Abstract

In Internet based commerce, sellers often use multiple distribution channels for the sale of standard consumer goods. We study a model of second degree price discrimination in which a monopolist sells to risk-averse buyers. The seller uses two channels that differ in their risk attributes. In one channel prices and qualities are fixed and availability is assured. In the second channel, the seller offers a joint-distribution of prices and qualities and may not guarantee availability. We characterize optimal two-channel selling policies. We show that it can be optimal to offer multiple identical items in a random sale event. However, the seller cannot benefit by offering two distinct quality levels in a sale event that is held with a probability less than one.

1. Introduction

In Internet based commerce, sellers often use multiple distribution channels for the sale of standard consumer goods. For example, within a two week period, Carnival Cruise Lines offered units of the same cabin class on one of its ships in three distinct ways: standard posted prices, ascending bid (“English”) auctions and “last minute” clearance sales. A possible explanation for such behavior is that sellers like Carnival are deliberately embedding price uncertainties into their sales channels in order to employ second-degree price discrimination among buyers who are risk-averse. Buyers who assign higher values to the offered product are typically more reluctant to risk compromising their surplus and are therefore prone to purchase at a higher posted price earlier, while buyers with lower values may wait and attempt to acquire the product at a bargain price.

According to our observations, US based air carriers such as American and Delta Airlines offer special “last-minute” Internet fares only on travel in economy-class. However, some retail travel specialists such as lastminute.com and site59.com offer also high-end hotel accommodations and luxury cruise vacations in “last-minute” promotions. Can airlines benefit by offering special last-minute promotions on travel in business-class, and if so, how?

This paper provides a simple framework that permits the investigation of these issues. The analysis provides insights into the determination of profit-maximizing selling policies that involve the offering of products of distinct qualities though a mechanism that involves both advanced-purchase transactions and random “last-minute” sales.

A substantial body of literature examines the provision of quality by a discriminating monopolist. Of particular relevance to our discussion are works by Mussa and Rosen [12], Deneckere and McAfee [5], Johnson and Myatt [8], and Anderson and Dana [2]. A basic result that follows from all of these models is that when products' quality is endogenous and buyers self-select among quality-levels a discriminating monopolist will distort the provision of quality at the lower end of the quality spectrum. Interestingly, our theoretical results indicate that when buyers are risk averse the use of random “last-minute” sales can either alleviate or exacerbate such quality distortions, depending on the circumstances. Nonetheless, we show that when the degree of buyers’ risk aversion exceeds a threshold, a monopolist who uses “last-minute” sales can maximize her profits by offering only products of efficient quality.

A number of papers in the price-discrimination literature assume, as we do in this paper, that consumers rationally anticipate price reductions. Coase [4] has famously conjectured that when consumers have perfect foresight and are sufficiently patient a monopolist will set its price arbitrarily close to marginal cost. The reason is that at any given time period buyers understand the ex post profitability of reducing the price in subsequent periods and optimally wait for such reductions to materialize. Stokey [16] formalizes this result and shows that when consumers differ in their degrees of impatience a monopolist will
commit to a decreasing pricing path that induces the most impatient consumers to purchase first. Besanko and Winston [3] argue that a decreasing price path will result also in an alternative setting with subgame perfect equilibrium (SPE) that does not involve an explicit commitment to prices. Su [17] analyzed an SPE where consumers arrive at a constant rate over time to the point of sale and individually decide whether to purchase immediately, or wait. He shows that the monopolist will dynamically adjust its prices downward over time whenever buyers’ waiting costs are positively correlated with their values for the good.

Our model, however, does not consider buyers’ disutility from waiting. Rather, we describe a setting where the consumption occurs at some point in time that is common to all buyers, and does not depend on the time of purchase; airline tickets and cruise vacations, for example, fit into this category. Accordingly, our results may apply to a lesser extent to goods where the product is not consumed immediately after purchase.

