Designing Hub Networks with Connected and Isolated Hubs

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Abstract
Modern supply chains depend on efficient, reliable and extensive freight transportation systems. Many of these systems utilize hub-and-spoke networks to provide transport between many origins and many destinations. These networks include hub facilities that provide a connecting, sorting and consolidation function which concentrates flows on the inter-hub links, called hub arcs, to exploit the economies of scale in transportation. This paper addresses the design of time definite hub and spoke networks using both connected hubs, that are adjacent to the hub arcs, and isolated hubs, that are not adjacent to any hub arcs. It also highlights the role of isolated hubs in expanding networks. Computational results are provided for time definite trucking in North America and the results demonstrate the role and the value of isolated hubs.

1. Introduction
Modern supply chains rely on efficient large-scale freight transportation systems. Many freight carriers, and some passenger carriers, have developed hub-and-spoke networks to provide transport between many origins and many destinations. Hub facilities (transportation terminals, sorting centers, etc.) provide a sorting, switching and connecting (SSC) function to link many origins and many destinations with fewer vehicles and routes than would be needed with a direct linkage between every origin-destination pair. Hub facilities can also provide a consolidation and break-bulk (CB) function to allow transshipment between different modes of transport (such as between ocean transport and rail transport, or between rail transport and truck transport) or between different sizes of vehicles for the same mode (such as between large and small trucks). Inter-hub links that provide a reduced per unit transportation cost are termed hub arcs. Hub and spoke networks are a key strategy for facilitating efficient transportation across large regions. Both the SSC and the CB hub functions are important to provide access to transportation economies of scale via the use of more efficient (usually larger) vehicles and modes.

A hub that provides only the SSC function and is not adjacent to any hub arcs is termed an isolated hub. In contrast, a hub that provides the CB function and is adjacent to a hub arc is a connected hub. Figure 1 provides examples of three hub networks to serve eight origins/destinations (e.g., cities). Origins/destinations are shown as black squares, hubs are shown as larger circles superimposed on the origins/destinations, and the heavier lines represent travel on the reduced cost hub arcs between two hubs.

Figure 1(a) shows a network with a single isolated hub. Because there is only a single hub in Figure 1(a), there are no hub arcs and this hub provides only the SSC function. All origin-destination paths in Figure 1(a) are one-stop paths (origin-hub-destination). Figure 1(b) provides an example of a three-hub network with three hub arcs. All hubs in Figure 1(b) are connected hubs that provide both the SSC function and the CB function for shipments traveling on a hub arc. Figure 1(c) provides an example of a three-hub network with one hub arc, two connected hubs and a single isolated hub. In Figures 1(b) and 1(c) some origin-destination paths are one-stop paths, while others are two-stop paths (origin-hub-hub-destination) that include travel on a reduced cost hub arc. Note how removal of the isolated hub in Figure 1(c) would result in much longer and more expensive paths for the origins/destinations in the lower left portion of Figure 1(c).

Isolated hubs may also be useful to respond to expanding demand. For example, consider a carrier with a hub and spoke network who wishes to expand its geographic service region. Origins and destinations in the new service region that are far from the existing hubs would incur large transport costs to use the existing hub network. Adding an isolated hub in the new region would allow the new origins/destinations to exchange shipments (or passengers) without the long trip to and from the existing hub network. Hall [9] describes how large US overnight express carriers established such isolated hubs on the east and west coasts as their business and networks expanded. Isolated hubs may also offer a lower cost alternative to
designing a network to serve an increasing demand in a fixed service region.

This paper explores hub network models that include both connected and isolated hubs. The following section provides some background and motivation for this research. The next section presents a mixed-integer programming model for designing multiple allocation hub networks with connected and isolated hubs. Section 4 provides results with the CAB data set and Section 5 illustrates how isolated hubs are useful to serve expanding demand. The final section is the conclusion.

2. Background and Motivation

One motivation for this research has been our work with a major North American time definite motor carrier. Time definite trucking firms offer very reliable, scheduled service between terminals generally located at major cities. Delivery schedules are usually coordinated with typical business operating cycles with pickup from the origin terminal occurring in the early evening and delivery to the destination terminal occurring in the morning. Delivery schedules usually conform to either overnight, second day, third day, etc. delivery. Time definite carriers can charge a premium for their high level of service, and time definite motor carriers can often provide service levels nearly equivalent to air carriers – at a fraction of the cost. Isolated hubs may be useful to help achieve high service levels by allowing shorter origin-destination paths. Time definite hub location models can provide insights into how time definite carriers can design networks to be efficient while delivering premium service.

