$ processing cost for product $z$ of an implemented function $f$ in a facility at candidate location $i$ at tier $s$
Flows entering a facility

\[ x_{zsjfg\omega}^j = \sum_{k=1}^{i} \sum_{(j,k) \in \mathcal{SF}_{i,j}^k} x_{zsjfg\omega}^{j} \quad \forall z \in Z, s = 2,3, j \in \text{LOC}_s, g \in \text{FCT}_s, \omega \in \Omega \]  

Demand

\[ \text{Dem}_{z\omega} = \sum_{k=1}^{i} \sum_{(j,k) \in \mathcal{SF}_{i,j}^k} x_{zsjfg\omega}^{j} \quad \forall z \in Z, s = S, j \in \text{LOC}_s, g \in \text{FCT}_s, \omega \in \Omega \]  

Flows leaving a facility

\[ x_{zsjfg\omega}^j = \sum_{k=1}^{i} \sum_{(j,k) \in \mathcal{SF}_{i,j}^k} x_{zsjfg\omega}^{j} \quad \forall z \in Z, \]  

Supply

\[ \text{Sup}_{z} \geq \sum_{f \in \text{FCT}_s} x_{zsjfg\omega}^j \quad \forall z \in Z, s = 1, i \in \text{LOC}_s, \omega \in \Omega \]  

Capacity constraints

Transport capacity

\[ \sum_{z \in Z} \frac{x_{zsjfg\omega}}{U_{zsj}^{ij}} \leq 1 \quad \forall s \in \{1, 2, \ldots, S - k\}, (i, j) \in \mathcal{SL}_i^k, (f, g) \in \mathcal{SF}_x^k, k = 1, 2, 3, \omega \in \Omega \]  

Facility capacity

\[ \sum_{z \in Z} \frac{x_{zsjfg\omega}}{U_{zsj}^{ij}} \leq y_{sij} \quad \forall s \in \{1, 2, \ldots, S - 1\}, i \in \text{FCT}_s, l \in L_i, \omega \in \Omega \]  

Furthermore, there exist non-negativity constraints for all decision variables, namely the variables \( x_{zsjfg\omega}^{j} \) with \( k = 1, 2, 3 \) as well as \( x_{zsjfg\omega}^{j} \). The variables \( y_{sij} \) are binary variables.

Objective function (1) consists of three cost types which are all assumed to be linear. At first, the objective function encompasses variable shipping costs between the locations of facilities. A further type of costs in the objective function is induced by variable processing costs at the locations of the facilities. Activities taken into account that could be implemented at the considered facilities are assembly, packaging, both assembly and packaging in combination, as well as a simple handling of products in a facility without executing any further activities. The third cost type represents infrastructure costs with a fixed cost character for establishing the considered functions in the facilities. The decisions regarding these functions represent decisions of the first stage in the stochastic model, whereas the decisions with respect to the variable costs represent decisions of the second stage.

Constraint set (2) represents material balances. The flow entering a facility must equal the quantity that is processed in this facility. Analogously, constraint set (4) declares material balances which ensure that the flow leaving a facility has to be as high as the quantity that is processed there. Constraint set (3) assures that
the demand of every customer zone for each product type is met. The demand is uncertain, hence it represents the stochastic parameter in the developed model. Constraint set (5) restricts the material flows leaving the factories to their maximum quantity they are able to supply. With constraint set (6), the maximum quantity of products that can be shipped from one location to another one is incorporated into the distribution network model. The total capacity consumption by shipping different products along the same link is calculated by a linear combination of the capacity consumption of individual products. Constraint set (7) represents a similar approach regarding the maximum quantity that can be processed by an implemented function within a facility. The binary variables in constraint set (7) indicate whether a specific function should be established in a facility or not. Furthermore, the constraints also permit calculating the capacity consumption in a facility if the products processed with the implemented function in the facility feature different completion status (see Figure 3 and its explanations in Section 3).

5. Input data

Pilot tests of the developed model have shown that solutions are highly dependent on the values of the parameters, and thus, on the considered problem. This means that it is impossible to make a global statement about the benefit of postponement and whether postponement strategies should be pursued or not.