The potential segmentation benefits that may arise from incentive schemes yielding random outcomes have long been recognized. In two independent seminal studies, Matthews [11] and Maskin and Riley [10] characterize optimal auctions with risk-averse buyers under different sets of assumptions. While assuming that buyers have uniform utility functions and differ only in their valuation of a single item, both studies establish that the seller can devise a truth-revelation mechanism that strictly dominates any one-price scheme while inducing an equilibrium in which almost all buyers are faced with risk. With such an “optimal auction” every buyer is induced to reveal his value of the good; he is then assigned a schedule that includes a “bid submission” fee, a probability of winning the item, and an “acquisition price” to be paid only if the item is won.

In a related study, Marom and Seidmann [9] characterize optimal auctions with risk-averse buyers under different sets of assumptions. While assuming that buyers have uniform utility functions and differ only in their valuation of a single item, both studies establish that the seller can devise a truth-revelation mechanism that strictly dominates any one-price scheme while inducing an equilibrium in which almost all buyers are faced with risk. With such an “optimal auction” every buyer is induced to reveal his value of the good; he is then assigned a schedule that includes a “bid submission” fee, a probability of winning the item, and an “acquisition price” to be paid only if the item is won. In a related study, Marom and Seidmann [9] characterize optimal auctions with risk-averse buyers under different sets of assumptions. While assuming that buyers have uniform utility functions and differ only in their valuation of a single item, both studies establish that the seller can devise a truth-revelation mechanism that strictly dominates any one-price scheme while inducing an equilibrium in which almost all buyers are faced with risk. With such an “optimal auction” every buyer is induced to reveal his value of the good; he is then assigned a schedule that includes a “bid submission” fee, a probability of winning the item, and an “acquisition price” to be paid only if the item is won.

A monopolist sells two distinct products: a high-quality product A and a low-quality product B. The per-unit marginal production cost of product B is fixed and normalized at zero; the per-unit marginal production cost of product A is a positive constant $c$.

Customers are divided into two types H and L. There are $q_H > 0$ customers of type H and $q_L > 0$ customers of type L. A type-i customer’s demand ($i \in \{H, L\}$) is either for one unit of product A, one unit of product B, or for no product at all. The reservation values of type-H and type-L customers for a unit of product B are denoted by $V_H$ and $V_L$ respectively ($V_H \geq V_L$). The corresponding reservation values for product A are $V_H + \delta_H$ and $V_L + \delta_L$.

We make the following model assumptions:

1. $\delta_H \geq \delta_L > 0$.
2. $\delta_L \geq c > 0$.

This implies that if the monopolist were serving only the low types she would find it more profitable to sell them high quality. So the high-quality product A is efficient for both buyer types H and L.

Buyers care only about product and price and are completely indifferent as to the time of purchase. The utility function $U_i$ ($i \in \{H, L\}$) is defined as follows.

If a type-i customer buys A: $U_i = (V_i + \delta_i - p_A)^{1-\rho} \times V_i + \delta_i - p_A \geq 0$. 

Section 3 includes a few concluding remarks.

2. Model

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If a type-i customer buys B: \( U_i = (V_i - p_B)^{1-\rho} \), \( V_i - p_B \geq 0 \).

If a type-i customer buys no product at all: \( U_i = 0 \).

Customers of both types thus exhibit the same degree of constant relative risk aversion \( \rho \) (0 \( \leq \rho < 1 \)).

Suppose that the seller initially offers the good \( j \) (\( j \in A, B \)) at price \( p_{1j} \). If potential demand and production capacity are not totally exhausted, it pays for the seller to continue and then offer additional units at a lower price \( p_{2j} < p_{1j} \), targeting potential buyers that have so far chosen not to buy at \( p_{1j} \). Since buyers understand this, they anticipate the subsequent discount, and optimally wait for it. In the absence of a time deadline, similar considerations apply indefinitely. The seller's power to attain strictly positive profits depends on the possibility of a commitment, whereby some potentially profitable future decisions are inhibited. The ex-post opportunity loss (which appears to violate subgame perfection) is amply justified by the ex-ante profitability of the initial commitment.