Another motivation for this research is to address issues of designing hub networks for dynamic (expanding) demand. Nearly all hub location research has considered a static setting with one given set of demand for transport between many origins and destinations. In response to expanding demand, either geographic expansion or an increasing level (or intensity) of demand in a given region, hub networks would need to expand by adding hubs and hub arcs to continue to provide efficient (low cost) and effective service.

Hub location researchers have analyzed a variety of hub-based transportation systems as summarized in recent literature reviews [1,5]. The majority of research has addressed cost minimizing (e.g., hub median) models and most researchers have considered networks with connected hubs in which origin-destination paths consist of up to three arcs, including one hub arc. These networks allow origin-destination paths to include one or two stops at hubs, as in Figure 1(b). Some research has limited origin-destination paths to a single hub stop [12,14,15] and thus considered networks with multiple isolated hubs.

In the classical hub median and hub arc location models [3,6,7,11], all hub nodes are connected hubs that perform both the SSC and the CB functions. Travel on the hub arcs has a discounted cost to reflect the economies of scale in transportation that provide a major incentive for hub networks. While most hub location models locate hub nodes, the hub arc location models locate the discounted hub arcs, whose
endpoints are connected hubs, to minimize the total transportation cost. This allows the hub arcs to be used where appropriate to achieve the specified objective. In this paper we extend the hub arc location model to include isolated hubs and time definite transportation.

Designing transport networks to achieve a high level of service – and a low cost – is an area of growing practical importance and research interest [4,8,10,12]. This paper models time definite transportation as in [4] where each origin-destination pair must be served by a path through the network that is shorter than a specified distance that depends on the direct distance between the origin and destination. Thus, rather than having a single common time limit or distance limit for all o-d pairs, as in a hub covering model [1,3], we employ distance limits designed to provide one-day, two-day, three-day and four-day service.

There has been some research that discusses the role of isolated hubs. Hall [9] provides different routing strategies that include use of isolated hubs to allow one-stop routings for shipments in defined geographic regions. In a similar spirit, O’Kelly and Lao [13] consider isolated hubs (termed “mini-hubs”) in a model that uses given hub locations to explore the allocation of non-hubs to the hubs and the use of truck or air transport. O’Kelly [12] extended this work to allow multiple isolated hubs and to find optimal hub locations for small problems.

Two key differences in our research are: (1) the ability to model more complex networks with isolated hubs, connected hubs and hub arcs that allow two-stop paths and more intricate network topologies; and (2) the use of time definite service levels to ensure all origin-destination paths provide a high level of service.

3. Hub Location Models

To formulate the time definite hub arc location models we will use a complete graph $G = (V,E)$ with node set $V = \{1,2,...,N\}$. The nodes correspond to origins and destinations (e.g., transportation terminals at cities) and potential hub locations. Let $d_{ij}$ be the length of arc $(i,j)$ and let $W_{ij}$ be the flow (e.g., quantity of freight) to be transported from $i$ to $j$. Distances are assumed to satisfy the triangle inequality.

The hub network includes two types of arcs: hub arcs between two hubs with a discounted unit transportation cost, and “regular” arcs that do not have a discounted cost. Without loss of generality we can let the transport cost per unit distance on the regular arcs be 1. Then, the cost for transportation along a hub arc is discounted by the parameter $0<\alpha<1$ to capture the economies of scale from consolidated transportation. A connected hub is a hub adjacent to at least one hub arc and an isolated hub is a hub not adjacent to any hub arcs. Because our model allows isolated and connected hubs, we may have regular (non-hub) arcs between two isolated hubs or between an isolated and a connected hub, as well as between origin/destinations and hubs.

