Integration of Multi-Criteria Decision Analysis and Negotiation in Order Promising

Tung Bui
University of Hawaii
tung.bui@hawaii.edu

Hans-Jürgen Sebastian
RWTH Aachen University
sebastian@or.rwth-aachen.de

Abstract
In an inter-firm supply chain, it is rather an exception than a rule that a producer can satisfy the initial request of a customer. Instead of rejecting the offer that would yield substantial immediate and long-term opportunity costs, it is important for the producer to immediately engage in an iterative search for a counter-offer that both meets the negotiated needs of the customer and the adjusted production capacity. In this paper, we deal with order promising in the Supply Chain Management. We consider the Available-to-Promise and the Capable-To-Promise functions as they are part of a production-distribution network. We present a hybrid and iterative method that combines an optimization technique based on a MILP model with a negotiation approach using multi-attribute utility functions.

Keywords: Operational supply chain management, order promising, available-to-promise (ATP), capable-to-promise (CTP), negotiation, multi-criteria decision analysis, multi-attribute utility, multi-agent system

1. Introduction

The Capable-To-Promise (CTP) function supports order promising in a short-term, order-based production environment. For incoming customer orders, CTP decides whether or not it is possible to fulfill the desired order quantity, and if so, the delivery date. Under scarcity of raw materials and limited production capacity, orders may be turned down or not fulfilled based on the initial terms of the customers. Yet, a current major weakness of the CTP function is its lack of support of any kind to help find an alternate and mutually agreeable solution between the producer and the customer upon rejection of the initial order. In practice, this follow-up and negotiation task is typically done “manually,” and the quality of the outcomes relies on the skills and experience of the person in charge of processing the order. In addition to the difficulty of finding the right person at the right time for the right problem, negotiating a complex CTP manually is often time consuming.

The purpose of this paper is to propose a multi-attribute negotiation process in the CTP in a supply chain network. Our objective is to automatically derive alternative offers in case a customer order must be denied based on its initial terms. The paper is organized as follows. First, we briefly describe the order promising process within a make-to-order production environment and suggest how negotiation can be applied to the problem at hand. We next discuss in detail how optimization and multi-attribute production-distribution network can be integrated to improve both the efficiency and effectiveness of the ATP/CTP functions. Using a simple example with two players, we present an implementation of the proposed concepts is presented using multi-stakeholders, multi-attribute utility functions.

2. Order Promising in Operational Supply Chain Management

2.1. The standard case of ATP/CTP functions
Available-To-Promise (ATP) and Capable-To-Promise (CTP) are critical activities in the management of supply chains. Within a make-to-order or configure-to-order production environment, production or configuration is not initiated until the producer receives a customer order requesting a specific product. Since the quantity of materials or components in stock or the production resources is often limited at a given point in time and cannot be replenished or extended by the requested date of delivery the producer has to decide on the quantity, due date and price to commit to each customer order (Kilger and Schneeweiss, 2005). Until this point we
considered a pull-based make-to-order production environment where one single producer with an integrated stock receives customer orders (Figure 1).

Figure 1. Basic scenario: An aggregated view of a supply chain for ATP/CTP functionality

ATP is generally understood as a simple function that looks up the producers finished products inventory and reserves the quantity ordered by the customer (e.g., Ball et al., 2004; Kilger and Schneeweiss, 2005; Stadtler, 2005). In turn, CTP takes the whole production process into consideration to look ahead what quantity may be available within a certain time frame. (Some authors denote the functionality of CTP as Advanced ATP (Chen et al., 2001)). Figure 2 shows a basic workflow of the ATP and CTP functions. First, customer orders are received by the producer. An order usually contains a set of ordered products (or order positions) and the desired quantities and a delivery due date. Sometimes, a price is specified with the order as well. To check whether the order can be fulfilled, the ATP and CTP functions are executed consecutively. If the ATP function is able to reserve the ordered quantity from existing stock, the contract is settled and the products are delivered on time. If not, the CTP function checks if the production of an appropriate amount of the ordered products is possible on time. In case that the CTP function can fulfill the order, production and delivery will be executed as planned. Otherwise, the customer order has to be turned down.