With any non-degenerate pure strategy, all sales take place at the same time period. Recognizing the possibility (indeed inevitability) of commitment, we consider an alternative "two-period" selling strategy that involves randomization. We assume that the seller can commit to a strategy of the form

\[
S = (p_{1A}, p_{1B}, p_{2A}, p_{2B}; \alpha, \beta, \gamma)
\]

where:

- \( p_{kj} > 0 \) is the price of product \( j \) in period \( k \) (\( k = 1,2 \)).
- \( 0 \leq \alpha \leq 1 \) is the probability that product A alone is offered in period 2 (at price \( p_{2A} \)).
- \( 0 \leq \beta \leq 1 - \alpha \) is the probability that product B alone is offered in period 2 (at price \( p_{2B} \)).
- \( 0 \leq \gamma \leq 1 - \alpha - \beta \) is the probability that both products are offered in period 2 (at prices \( p_{2A}, p_{2B} \)).

In what follows, our main goal is to solve for the optimal selling strategy \( S \).

### 2.1. Randomization with a single quality level

In this section we consider a case where only one product is offered, without loss of generality let us assume that it is product B. In period 1, the price of product B is denoted by \( p_{1B} \). The seller commits to holding a second-period sale (at price \( p_{2B} \)) with probability \( \beta \). To save on notations we represent the seller's strategy \( S \) in this case by \( S = (p_{1B}, p_{2B}; \beta) \).

Let \( S \) be a strategy with profits \( \Pi = \Pi(S) \). If at the corresponding equilibrium both customer types purchase in the same period the seller can trivially attain or exceed \( \Pi \) with some strategy \( S' \) where \( \beta = 0 \). It therefore suffices to consider the case where one type (i.e. H) buys in period 1 and the other buys in period 2. The seller’s profit-maximization problem is thus equivalent to:

\[
\text{Max}_{S} \quad p_{1B}q_H + \beta p_{2B}q_L \quad \text{s.t.}:
\]

\[
\begin{align*}
V_H - p_{1B} & \geq 0, \\
V_H - p_{1B} & \geq \beta^{1-\rho}(V_H - p_{2B}), \\
V_L - p_{2B} & \geq 0, \\
\beta^{1-\rho}(V_L - p_{2B}) & \geq V_L - p_{1B}, \\
\beta & \in [0,1].
\end{align*}
\]

Conditions (1.1) and (1.3) are individual-rationality constraints for type H and type L customers, respectively. Condition (1.1) means that a type-H customer prefers to purchase product B in period 1 to not purchasing any product at all. Condition (1.3) implies that a type-L customer prefers to purchase product B whenever a sale is held in period 2 to not purchasing at all. Conditions (1.2) and (1.4) are incentive-compatibility constraints for the two customer types. Condition (1.2) implies that a type-H customer prefers to purchase product B in period 1 to deferring his purchasing decision to period 2. Condition (1.4) implies that the opposite holds true for a type-L customer.

In this problem condition (1.1) never binds at the optimum since it is necessarily satisfied by conditions (1.2) and (1.3):

\[
\frac{1}{\rho} (V_H - p_{2B}) \geq V_L - p_{1B}. \quad \beta \in [0,1].
\]

Where the second inequality sign holds because \( V_H \geq V_L \). With no loss of optimality, therefore, we omit condition (1.1). Similarly, condition (1.4) is always satisfied by condition (1.2) and is omitted. Condition (1.2) always binds at the optimum; otherwise the seller can raise \( p_{1B} \) until it does bind; this step does not affect the remaining constraints (1.3) and (1.5) while improving the maximand. As a result, it is always optimal to set,

\[
p_{1B} = V_H - \beta^{1-\rho}(V_H - p_{2B}) \leq V_H, \quad \beta \in [0,1].
\]

Note that the seller can always exclude L-type customers by setting \( \beta = 0 \) and \( p_{1B} = V_H \). With no loss of optimality, therefore, we restrict our attention to
strategies $S$ with $p_{2B} = V_L$. The seller’s optimization problem is thus equivalent to

$$\text{Max}_{\beta \in [0,1]} \pi(\beta) = \left( V_H - \beta^{1-\rho}(V_H - V_L) \right) q_H + \beta V_L q_L. \quad (4)$$

Under what circumstances should the monopolist use randomization (i.e. set $\beta$ in the interior), and how?