The following assumptions are made for choosing the parameter values:
1. It is more favorable to execute several production steps at one site than at different sites (see Yang and Burns [43]).
2. The costs for executing one task also increase with augmented customer focus (see Lee et al. [26]).
3. The longer the production process is proceeded the higher are the transport costs, e.g., because of impractical package proportions.
4. Transport capacities are infinite, i.e., the transport amounts are not limited.
5. Transport itineraries exist between all sites and functions.
6. Distances between tiers are assumed to be one.
7. The demand follows a normal distribution.

The mean of demand and the quantity of supply in every factory, respectively, is identical for each product in all scenarios. The variable costs for combined assembly and packaging activities exceed the variable costs for separate assembly or packaging activities. These variable costs as well as the variable transport costs increase with the degree of completion of the products.

For performing the test-runs different scenarios are generated. Each scenario can be characterized by different normally distributed demands with mean \( \mu \) and standard deviation \( \sigma \). The generation of demands takes place on the basis of a discretized normal distribution. The assumption of the normal distribution can be constituted by describing random variables which are grouped around an average and independent from each other. Because of the different levels of discretization the normal distribution has different precise approximations, see Figure 5. The number of generated scenarios corresponds to the number of discretized steps (for theoretical background about statistics see, e.g., Ross [34]).

![Figure 5. Discretized normal distribution with 7 and 101 scenarios](image)

6. Numerical results

This section presents the solutions of the applied problem instances. The modeling language is the Mathematical Programming Language (MPL); data is saved in a Microsoft Access database. The scenarios are created by a VBA script which saves them in the database. Access and MPL interact in a way that the solution process of the problem instances can be directly solved by Access. The results of the optimization runs are inscribed by MPL directly into the data base. Finally, a tool created with C# 3.5 visualizes the solution. The solution is generated by the solver MOPS 9.29 (see [29]) on a Windows PC (Athlon 64 Dual Core 5200+, 2.7 GHz, 4 GB Ram, Windows Vista 64 Bit).

At first a deterministic problem instance is solved for mere comparative purposes. The solution of this problem instance is \( z^* = 2581000 \). The model comprises 475 decision variables and 157 restrictions. Only a small number of decision variables are binary variables and, therefore, integer variables. The import time of the model amounts to 0.637 seconds, the solution time by MOPS to 0.09 seconds.

In Table 1 the cost components of the solution are depicted. In addition, the solution of the same problem instance without postponement is depicted, too. By comparing these solutions it becomes obvious that a postponement strategy is beneficial. Nonetheless, it
should be noted that a different ratio between fixed and variable costs might lead to an alternative decision, i.e., that assembly and packaging remain in the factories and no postponement strategy is implemented. This assumption was validated by the computation of a problem instance with reduced shipping costs per unit distance.

**Table 1. Solutions and their cost components for problem instances incl./excl. postponement**

<table>
<thead>
<tr>
<th>Postponement</th>
<th>Fixed Costs</th>
<th>Processing Costs</th>
<th>Shipping Costs</th>
<th>Solution Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incl.</td>
<td>1301000.00</td>
<td>480000.00</td>
<td>800000.00</td>
<td>2581000.00</td>
</tr>
<tr>
<td>Excl.</td>
<td>1000000.00</td>
<td>320000.00</td>
<td>1440000.00</td>
<td>2760000.00</td>
</tr>
</tbody>
</table>

Next, the solution of a problem instance that takes into account stochastic demands is presented. The demands correspond to an approximated normal distribution with the mean $\mu=20000$ and the standard deviation $\sigma=100, 1000, 2000$ and 4000. This means that an increased demand variability with a higher $\sigma$ effects higher maximum demands (with low probabilities) than a light demand variability with a lower $\sigma$. The solutions are given in Table 2. Due to the uniformity of the results and limited space, only the solutions of test instances that are generated by a discretization of a normal distribution with 51 scenarios are shown. For all solutions of these test instances postponement strategies seem to be beneficial.

**Table 2. Solution of four problem instances with 51 scenarios and different $\sigma$, only packaging is allowed to be postponed**

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Fixed Costs</th>
<th>Processing Costs</th>
<th>Shipping Costs</th>
<th>Solution Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1301000.00</td>
<td>480000.28</td>
<td>800005.47</td>
<td>2581008.75</td>
</tr>
<tr>
<td>1000</td>
<td>1301000.00</td>
<td>480005.52</td>
<td>800009.21</td>
<td>2581014.73</td>
</tr>
<tr>
<td>2000</td>
<td>1301000.00</td>
<td>479995.39</td>
<td>799999.32</td>
<td>2580987.71</td>
</tr>
<tr>
<td>4000</td>
<td>2301000.00</td>
<td>479552.27</td>
<td>801828.67</td>
<td>3582830.94</td>
</tr>
</tbody>
</table>