To include a high level of service in our hub location models, we adopt the time definite hub arc model from [4]. This is based on the first type of hub arc location model presented in [6], denoted HAL1. As with most hub location research, in this model each origin-destination flow must be routed via at least one hub and every origin-destination path includes at most three arcs and at most one hub arc. (Other hub arc location models in [6,7] allow longer and more complex paths.) We can then view each origin-to-destination path as including collection from the origin to a hub via regular arc $(i,k)$ and distribution from a hub to the destination via regular arc $(m,j)$. Paths may also include a transfer component on hub arc $(k,m)$ between two hubs. Note that HAL1 requires that the collection and distribution components of an origin-destination path occur on arcs other than hub arcs. This can lead to the use of non-discounted regular arcs that join two hubs and may coincide with hub arcs. By relaxing the restriction in classical hub median (and related) models that the hub arcs form a complete subgraph on the hub nodes, the hub arc model has greater flexibility to utilize the hub arcs where appropriate to produce a network with lower transportation costs (for a given number of hub arcs).

Because of the three-part path for collection, transfer and distribution (even though one or two parts may be the degenerate arc from a node to itself), we write the cost for an origin-destination path as

$$C_{ijkm} = d_{ik} + \alpha d_{km} + d_{mj}$$

where $(k,m)$ is a hub arc for the transfer component and $(i,k)$ and $(m,j)$ are regular (possibly degenerate) arcs for collection and distribution. Note that each arc in the model may correspond to travel on multiple arcs (e.g., roads) in the underlying physical transport network.

To model the time definite nature of transportation we wish to prevent excessively long travel paths via the hub network. This effectively eliminates potential paths and variables in the model. We use the set $xfeas$ to represent the set of indices (i.e. paths) that are feasible for a particular problem. This set is defined based on the direct origin-destination distances $d_{ij}$ and the path distances $d_{ix} + d_{km} + d_{mj}$ associated with the $i$-$k$-$m$-$j$ path from origin $i$ to destination $j$. As in [4] we define four pairs of distances corresponding to overnight, second day, third day and fourth day service. Each distance pair includes the maximum
direct origin-destination distance and the maximum allowable travel distance via the hub network. These distance values are derived from the service schedules of a major North American time definite motor carrier. Thus, (1) all origins and destinations within 400 miles of each other need to be served by a path \((i-k-m-j)\) of length at most 600 miles, (2) all origins and destinations between 400 and 1000 miles of each other need to be served by a path \((i-k-m-j)\) of length at most 1200 miles, (3) all origins and destinations between 1000 and 1800 miles of each other need to be served by a path \((i-k-m-j)\) of length at most 2000 miles; and (4) all origins and destinations separated by more than 1800 miles need to be served by a path \((i-k-m-j)\) of length at most 4000 miles. In the formulations below we use the set \(xfeas\) to limit the size of the formulation by restricting sets of constraints and summations to only the feasible flow variables.

Let \(p\) be the given number of hubs to locate and \(q\) be the given number of hub arcs to locate. To formulate the hub arc location model with isolated and connected hubs (HALIC) we use three sets of variables:

\[X_{ijkm} = \text{the fraction of flow on path } i-k-m-j,\]
\[Z_{km} = 1 \text{ if there is a hub arc } k-m; 0 \text{ otherwise, and}\]
\[Y_k = 1 \text{ if node } k \text{ is a hub; } 0 \text{ otherwise.}\]

The mixed integer programming formulation HALIC is then:

**Minimize** \[\sum_{i<j,k,m \in xfeas} (W_{ij} + W_{ji}) C_{ijkm} X_{ijkm} \quad (1)\]

**Subject to**
\[\sum_k m \sum_m X_{ijkm} = 1 \quad \forall (i, j, k, m) \in xfeas, i < j, \quad (2)\]
\[\sum_k Y_k \leq p \quad (3)\]
\[X_{ijkm} + \sum_m (X_{ijkm} + X_{ijmk}) \leq Y_k \quad \forall (i, j, k, m) \in xfeas, i < j, \quad (4)\]
\[X_{ijkm} \leq Z_{km} \quad \forall (i, j, k, m) \in xfeas, i < j, k \neq m, \quad (5)\]
\[\sum_k m \sum_m Z_{km} = q \quad (6)\]
\[Y_k \in \{0,1\} \quad \forall k \in V, \quad Z_{km} \in \{0,1\} \quad \forall k, m \in V, \quad (7)\]
\[X_{ijkm} \geq 0 \quad \forall (i, j, k, m) \in xfeas, i < j, \quad (8)\]

The objective (1) minimizes the total transportation cost. Constraint (2) ensures that each origin-destination flow is sent via some hub “pair”, which may be a single hub as in \(X_{ijk}\). Constraint (3) limits the number of hub nodes. Note that the inequality allows connected hub arcs (with fewer than \(p\) hubs) if that reduces the cost. Constraint (4) ensures that hubs are opened for all routings of flows. Constraint (5) ensures there is a hub arc established for each transfer flow. Constraint (6) requires exactly \(q\) hub arcs to be selected. Constraints (7,8) restrict the hub arc variables appropriately.