We first observe that setting $\beta = 0$ is never optimal. This holds true since $\pi(\beta)$ is a continuously differentiable function whereas at the point $\beta = 0$ we have

$$\pi'(0) = V_L q_L > 0. \quad (5)$$

Hence, whenever the seller does not randomize her strategy she will set $\beta = 1$. The following lemma argues that randomized selling is profitable if and only if risk aversion ($\rho$) exceeds a threshold value.

**Lemma 1:** At the optimum, $0 < \beta < 1$ if and only if:

$$\rho > 1 - \frac{V_H - V_L}{V_L} \cdot \frac{q_H}{q_L}.$$ 

All proofs in this version of the paper are omitted for the sake of brevity.

It is an intuitive result that an increased degree of differentiation between the two types (i.e. a higher ratio $\frac{V_H - V_L}{V_L}$) always facilitates profitable segmentation. In addition, lemma 1 shows that a higher proportion of type-H customers (i.e. a higher ratio $\frac{q_H}{q_L}$) necessarily results in the same effect. We explain this model outcome as follows. By setting a probability $\beta$ that is strictly lower than 1 the seller can charge for each unit of the quantity sold in period 1 (i.e. $q_H$) a higher price ($p_{1A} > p_{2B}$). However, at the same time the seller forgoes potential revenues of $(1 - \beta)p_{2B}q_L$ in period 2. A higher ratio $\frac{q_H}{q_L}$ increases the relative magnitude of the positive effect on profits (in period 1) as compared to the negative effect (in period 2); thus, the seller’s motivation to segment the market is indeed increased.

Whenever the condition of lemma 1 is satisfied, the optimal point $S^*$ involves:

$$p_{1B}^* = V_H - \beta^{1-\rho}(V_H - p_{2B}) \quad , p_{2B}^* = V_L. \quad (6)$$

$$\beta^* = \left( 1 - \rho \cdot \frac{V_H - V_L \cdot \frac{q_H}{q_L}}{V_L} \right)^{\frac{1}{\rho}} < 1 \quad , V_H \geq V_L. \quad (7)$$

$\geq 0 \quad , 0 < \rho \leq 1.$

The above expression for $\beta^*$ is an increasing function of risk-aversion ($\rho$) and a decreasing function of both ratios $\frac{q_H}{q_L}$ and $\frac{V_H - V_L}{V_L}$. We explain these results as follows.

When the ratio $\frac{V_H - V_L}{V_L}$ increases, a type-H customer observes a period 2 price ($p_{2B} = V_L$) that is lower relative to his own value ($V_H$). As a result, he has a greater incentive to defer his purchase to period 2. The seller’s best response in this case is to reduce the probability $\beta$ so as to keep type-H customers indifferent between making early and late purchases.

When the ratio $\frac{q_H}{q_L}$ increases, a greater advantage arises to the seller from incrementally increasing the price $p_{1A}$ while simultaneously reducing the probability $\beta$ such that type-H customers remain indifferent. Accordingly, we find that the optimal $\beta$ is a decreasing function of the ratio $\frac{q_H}{q_L}$.

Finally, we note that whenever the condition of lemma 1 is not satisfied any strategy that results in selling B at a price $V_L$ to both types with probability 1 is optimal.

### 2.2. Two quality levels and no randomization

In this section we consider a case where the seller offers two products but does not use randomization. For convenience, let us assume that all sales take place in period 1.

Obviously, it suffices for the seller to consider the following three strategies:

- **$S_I$** ("Sell product A to type H and no product to type L"): $p_A = V_H + \delta_H \cdot p_B > V_H$.
- **$S_{II}$** ("Sell product A to both types H and L"): $p_A = V_L + \delta_L \cdot p_B > V_H$.
- **$S_{III}$** ("Sell product A to type H and product B to type L"): $p_A = V_L + \delta_H \cdot p_B = V_L$.