2.2. Order promising in a production-distribution network

When Order Promising is executed in the context of customer orders through the whole logistics network (i.e., inter-organizational supply chain), all participants need to be consulted. These typically include a huge number of logistics networks characterized by their number and type of products (functional or innovative products), type of production (make-to-stock, make-to-order and configure-to-order), and the location of the push-pull boundary within the networks. In 2.2.1, we will develop a decision model of the supply chain (SC), which allows optimization for different objective functions.

Figure 2. Basic workflow of Available-to-Promise (ATP) and a Capable-to-Promise (CTP) functions
2.2.1. A dynamic model of a production-distribution network.

For the sake of clarity, we focus on one production-distribution networks only, meaning that both the supplier stage and the customer stage of the whole SC are not considered explicitly. Such a production-distribution system is shown in Figure 3.

![Figure 3. An example of a production-distribution network](image)

The configuration of the production-distribution subsystem of the Supply Chain can be structured as follows (The network in fig.3 is a special instance of the more general network defined in the following):

- There is only one producer $P_{\text{local}}$ (the focal or main enterprise in the SC) with $n$ manufacturing facilities at different production locations $PL_i$, $i = 1, \ldots, n$. We consider one commodity (product) manufactured at all $n$ locations. The idea is to focus on a functional product, such as a washing machine, a refrigerator or a lawn-mower. Of course, there can be several variants of the considered product, but we do not deal with a configure-to-order production environment in this paper. We further assume to have a make-to-stock production environment and that the order penetration point within the SC is given. (For example, generic products are manufactured at the production sites, and the places where the variants are assembled are the warehouses.) If we go more into details, we will recognize that within the network in Figure 3 we have a combination of make-to-stock and make-to-order.

- The distribution system consists of warehouses $WL_j$, $j = 1, \ldots, m$ and transportation links. The inventory level of the considered product in warehouses $WL_j$ at time $t$ is given by $I_t(WL_j)$, $j = 1, \ldots, m$, measured in number of items. (At this point we do not distinguish between the variants in order to keep the quantitative model simple. However, an extension of the model is possible.)

- There are retailers $RT_k$, $k = 1, \ldots, l$ which represent “the customers” in the SC. Each retailer predicts the demand of its physical customers and generates “customer orders” for the SC.

---

1 In the following $t$ denotes a continuous time parameter: later we will restrict on discrete time because of simplicity reasons.
If we denote $P=\{PL_i| i=1,\ldots,n\}$, $W=\{WL_j| j=1,\ldots,m\}$ and $R=\{RT_k| k=1,\ldots,l\}$, we can represent the transportation relations as follows:

1) $(PL_i, WL_j) \in (P \rightarrow W) \subseteq P \times W$ (producer-warehouse links)

2) $(WL_j, RT_k) \in (W \rightarrow R) \subseteq W \times R$ (warehouse-retailer links)

3) $(WL_j, RT_k) \in (W \rightarrow R) \subseteq W \times R$ (warehouse-retailer links)

4) $(PL_i, RT_k) \in (P \rightarrow R) \subseteq P \times R$ (direct transportation link)

Each transportation link has an assigned transportation time, which is assumed to be deterministic and known.

The set of retailers $R$, which representing the customers, generates a stream of orders at time $t$ for the SC. The SC, set up to support the focal enterprise $P_{focal}$, receives this stream of orders and has a number of possible alternatives:

- Accept only a subset of orders and reject another subset of orders,
- Start negotiating with the retailers about a third subset of orders.

We propose a quantitative decision model that describes both the production-distribution system introduced above and the order-stream. The detailed consideration of the network offers much more degrees of freedom (many decision variables) for such a network-CTP in comparison to a standard-CTP described in 2.1.