Note that whenever \(p > 2q\), isolated hubs are required. However, isolated hubs may also be advantageous when \(p \leq 2q\). To prevent the use of isolated hubs we can add the following constraint to ensure that each hub node has an adjacent hub arc:

\[Y_k \leq \sum_{m<k} Z_{mk} + \sum_{m>k} Z_{km} \quad \forall k \in V. \]

Note that when \(q = p(p-1)/2\), then the hub arcs fully connect the hub nodes and additional hub arcs provide no benefit.

If we have fixed costs for the hubs and hub arcs, then rather than specifying the number of hubs and hub arcs via \(p\) and \(q\), we can make that an outcome of the model, by minimizing the sum of the transport and fixed costs. The objective would be

**Minimize** \[\sum_{i<j, k, m \in xfeas} (W_{ij} + W_{ji}) C_{ijkm} X_{ijkm} \]
\[+ \sum_k FH_k Y_k + \sum_{k,m} FA_{km} Z_{km}\]

where \(FH_k\) and \(FA_{km}\) are the (annualized) fixed costs for hub \(k\) and hub arc \((k,m)\), respectively. With this objective we would use constraints (2), (4), (5), (7) and (8) to determine the optimal hub network design.

**4. Results**

This section provides results for time definite hub network design with isolated and connected hubs from the HALIC model using the CAB dataset commonly used in hub location research. This dataset consists of flows between 25 major cities in the USA (the cities are shown in Figures 3 and 4). We denote the time definite level of service defined by the set \(xfeas\) by “High SL” in contrast to the case with no limits on the level of service, denoted “No SL”. Because the high service levels are based on those for a major time definite carrier in North America, results based on the CAB dataset should represent a realistic application. We solved the HALIC model using equations (1-8) to allow isolated and connected hubs. We also solved this model with additional constraint (9) (that prevents
isolated hubs) to assess the value of allowing isolated hubs.

Computational results include two levels of inter-hub transportation discount: \( \alpha = 0.2 \) and \( \alpha = 0.6 \). The small value of \( \alpha \) reflects a strong degree of consolidation and economies of scale to generate large relative cost savings on hub arcs. The value \( \alpha = 0.6 \) reflects a lesser degree of consolidation and may be a reasonable value for truck transportation where the trucks used on hub arcs may be one-and-a-half to two times larger than the trucks used on the non-hub arcs. All problems were solved to optimality using the GAMS integrated development environment with CPLEX 10.1.1 on a 3.2GHz Pentium processor with 1 GB RAM. Preprocessing was used to eliminate some variables by setting one of the flow variables \( X_{ijkm} \) or \( X_{ijn} \) equal to zero based on whichever of \( C_{ijkm} \) or \( C_{ijn} \) was smaller.

We solved 284 problems with the number of hubs ranging from \( p = 1 \) to \( p = 12 \) and the number of hub arcs ranging from \( q = 0 \) to \( q = (p(p-1))/2 \). With the high service level a minimum of nine hubs were required to provide short enough routes for all origins and destinations using the CAB data. Table 1 provides selected results for nine hubs with \( \alpha = 0.2 \). The left half of the table is results with High SL and the right half is the corresponding results for No SL. For each service level, the first two columns provide the optimal cost and the number of isolated hubs, and the last two columns provide the cost and the percentage increase when constraint (9) is imposed to prevent the use of isolated hubs.

The results in Table 1 show that isolated hubs are commonly used in optimal solutions. Note that with nine hubs, when \( q > 4 \) isolated hubs are not required (since nine hubs can be connected by five hub arcs), but isolated hubs are used to reduce the cost. As shown in Table 1, isolated hubs are used for \( 5 \leq q \leq 14 \) with High SL and for \( 5 \leq q \leq 13 \) with No SL. Isolated hubs are most useful when there are few hub arcs, and the isolated hubs can provide good service to an area remote from the hub arcs. When there are many hub arcs, then there is likely to be a hub relatively near all the origins/destinations, so isolated hubs are not useful.