Let $\pi_i$ represent the equilibrium profit that corresponds to strategy $S_i$, ($i \in I, II, III$). We have:

- $\pi_I \geq \pi_{II}$ if and only if, $V_H + \delta_H \geq \frac{q_H + q_L}{q_H} (V_L + \delta_L) - \frac{q_L c}{q_H}$.
- $\pi_I \geq \pi_{III}$ if and only if, $V_H \geq \frac{q_H + q_L}{q_H} V_L$.
- $\pi_{II} \geq \pi_{III}$ if and only if, $\delta_H \leq \frac{q_H + q_L}{q_H} \delta_L - \frac{q_L c}{q_H}$.
2.3. Two quality levels and randomization

This section studies optimal selling policies that involve both vertical-differentiation and randomized selling. We argue when it would be optimal for the seller to offer the low-quality product B alone in a “last-minute” sale. We also show the different condition under which it would be optimal to offer the high-quality product A alone, or alternatively, offer both products A and B in a sale.

The seller’s strategy space consists of all strategies of the form,

\[ S = (p_{1A}, p_{2B}, p_{2A}, p_{2B}; \alpha, \beta, \gamma) \]

We introduce the following results.

**Lemma 2:** Without loss of optimality, the seller can restrict herself to strategies that induce customers of type H to buy (either product A or product B) in period 1, and customers of type L to buy (either product A or product B) in period 2.

**Corollary:** There always exists an optimal strategy with \( p_{1A} > p_{2B} > p_{2A} = p_{2B} = V_L \).

**Lemma 3:** Without loss of optimality, the seller can restrict herself to strategies with \( \gamma = 0 \).

**Theorem 1:** In equilibrium, customers of type H buy (product A) with certainty whereas customers of type L buy (either product A or product B) with non-zero probability.

Notably, theorem 1’s assertion does not generally hold true in the pure vertical-segmentation case (§2.2). Our analysis thus indicates that a monopolist that uses randomization excludes fewer buyers from consumption with certainty\(^1\). The next theorem contains our main result.

**Theorem 2:** Without loss of optimality, the seller can restrict herself to strategies with either \( \alpha = 0 \) or \( 1 - \alpha = \beta = 0 \).

Simply put, the seller cannot increase its profits by offering more than one product with positive probability in a random (i.e., \( \alpha + \beta < 1 \)) sale event.

We proceed to describe optimal strategy choices in various model settings. Following theorem 2, we restrict our attention to strategies \( S = (p_{1A}, p_{1B}, p_{2A}, p_{2B}; \alpha, \beta, \gamma) \) (with \( p_{1B} > V_H, p_{2A} = V_L + \delta_L, p_{2B} = V_L \) and \( \gamma = 0 \)) that correspond to one (or more) of the following three cases.

**Case I:** At the optimum \( \alpha = 0 \)

In this case the seller optimally commits in advance not to sell product B in period 2. Let \( s_1 = (1p_{1A}, 1p_{1B}, 1p_{2A}, 1p_{2B}; 1\alpha, 1\beta, 1\gamma) \) with \( 1p_{1B} > V_H, 1\beta = 0 \) and \( 1\gamma = 0 \).

\[ s_1 = (1p_{1A}, 1p_{1B}, 1p_{2A}, 1p_{2B}; 1\alpha, 1\beta, 1\gamma) \]

\[ s_1 = (1p_{1A} - c)q_H + 1\beta V_L q_L. \quad (8) \]

Whereas \( \pi_1 \) exceeds \( \pi_{hl} \) (i.e., \( 0 < 1\beta < 1 \)) if and only if

\[ \rho > 1 - \frac{V_H - V_L}{V_L} q_H 0 < \rho < 1. \quad (9) \]

Surprisingly, condition (9) is identical to lemma 1’s condition. That is, we find that the optimality of randomization in this case is completely independent of the attributes of product A (i.e., \( \delta_H \) and \( \delta_L \)).