### Table 1. Order set

<table>
<thead>
<tr>
<th>$O_r$</th>
<th>“new set” of orders at time $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{r&lt;ct}$</td>
<td>accepted orders at time $t'&lt;t$, $\bigcup_{t'&lt;t} O_{r&lt;ct}'$ – accepted orders before $t$</td>
</tr>
<tr>
<td>$O_{r&lt;ct}$</td>
<td>rejected orders at time $t'&lt;t$, $\bigcup_{t'&lt;t} O_{r&lt;ct}'$ – rejected orders before $t$</td>
</tr>
<tr>
<td>$O_{r&lt;ct}$</td>
<td>orders arriving at time $t'$ and selected for negotiation, $\bigcup_{t&lt;ct} O_{r&lt;ct}'$ – orders selected for negotiation before $t$</td>
</tr>
</tbody>
</table>

### Table 2. Decision variables

<table>
<thead>
<tr>
<th>Decision</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery from warehouse stock</td>
<td>$w_{j,k}(t)$</td>
<td>quantity sent from warehouse $WL_j$ to $RT_k$ starting at time $t$ (If transportation time is $d_{j,k}^<em>$, then this quantity arrives at the retailers location at time $t + d_{j,k}^</em>$)</td>
</tr>
<tr>
<td>Direct delivery from the manufacturer’s stock</td>
<td>$p_{i,k}(t)$</td>
<td>quantity sent from production facility $PL_i$ to $RT_k$ directly starting at time $t$ (If transportation time is $d_{i,k}$, then it arrives at time $t + d_{i,k}$ at the retailers location)</td>
</tr>
<tr>
<td>Production orders for the manufacturers</td>
<td>$x_i(t)$</td>
<td>quantity which production facility $PL_i$ starts to manufacture at time $t$ (With $C_i$ number of items produced by $PL_i$ per time unit, the manufacturing time for any single order can be computed.)</td>
</tr>
<tr>
<td>Distribution (replenishment) decisions</td>
<td>$y_{i,j}(t)$</td>
<td>quantity sent from production facility $PL_i$ to warehouse $WL_j$ (in order to store the amount of goods there) at time $t$.</td>
</tr>
</tbody>
</table>

**Constraints:**

\[
\sum_{j=1}^{m} y_{i,j}(t + t_m(x_i(t))) \leq x_i(t), \text{ where } t_m(x_i(t)) = \frac{x_i(t)}{C_i} \quad \text{for all } i \text{ and } t \tag{1} \]  
warehouse replenishment

\[
\sum_{j=1}^{m} w_{j,k}(t) + \sum_{i=1}^{n} p_{i,k}(t) \leq Q_{t}^{(k)} \quad \text{for all } k \text{ and } t \tag{2} \]  
delivery assignment of retailers

\[
I_{t+1}(WL_1) = I_t(WL_1) + y_{11}(t-d_{11}) - w_{11}(t) \\
I_{t+1}(WL_2) = I_t(WL_2) + y_{12}(t-d_{12}) + y_{22}(t-d_{22}) - w_{22}(t) \\
I_{t+1}(WL_3) = I_t(WL_3) + y_{23}(t-d_{23}) - w_{32}(t) - w_{33}(t) \tag{3} \]  
inventory dynamics (warehouses)

\[
I_{t+1}(PL_1) = I_t(PL_1) - y_{11}(t) - y_{12}(t) - p_{11}(t) + x_1(t) \tag{4} \]  
inventory dynamics (production facilities)

\[
w_{11}(t) \leq I_t(WL_1) + y_{11}(t-d_{11}) \\
w_{22}(t) \leq I_t(WL_2) + y_{12}(t-d_{12}) + y_{22}(t-d_{22}) \\
w_{32}(t) + w_{33}(t) \leq I_t(WL_3) + y_{23}(t-d_{23}) \tag{5} \]  
distribution decisions restriction

\[
y_{11}(t) + y_{12}(t) + p_{11}(t) \leq I_t(PL_1) + x_1(t) \\
y_{22}(t) + y_{23}(t) + y_{24}(t) + p_{21}(t) + p_{22}(t) + p_{23}(t) + p_{24}(t) \leq I_t(PL_2) + x_2(t) \tag{6} \]  

Equations (3) to (6) have to be fulfilled for each time $t$.

The constraints (1) and (2) are valid for the general production-distribution system described above, but the constraints (3)-(6) represent the special instance introduced in figure 3 only.

Now, we discuss the constraints (1) to (6):

$PL_i$ produces a quantity $x_i(t)$ starting at time $t$. Then, the manufacturing time $t_m$ of this quantity $x_i(t)$ (denoted by $t_m(x_i(t))$) becomes $t_m(x_i(t)) = x_i(t)/C_i$.