The “% Increase in Cost” column shows that not allowing isolated hubs can increase the cost up to 4.4% with the High SL – and that the cost increase decreases as the number of hub arcs increases. Table 1 also shows how the service level affects the costs: as expected, a high level of service can increase costs substantially, especially as the number of hub arcs increases.

Figure 2 displays a wider range of results for two to nine hubs for \( \alpha = 0.2 \) with No SL. This chart includes eight pairs of curves, where the upper curves provide the transportation cost and the lower curves show the number of isolated hubs in each solution. Each optimal solution provides one point in a cost curve and a corresponding point in the lower curve showing the number of isolated hubs. Each of the eight separate upper curves shows the cost as the number of hub arcs increases for a constant number of hubs. Each lower curve shows the number of isolated hubs used as the number of hub arcs increases for a constant number of hubs. Thus, the rightmost upper curve shows the decrease in cost with nine hubs as the number of hub arcs increases from zero to the maximum of 36. Collectively the eight upper curves show how the cost decreases as the number of hubs increases from two to nine. The lower curves show how the number of isolated hubs drops from a maximum of \( p \) (with no hub arcs) to zero as hub arcs are added.

Figure 3 shows the locational patterns for three optimal solutions with nine hubs and five hub arcs with High SL. In this Figure, the origin/destination cities are shown as small squares, the hubs are shown as large circles, and the hub arcs are shown as thick lines. The regular arcs connecting origins/destinations to hubs are not shown to maintain clarity. The map in Figure 3(a) shows that the optimal solution with \( \alpha = 0.2 \) uses two centrally located isolated hubs (in Cincinnati and Memphis), along with seven connected hubs. Four of the five hub arcs are adjacent (at New York). The map in Figure 3(b) shows a corresponding optimal solution with \( \alpha = 0.2 \) when isolated hubs are not allowed. In this case, the five hub arcs are required to use all nine hubs and the cost necessarily increases (by 4.4%). Note that the hub nodes in these maps are identical, though there are no isolated hubs in Figure 3(b). The map in Figure 3(c) shows that the optimal solution with \( \alpha = 0.6 \) uses three isolated hubs (in Cincinnati, Memphis and Houston). As in the other two maps, the same set of nine hubs is used, but several different hub arcs are used. The use of the same set of hubs in these three solutions is driven by the high service level and the relatively small number of hubs (the high service level cannot be met with fewer than nine hubs). With lower levels of service, or with more hubs, there is more flexibility to locate hubs and the locations would not all coincide as in this example. Note also that the five hub arcs are connected in Figure 3(c), but not in the other two maps. This illustrates the types of results from the different problems in which the discount for hub arc transport (\( \alpha \)) and the use of isolated hubs varies. The complete set of results show that isolated hubs are more useful with smaller \( \alpha \) values and with higher service levels.
Table 1. Selected results with 9 hubs and $\alpha=0.2$.

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5. Network Expansion

In practice, hub networks are dynamic and they will grow or shrink over time in response to changing demand and changing cost structures. This section illustrates the role of isolated hubs in response to increasing demand.

If the level of demand increased uniformly for every origin-destination pair then the transportation cost for a given hub network would increase by an identical percentage. One response to the increasing transportation costs would be to expand the network by adding new hubs and hub arcs to reduce the transportation costs. We consider four fundamental types of network expansions: (i) adding an isolated hub; (ii) adding a hub arc that connects two existing isolated hubs; (iii) adding a hub arc that connects an existing isolated hub and an existing connected hub; and, (iv) adding a hub arc that connects two existing connected hubs. Composite network expansions such as adding a new connected hub and new hub arc can be viewed as combinations of these four fundamental alternatives. Each of these alternatives would introduce different costs depending on the particular application. For example, if hubs are air express sorting centers, then adding a new isolated hub could involve a large fixed cost. Similarly, adding a new hub arc might correspond to adding a new flight service with an expensive large aircraft (e.g., Boeing 747), or adding a inexpensive new large truck trip. Note that adding a hub arc incident to an isolated hub may be more expensive than adding a hub arc between connected hubs because additional activities and resources may be required at the hub to implement the consolidation function.