Whenever condition (9) is satisfied we have

\[ 1\beta = \left( \frac{1}{1 - \rho} \cdot \frac{V_H - V_L}{V_L} q_H \frac{\rho - 1}{\rho} \right)^{\frac{1}{1 - \rho}} 0 < \rho < 1 \]

\[ 1p_{1A} = V_H + \delta_H - 1\beta^{\frac{1}{1 - \rho}} (V_H - 1p_{2B}); \quad 1p_{2B} = V_L. \quad (11) \]

The optimal probability \( 1\beta \) is identical to what is described by equation (7) (!). In particular, it is an increasing function of risk-aversion (\( \rho \)) and a decreasing function of both ratios \( q_H q_L \) and \( V_H - V_L \).

Whenever condition (9) is satisfied any strategy that results in selling A at a price \( V_H + \delta_H \) to type H, and B to type L at a price \( V_L \) with probability 1 is optimal.

**Case II:** At the optimum \( \beta = 0 \)

In this case the seller optimally commits in advance not to sell product B in period 2. Let \( s_2 = (2p_{1A}, 2p_{2A}; 2\alpha) \) be the optimal strategy. The corresponding equilibrium profit is

\[ \pi_2 = 2p_{1A} q_H + 2\alpha (V_L + \delta_L) q_L - (q_H + 2\alpha q_L) c. \quad (12) \]

We find that \( \pi_2 > \pi_{hl} \) (i.e., \( 0 < 2\alpha < 1 \)) if and only if buyers are sufficiently risk averse

\[ \rho > 1 - \frac{V_H - V_L + \delta_H - \delta_L}{V_L + \delta_L - c} q_H \quad 0 < \rho < 1. \quad (13) \]

Whenever condition (13) is satisfied we have

\[ 2\alpha = \left( \frac{1}{1 - \rho} \cdot \frac{V_H - V_L + \delta_H - \delta_L}{V_L + \delta_L - c} q_H \frac{\rho - 1}{\rho} \right)^{\frac{1}{1 - \rho}} 0 < \rho < 1 \]

\[ 2p_{1A} = V_H + \delta_H - 2\alpha^{\frac{1}{1 - \rho}} (V_H + \delta_H - 2p_{2A}); \quad 2p_{2A} = V_L + \delta_L. \quad (14) \]
As in the previous case (equations (7) and (10)), the optimal probability of sale \( \pi_2 \) here is also an increasing function of risk-aversion (\( \rho \)) as well as a decreasing function of both ratios \( \frac{q_H}{q_L} \) and \( \frac{v_H-v_L}{v_L} \).

Whenever condition (13) is not satisfied any strategy that results in selling A at a price \( V_H + \delta_H \) to type H, and A to type L at a price \( V_L + \delta_L \) with probability 1 is optimal.

Should the seller offer product A or product B in a random sale event (i.e., when at the optimum \( \alpha + \beta < 1 \))?

We find that \( \pi_2 > \pi_1 \) holds true if and only if\(^3\)

\[
\left( \frac{V_H-V_L}{V_H-V_L + \delta_H - \delta_L \frac{1-\rho}{\rho}} \right) \geq \frac{V_L}{V_L + \delta_L - c}.
\]

(16)

Thus we find that an incremental increase in risk aversion (\( \rho \)) influences the seller to offer product A (alone) rather than product B (alone) in a period 2 random sale event. Surprisingly, condition (16) does not incorporate the quantities \( q_H \) and \( q_L \). This result implies, for example, that an increase in the relative number of type-L buyers does not necessarily influence the seller to offer a higher-quality product to them. Quite expectedly (see §2.2.), condition (16) implies that a greater difference between the two types’ valuations of product B \( (V_H - V_L) \) increases the seller’s motivation to offer product A whereas a greater difference in their valuation of product A \( (V_H - V_L + \delta_H - \delta_L) \) influences the firm to offer product B in period 2.

Thus, we have established that a greater difference between the valuations of a given product by the two consumer types influences the seller to offer the other product in a random sale event. The reason for this is that an increase in the ratio \( \frac{\delta_H - \delta_L}{v_H-v_L} \) allows the seller to charge a relatively higher price for product A in the first period and thus recover lost potential revenues that are incurred whenever the inefficient product B (alone) is sold in the second period.