We further assume that there is no inventory available at the production site and that transportation of the produced commodity to the warehouses does not start before the whole production order quantity $x_i(t)$ has been produced. Then, we get the constraint (1). Formula (1) shows that constraints are becoming complicated, if we do have a continuous time parameter $t$ and if we consider explicitly time lags caused by production capacity constraints and the missing producers inventories. Therefore, in the following, because of simplification, we assume discrete time $t = 0, 1, 2, \ldots$ and inventories integrated at the production locations $PL_i$.

Constraint (2) requires that delivery from warehouse stock or direct delivery from manufacturers stock to a retailer $RT_k$ should not be bigger than the ordered quantity $Q_{t}^{(k)}$ of this retailer. Formula (2) assumes that the respective links in the network exist. (If a link does not exist, the respective decision variable is set to 0). The delivery starts at time $t$ provided that the inventory levels at time $t$ allow to do this:

\[
w_{j,k}(t) \leq I_t(WL_j) \quad \text{for all } j \text{ and } k \\
\]

for manufacturers inventories $PL_i$.

Formula (2) assumes that the respective links in the network exist. (If a link does not exist, the respective decision variable is set to 0). The delivery starts at time $t$ provided that the inventory levels at time $t$ allow to do this:

\[
w_{j,k}(t) \leq I_t(WL_j) \quad \text{for all } j \text{ and } k \\
\]

Now we discuss the constraints related to an order quantity $Q_{t}^{(k)}$.

Our model is dynamic in nature. Since we are considering discrete time $t$, we have to model the state transformation for state variables. The inventory variables are states in the sense of such a dynamic system or a multi-stage decision making process. Therefore, they have to become transformed over time.

Formulas (3)-(6) relate to the example network in Figure 3, but can be easily generalized. Formula (3) shows how the inventory levels of the warehouses are changing over time.
If we use stock variables $I_t(PL_p)$ and $I_t(WL_j)$ to model producers and warehouses inventories we have to make sure that these variables remain non-negative over time. Therefore, from (3) and (4) we get (5) and (6) if we require $I_{t+1}(WL_j) \geq 0$ and $I_{t+1}(PL_p) \geq 0$. If initial values $I_0(WL_j) \geq 0$, $I_0(PL_p) \geq 0$ are given and inequalities (5) and (6) are fulfilled, then non-negative inventory levels are guaranteed.

Also, we have to take into account the capacity constraints:

$$0 \leq x_1(t) \leq C_1$$
$$0 \leq x_2(t) \leq C_2$$

where $C_1$ are maximal production capacities per time interval.

The model developed above is a dynamic LP-model in case of a continuous product. If the product is discrete, also the model becomes an integer LP model.

### 2.2.2. A strategy of a network CTP based on a sequence of submodels

In 2.2.1, we developed a dynamic model for the production-distribution network as a subsystem of a SC. This model consists of decision variables and a set of time dependent constraints. It might become very large in terms of the number of variables and constraints. This model can be used in order to simulate the processes (production, storage, transportation) of the SC. For a given set of orders $O_t=\{O_t^{(1)}, \ldots, O_t^{(l)}\}$ at time $t$ we can use the respective data $O_t^{(k)}=\{Q_t^{(k)}, RLT_t^{(k)}\}$ in particular the ordered quantity and requested lead time to define situations which are more or less complicated. Then, we define a strategy, which means mainly a sequence in which the characteristics of the current situations are checked. A most intuitive (and therefore “the basic”) strategy related to the network is:

I. *Try to fulfill the order-set from warehouse-inventories only.*

II. *If I. is not possible, try to fulfill the order-set by using both the warehouse inventories plus the goods which are in transport from the producers to the warehouses (regular replenishment processes of the warehouses)*

III. *If II. is not possible, select direct delivery from producers to the retailers in addition.*

IV. *If III. is not possible, start new production orders at time $t$ in addition.*

Each situation (I – IV) corresponds with a model which can be built from the set of decision variables and constraints we have introduced before. It is obvious that the model corresponding to I. is the simplest one (the network-ATP) and the other models become more and more complex in the sequence I$\Rightarrow$II$\Rightarrow$III$\Rightarrow$IV. In order to illustrate the approach we will consider situation I (see Figure 5)
Situation I is given, if \[ \sum_{j=1}^{m} I(WL_j) \geq \sum_{k=1}^{l} Q^{(k)}_t \] (9)