The table shows that different $\sigma$ of about 100, 1000 and 2000 do not effect any alteration of the solution compared to the deterministic scenario, which means that in one regional warehouse a combined assembly and packaging for all products takes place. Only with $\sigma=4000$ the (highlighted) solution is altered. This alteration can be explained by capacity bottlenecks in a production site that occur for some instances with extraordinary demand – and for all instances there have to be feasible solutions. These bottlenecks would have to be mitigated by establishing an additional production site. This production site incorporates both assembly and packaging activities. According to Table 2 there will be a considerable increase of fixed costs resulting from this circumstance. The marginal divergences in the solution are roundoff errors of the solution process and the discretization of the normal distribution.

In our next analysis we limit the postponement concept to packaging activities. The results of this study are given in Table 3. Apart from roundoff errors, the computational results for $\sigma=100, 1000$ and 2000 are identical. Despite the possibility to establish the packaging activities somewhere else than in the production sites, the production sites both incorporate assembly and packaging activities. No postponement takes place in this situation. In case of $\sigma=4000$, the solutions changed analogously to the former experiment. Again, due to potential bottlenecks that may be effected by the increased variation of demand, another production site is established. Furthermore, Table 3 shows that – compared to Table 2 – the fixed costs as well as the processing costs decreased whereas the shipping costs and the total costs increased.

**Table 3. Solution of four problem instances with 51 scenarios and different $\sigma$, only packaging is allowed to be postponed**

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>Fixed Costs</th>
<th>Processing Costs</th>
<th>Shipping Costs</th>
<th>Solution Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1000000.00</td>
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<td>1440009.84</td>
<td>2760012.03</td>
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<tr>
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<td>320003.68</td>
<td>1440016.57</td>
<td>2760020.26</td>
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<tr>
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<td>319996.93</td>
<td>1439986.17</td>
<td>2759983.10</td>
</tr>
<tr>
<td>4000</td>
<td>2000000.00</td>
<td>320004.44</td>
<td>1440019.98</td>
<td>3760024.42</td>
</tr>
</tbody>
</table>

The problem sizes as well as the solution times of the problem instances are presented in Table 4. The column MPL-Time specifies the loading time of the model by MPL. In column Sol-Time the solution time of the MOPS 9.29 solver is presented with its standard options. It becomes visible that a problem like the given one can also be solved under consideration of stochastic influences like stochastic demands in a viable time. Furthermore, it becomes clear that the solution time, nevertheless, increases while using a finer discretized distribution function, but that the solution quality does not increase.

**Table 4. Problem sizes and solution times of four problem instances**

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Variables</th>
<th>Restrictions</th>
<th>MPL-Time</th>
<th>Sol-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3127</td>
<td>1099</td>
<td>3.799</td>
<td>0.619</td>
</tr>
<tr>
<td>11</td>
<td>4895</td>
<td>1727</td>
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</tr>
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</table>

7. Conclusions

In the presented paper the problem of establishing postponement strategies in a distribution network made
up of four tiers and with stochastic demand is studied. The developed mixed integer programming model is a stochastic two-stage model that enables the decision where to establish central and regional warehouses, where to implement potentially postponed functions for assembly and packaging in the distribution network (decisions of the first stage) and which quantities should be shipped from one facility to another one (decisions of the second stage). Several scenarios are conceived and the corresponding test instances are solved.

The presented model enables decision-makers for the first time to evaluate the question of whether and at which location in a distribution network resources for the realization of a postponement strategy shall be implemented with the help of a quantitative model which also gives insight to stochastic demands. The significant virtue of this approach is the possibility to calculate with data of real problems. Decision-makers are no longer dependent on trend statements about profit and applicability of the corresponding concepts. This is a fortiori important regarding the large differences of the solutions with subject to their cost structure.

The proposed stochastic model can be regarded as a basis for further research. Thus, several aspects should be taken into account in future work, e.g., nonlinearity of costs, diverse capacity levels, alternative technologies for shipping as well as processing in the facilities, an observation across several periods (dynamic model) and thus considering inventories, explicitly taking into account bills of material, capital commitment, as well as insurance contributions. Moreover, implications of incorporating our model into advanced planning systems need to be explored.

8. References