Case III: At the optimum \( 1 - \alpha - \beta = 0 \)

In this case the seller holds a sale in period 2 with certainty. Let \( s_3 = \left( 3p_{1A}, 3p_{2A}, 3p_{2B}; 3\alpha, 3\beta \right) \) be an optimal strategy. The corresponding equilibrium profit is

\[
\pi_3 = 3p_{1A}q_H + \left( 3\alpha 3p_{2A} + 3\beta 3p_{2B} \right)q_L - (q_H + 3\alpha q_L)c.
\]

(17)

For all \( 0 < \rho < 1 \), let

\[
\omega(\rho) = \left( \frac{q_H}{q_L}, \frac{1}{\rho} \frac{1}{1-\rho} (V_H-V_L (\delta_H-\delta_L))^{\frac{1-\rho}{\rho}} \right) \frac{1}{\delta_L-c}.
\]

(18)

We find that \( \pi_3 > \max \{ \pi_{II}, \pi_{III} \} \) (i.e., \( 0 < 3\alpha \cdot 3\beta < 1 \), if and only if

\[
V_H - V_L < \omega(\rho) < V_H - V_L + \delta_H - \delta_L , 0 < \rho < 1.
\]

(19)

Whenever condition (19) is satisfied the optimal strategy \( s_3 \) entails:

\[
3\alpha = \frac{(\omega(\rho))^{1-\rho} - (V_H - V_L)^{1-\rho}}{(V_H - V_L + \delta_H - \delta_L)^{1-\rho} - (V_H - V_L)^{1-\rho}} \frac{q_H}{q_L},
\]

(20)

\[
\rho \leq 1, 0 < \rho < 1.
\]

\[
3\beta = 1 - 3\alpha < 1.
\]

(21)

\[
p_{1A} = \frac{V_H + \delta_H - \delta_L}{V_H + \delta_H - \delta_L (3\alpha - 3\beta)^{1-\rho} + 3\beta (V_H - \delta_H - \delta_L)^{1-\rho}}.
\]

(22)

\[
3p_{2A} = V_L + \delta_L , 3p_{2B} = V_L.
\]

(23)

We note that in this case the optimal probabilities \( 3\alpha \) and \( 3\beta \) are (while always continuous) not necessarily monotone functions of risk aversion (\( \rho \)).

If condition (19) is not satisfied then either it is optimal to sell A at a price \( V_H + \delta_H \) with probability one to type-L buyers or it is optimal to sell B at a price \( V_H \) with probability one to type-L buyers in period 2 (see also §2.2.).

Finally, we note that for each one of the three aforementioned cases (I, II and III) there indeed exist model settings in which all optimal strategies fall exclusively under this case.

2.4. The effects of buyers’ risk aversion on product-line efficiency

In this section we analyze two specific model scenarios in order to argue that an increase in buyers’ risk aversion (\( \rho \)) can lead either to an increase or to a

\[\text{\footnotesize\cite{6}}\]

This condition is developed in the proof of theorem 2.
Case I: Higher buyers’ risk aversion has a moderating effect on quality distortion

Using the framework of section 2.3, we consider a scenario with the following parameters: \( V_H = 1.1 \), \( \delta_H = 3; V_L = 1; \delta_L = 1 \) and \( c = 0.5 \). Let also \( q_H = q_L = q \ (q > 0) \).

Let us assume for a moment that buyers are risk neutral and analyze the seller’s corresponding optimal strategies. Previous research (Riley and Zeckhauser [13], Marom and Seidmann [9]) establishes that whenever buyers are risk neutral \((\rho=0)\) there always exists an optimal strategy \( s_{RH}^* \) that does not involve randomization. In our specific case, there is an optimal strategy \( s_{RH}^* \in \Sigma_2 \) that entails selling product A to type-H buyers in the first period (at price \( p_{1A} = 4 \)) and selling product B with certainty \((\beta = 1)\) to type-L buyers in the second period (at price \( p_{2B} = 1 \)). In the corresponding equilibrium the seller’s profit is \( \pi = 4.5 \).