In this case, it is possible to fulfill the overall ordered quantity by the aggregated warehouse stock: \[ \sum_{j=1}^{m} I(WL_j) \text{ at time } t. \] However, inequality (9) does not check, whether the delivery from warehouse inventories is possible or partly possible within the requested lead times. Also, there is no decision which quantities should be delivered from which warehouse to which retailer. In order to prepare optimal decisions from the Supply Chains focal enterprise (the producer P) point of view we define two goals (objective functions):
- the cost \( K_t \) of delivery from the warehouses, and
- the lead time \( LT_t \) for this delivery process.

\[ K_t = \sum_{j=1}^{m} \sum_{k=1}^{l} K_{jk} \cdot w_{jk}(t) \quad (10) \]

where \( K_{jk} \) refers to transportation cost per item on the relation \( WL_j \rightarrow RT_k \). (If the link \( WL_j \rightarrow RT_k \) is not included in the network, we have \( w_{jk}(t) = 0 \) by definition.)

We define the lead time for order \( Q^{(k)}_t \) of retailer \( RT_k \) by:

\[ LT_t^{(k)} = \max_{j=1,m} (d^{*} \cdot w_{jk}(t) > 0, \sum_{j=1}^{m} w_{jk}(t) = Q^{(k)}_t) \quad (11) \]

(Lead time means the maximal transportation time from the warehouses to \( RT_k \) provided, the whole ordered quantity is delivered.)

In sum, if we do not aggregate lead times of the retailers, we have \( l+1 \) objectives. If we define an overall lead time by:

\[ LT_t = \max_{k} LT_t^{(k)} \quad (12) \]

we get two objectives which both need to be minimized. This situation can be formulated as a series of multiple criteria decision models. The solutions of these problems are then used as initial offers within a negotiation process between the producer \( P=P_{\text{focal}} \) (the Supply Chain Agent) on the one hand and the retailers \( RT_1, \ldots, RT_l \) (the Retailer Agents) on the other hand.

3. Outline of a Negotiation Approach within a CTP Environment

Bui and Shakun (2002) propose a negotiation approach called Evolutionary System Design (ESD) and a software tool (NEGOTIATOR) to illustrate its implementation. The principle behind ESD is that antagonists from an impasse solution might be able to find a consensus if they go through a number of systematic and iterative steps – that is to search for alternate solutions through an expanded “solution space”, and to define and continuously refine values and goals related to the problem at hand until a consensus is found. Although this evolutionary approach was originally applied to a complete different field (e.g., Bichler et al., 2003), it seems to be also applicable to a CTP environment in the SCM (Dudek and Stadler, 2005). In this section, we demonstrate how ESD might be used in SCM. As such it is a more generalized model than the one proposed by Julka et al. (2002).

3.1. Definition of values and goal variables

We consider the production-distribution network example from 2.2 with only one retailer, and we use the terminology introduced by Bui and Shakun (2002).

- General values are high performance, reliability in delivery, safety.
- Operational expressions of the general values are formulated by goal variables; delivered quantity (of the requested good), lead time, cost, price

\[ \begin{align*}
\text{Qt} & \quad \text{ordered quantity by the retailer (Party A of negotiation)} \\
\text{OQt} & \quad \text{offered quantity by the SC in response to the order} \\
\text{Kt} & \quad \text{cost of the ordered quantity Qt} \\
\text{Pt} & \quad \text{price of the ordered quantity Qt} \\
\text{LTt} & \quad \text{lead time of the ordered quantity Qt} \\
t & \quad \text{is the time index } t \in \{0, 1, 2, \ldots\}
\end{align*} \]

The SC-agent computes cost and lead time using an optimization submodel (introduced in 2.2.2) and calculates a prize \( P_t \).

3.2. Weighted control variables (attributes)

- Control variables of Party A (Retailer) are:
  - ordered quantity (number of items), price (e.g., in EURO), lead-time (e.g., days)
- Control variables of Party B (retailer) are:
  - offered quantity (number of items), price, lead-time

The first round of negotiation starts with initial offers. We consider in the following a special scenario because of simplification reasons:

The SC-agent offers the ordered quantity (and not a fraction of it). The negotiation focuses on price and lead-time only. Price and lead time are related to the
ordered quantity. Each agent simultaneously starts the negotiation with an initial offer.

Because the SC seeks to fulfill the entire ordered quantity of $Q_t$ ($O_{Q_t} = Q_t$) items (or to reject the order), it tries to solve Submodel I first and selects a solution which is closest to the requested-lead-time of party A. Of course, the buyer (party A) does not propose a price, but the SC-agent (party B) does (e.g., Faratin et al., 2002).

Bui and Shakun (2002) introduce weights that express the importance of the attributes, which are in our case price and lead-time. The weights of party A and party B for price and lead time are denoted by $w_A(P_t)$, $w_B(P_t)$, by $w_A(LT_t)$, $w_B(LT_t)$ respectively. These weights can be normalized.

3.3. Ranges of the values of control variables (decision attributes)

From Submodel I, we can derive two optimization problems by adding either a cost function or a lead time function as objective functions to the model introduced in Section 2 (see (10) and (11)).

The SC-agent solves both optimization problems with the ordered quantity as input and gets a minimal cost solution and a minimal lead-time solution as well. This generates the following ranges (or intervals):

$$K_t \leq K_t (\text{Max}) \text{ and } LT_t \leq LT_t (\text{Max})$$

for the control variables, provided, the ordered set is $Q_t$ and this quantity $Q_t$ can be realized within Submodel I. The SC-agent derives a price-interval $P_t (\text{Min}) \leq P_t \leq P_t (\text{Max})$ and a lead-time interval $LT_t (\text{Min}) \leq LT_t \leq LT_t (\text{Max})$ from the given cost-interval.

If we do not require that the ordered set of $Q_t$ can be completely realized by submodel I, we can define more general ranges for price and lead-time which are the basis for the following utility function definitions. We assume now that there are known intervals based on actual production constraints, i.e., ranges of the control variables:

$$P_t (\text{Min}) \leq P_t \leq P_t (\text{Max}) \text{ and } LT_t (\text{Min}) \leq LT_t \leq LT_t (\text{Max}).$$

3.4. Utility functions

3.4.1. Conditional utility functions. The method is based on the use of utility functions of each party A and B. Each party defines a conditional utility function (under the condition of the ordered quantity $Q_t$): $u_A(P_t, LT_t | Q_t)$, $u_B(P_t, LT_t | Q_t)$, $u_A$, $u_B$: utility of party A, B of a price $P_t$ and lead time $LT_t$ under the condition that the order is $Q_t$ (defined over the ranges introduced in 3.3.)

In this example, it is very important to note, that both utilities $u_A$ and $u_B$ depend on two variables $P_t$ and $LT_t$. That means, it is not possible to define, for example, the utility of a particular value of the price variable without knowing the lead-time values (Figure 6).

![Figure 6. Two-dimensional conditional utility function](image)

3.4.2. Weighted utility function. We get the weighted utility functions, if we introduce one-dimensional utility functions $u_A(P_t | Q_t)$ and $u_B(LT_t | Q_t)$ of the party A (and analogously for party B) with respect to only one variable $P_t$ or $LT_t$ respectively. If we do not require that the ordered set of $Q_t$ can be completely realized by submodel I, we can define more general ranges for price and lead-time which are the basis for the following utility function definitions. We assume now that there are known intervals based on actual production constraints, i.e., ranges of the control variables:

$$P_t (\text{Min}) \leq P_t \leq P_t (\text{Max}) \text{ and } LT_t (\text{Min}) \leq LT_t \leq LT_t (\text{Max}).$$

Then, the weighted utilities $u_A^w$ and $u_B^w$ can be obtained as below:

$$u_A^w (P_t, LT_t | Q_t) = w_A(P_t) \cdot u_A(P_t | Q_t) + w_A(LT_t) \cdot u_A(LT_t | Q_t)$$

$$u_B^w (P_t, LT_t | Q_t) = w_B(P_t) \cdot u_B(P_t | Q_t) + w_B(LT_t) \cdot u_B(LT_t | Q_t)$$

Of course, we cannot expect that the weighted utility functions are identical with the conditional (two dimensional) utility functions $u_A$, $u_B$. This weighted utility approach is used in Bui and Shakun (2002). If we deal with two-dimensional utilities, weights are, of course, not needed, but it seems to be easier to use one-dimensional utility functions and to aggregate them using weights.