Let us assume now that buyers are risk averse with a degree of constant risk aversion of \( \rho = 0.85 \). In this case, there is an optimal strategy \( s_{RH}^* \in \Sigma_3 \) that entails selling product A to type-H buyers in the first period (at price \( p_{1A} = 3.964 \)). In the second period the seller optimally randomizes its sales as follows. With probability \( \alpha = 0.081 \) the monopolist sells product A to type-L buyers (at price \( p_{2A} = 2 \)). With the complementary probability \( \beta = 1 - \alpha = 0.919 \) the firm sells product B to type-L buyers (at price \( p_{2B} = 1 \)). In the corresponding equilibrium, the seller’s profit is \( \pi = 4.505 \).

We observe that in this case a higher level of buyers’ risk aversion influences the seller to reduce quality distortion by offering the efficient product A with some likelihood in the second period to type-L buyers (whereas in \( s_{RH}^* \) only the inefficient product B was offered to them). Interestingly, the increase in \( \rho \) in this case also results in a Pareto improvement due to the decrease in the price \( p_{1A} \) paid by type-H buyers at the equilibrium corresponding to \( s_{RH}^* \).

Case II: Higher buyers’ risk aversion exacerbates quality distortion

We consider the following model settings: \( V_H = 1.03 \), \( \delta_H = 1.45; V_L = 1, \delta_L = 1 \), \( c = 0.5 \); and \( q_H = q_L = q \ (q > 0) \).

If buyers are risk neutral \((\rho = 0)\) then there is an optimal strategy \( s_{RH}^* \in \Sigma_2 \) that entails selling the high product A to both buyer types H and L. At price \( p_{1A} = 2 \) the seller’s resulting profit is \( \pi = 3 \).

If buyers are risk averse with \( \rho = 0.85 \) the optimal strategy \( s_{RH}^* \in \Sigma_3 \) entails selling product A to type-H buyers in the first period (at price \( p_{1A} = 2.287 \)). In the second period the monopolist sells product A to type-L buyers (at price \( p_{2A} = 2 \)) with probability \( \alpha = 0.622 \), and product B (at price \( p_{2B} = 1 \)) with a complementary probability \( \beta = 1 - \alpha = 0.378 \). The seller’s resulting profit is \( \pi = 3.099 \).

In case II, therefore, the monopolist will distort the proportion of quality by introducing the inefficiently low quality product B only if buyers are risk averse.

Thus we have shown that, in general, a higher degree of buyers’ risk aversion has an indeterminate effect on product-line efficiency.

3. Concluding remarks

This paper extends the framework of Marom and Seidmann [9] by considering two products rather than one. Even though the two models take different approaches and largely deal with separate issues, the main conclusions that arise from them are remarkably similar. Marom and Seidmann [9] argue that when the monopolist’s available capacity is sufficiently high she has a unique optimal strategy that entails fixing both prices \( p_1 \) and \( p_2 \) and holding a “sale” with a probability \( \alpha \) less than one. In this paper we show that it is always optimal for the seller to commit to selling but a single product (either A or B) in a random sale event (that is held with a probability less than one). Put together, the two models indicate that a monopolist cannot benefit by offering a non-degenerate joint-distribution of qualities and prices in a random sale event.

Two managerial implications of the model are worth emphasizing. First, we have shown that when buyers are highly risk averse it can be optimal for a monopoly seller to offer high quality products (such as business-class airline tickets) in “last-minute” sales. Interestingly, the sale of high-end products in a random event can optimally assume one of two forms. The first form is a random “sale” in which the high quality product alone can be offered. The second form resembles a “random upgrade” sequential transaction in which the seller first guarantees the supply of a basic quality product, and randomly offers an upgrade opportunity (for a fee) at a later time. This result can help to explain, for example, why some airlines offer upgrades to their frequent flyers club members only a short time before the flight’s departure.

A second managerial implication of the model is that an incremental increase in buyers’ risk aversion influences the seller to offer the higher quality product with a higher probability. In the airline industry, for example, business travelers’ willingness to incur transaction risks is generally considered to be lower than that of leisure travelers. Quite intuitively, therefore, it follows from the analysis that business-class tickets (or “last-minute” upgrades) should be
offered with a higher frequency on routes where the proportion of (risk averse) business travelers is higher.

4. References