3.4.3. Joint utility functions. Multiplying $u_A$, $u_B$ with normalized weights and adding the resulting curves result in “joint utilities of A and B with respect to each of the attributes price and lead-time (Figures 7 and 8).
\begin{align*}
  u_{\text{Price}}^{\text{Joint}} &= w_A(P_t) \cdot u_A(P_t \mid Q_t) + w_B(P_t) \cdot u_B(P_t \mid Q_t), \\
  u_{\text{Price}}^{\text{Joint}} &= 0.357 \cdot u_A(P_t \mid 500) + 0.555 \cdot u_B(P_t \mid 500), \\
  u_{\text{lead-time}}^{\text{Joint}} &= w_A(LT_t) \cdot u_A(LT_t \mid Q_t) + w_B(LT_t) \cdot u_B(LT_t \mid Q_t), \\
  u_{\text{lead-time}}^{\text{Joint}} &= 0.643 \cdot u_A(LT_t \mid 500) + 0.445 \cdot u_B(LT_t \mid 500)
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Joint utility for price}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Joint utility for lead-time}
\end{figure}

Bui and Shakun (2002) suggest the Pareto principle to determine the maximum of the joint utility functions in order to get a social solution that maximizes the sum of weighted utility functions with respect to each of the attributes. In our example as shown in Figures 7 and 8, the highest joint utility for price is given by the maximal price \( P_t(\text{Max}) \) resulting from Submodel I and the sub-sequential pricing procedure. The highest joint utility for lead-time lies in the interior the lead-time interval and is marked by Max! on the LT axis.

Another approach is to use the joint conditional two-dimensional utility functions by maximizing:
\[ u_A(P_t, LT_t \mid Q_t) + u_B(P_t, LT_t \mid Q_t) \]
with respect to \((P_t, LT_t)\) over:
\[ [LT_t(\text{Min}), LT_t(\text{Max})] \times [P_t(\text{Min}), P_t(\text{Max})] \]
and using the optimal solution \((P_t^*, LT_t^*)\) for further negotiation. In that case, weights are not needed.

In summary, the multi-attribute utility approach outlined in this section delivers a compromise solution with respect to the initial offers of both parties A and B. It might happen that the retailer does not accept that compromise solution and chooses to modify his initial offer. In this case, the SC-agent needs to look to the new situation and to identify another sub-model (e.g., Submodel III (if it is not possible to use Submodel I for the new situation)) to generate a “next” offer.

Depending on that offer, utility considerations might become needed again and this iterative or evolutionary process would continue. Also, other types of offers are possible. For example, we can consider splitting the ordered quantity into parts with different lead times or changing partners. As such, we include here in the iterative search for a solution, an inductive solution approach with incentive compatibility mechanism (Myerson, 1979).

In Figure 9, we outline an algorithmic approach of this combined optimization-negotiation method we have introduced above. Due to the length limitation of this paper, we do not discuss the known issues related to the applications of multi-attribute utility theory involving multiple players (e.g., necessity of partial utilities to be independent among them (no correlation); possible existence a multiple equilibria; possible risk of coalition, etc.). Our approach here is a hybrid one that uses MAUT as a framework to iteratively find a solution (any solution is better than a rejected order), and if possible enhance it. We do acknowledge that there is no guarantee for an optimal solution, in the process (see Figure 9).

\section{4. Conclusions and Future Work}

This paper briefly shows how to combine optimization and multi-criteria decision analysis based on a MILP model of a production-distribution network with MAUT in particular with iterative negotiation. The model is based on the Pareto concept that seeks to move toward maximizing the social utility function. We describe here a simple two-player scenario. The implementation will be done by a multi agent system. In an earlier study (Bui et al., 2009), we implemented a multi-agent simulation framework for automated
negotiation in order promising. The model presented in this paper will be integrated in this simulation to help study the effectiveness of the proposed combined optimization-negotiation approach.

**ACKNOWLEDGMENTS**

This research is supported by the DAAD (German Academic Exchange Service) through the PPP-program.

**SELECTED REFERENCES**