As an illustration of optimal networks with expanding demand in a fixed region, we use the optimal solutions for the CAB data set from the HALIC model for No SL with $\alpha=0.6$, but we increase the demand level uniformly for each origin-destination pair. For illustrative purposes the annualized fixed costs are taken as follows: (i) the cost of adding an isolated hub = 90; (ii) the cost to connect two existing isolated hubs = 80; (iii) the cost to connect an existing isolated hub and an existing connected hub = 70; and (iv) the cost to connect two existing connected hubs = 65. Let the “current” optimal network involve three hubs, two hub arcs (Los Angeles–Chicago and Chicago–New York) and no isolated hubs; for which the optimal transportation cost is 965.2.

Let $\beta$ be the relative increase in demand so the new demand is given by $\beta W_{ij}$ ($\beta=2$ indicates a doubling of demand). The eight maps in Figure 4 show how the optimal hub network expands as the demand increases. The transition from each optimal network to the next depends on the relative optimal transportation costs.
and the costs of adding network components. For example, the transitions from Figure 4(a) to Figure 4(h) occur at the following respective $p$ and $q$ values: 1.6, 2.9, 4.2, 4.3, 5.2, 8.1 and 8.2. Note that in some cases (e.g., 4(a) to 4(b), or 4(e) to 4(f)) the optimal network changes by adding a new isolated hub; while in other cases the change is adding a hub arc to connect existing connected hubs (4(d) to 4(e)) or to connect an existing connected hub and isolated hub (4(g) to 4(h)). The heavy use of isolated hubs in Figures 4(b) through 4(g) shows how they can help a network incrementally respond to increasing demand.

Another type of expansion to be considered is a geographical expansion. As an illustration of this situation, we compare optimal solutions for the full CAB dataset of 25 cities and for a subset of this dataset with the five western cities (Los Angeles, San Francisco, Seattle, Phoenix and Denver) removed. As the base case, suppose a carrier serves only the 20 eastern-most cities in the CAB data set with $p=4$ and $q=3$. The base case optimal solution has hubs in Chicago, Memphis, Miami and New York, with the three hub arcs being from New York to the other three cities. Now suppose the carrier expands its service region to include the five western cities, thus giving a total service region corresponding to the full CAB dataset. The cost to serve all 25 CAB cities with the base case optimal network is 1085.427. If we allow one isolated hub in addition to the base case optimal network, then the optimal solution is to add an isolated hub in Los Angeles and the cost is reduced to 913.746 (a reduction of 15.8%). If we allowed the three base case hub arcs to be moved while keeping the same four optimal hub nodes from the base case, and also allowing one new hub, then the optimal solution is still to add a hub in Los Angeles; but the three optimal hub arcs are Los Angeles – Chicago, Chicago – New York, and New York – Miami, with the isolated hub in Memphis. This solution has a cost of 864.147, which is a further reduction of 4.6% from repositioning the hub arcs. These optimal solutions show how an expanding service region can be well served by only a small expansion of the hub network that uses isolated hubs.

The discussion and illustrations in this section are not meant to be a definitive analysis of isolated hubs, but to demonstrate the utility of isolated hubs and to highlight the need for more comprehensive hub location models. Most existing models (see [1,5]) have focused on rather restricted topologies and routing assumptions that have limited their use in applied settings. Incorporating more real-world features (and costs) in hub network models is important to develop meaningful insights and applications.

6. Conclusions

This paper presents models for time definite hub location and network design with isolated and connected hubs. These models allow hub networks to be efficient (low cost) and to provide high levels of service, and they allow more flexibility than existing hub location models. This research highlights the importance of using isolated hubs to reduce transport and total network costs. The results show that isolated hubs can be very useful, especially when the network of hub arcs is sparse. Isolated hubs are also more useful with larger discounts for inter-hub transportation (such as from the use of much larger vehicles between hubs) and with higher service levels.

Isolated hubs can play an especially important role as hub networks expand. Such networks may expand by adding hub arcs between existing connected or isolated hubs, adding hub arcs with new connected hubs, or adding isolated hubs. Adding isolated hubs may provide a more cost effective way to improve cost and service than adding new connected hubs and new hub arcs.

Areas for future research include consideration of other hub arc location models from [6,7] that allow more complex paths, incorporation of fixed costs for connected hubs, isolated hubs and hub arcs, and models with multiple transportation modes (e.g., air and truck). Alternate forms of the service level restrictions could also be explored and service levels might also be relaxed or tightened for selected origin-destination pairs to reflect competitive pressures. Note though that service level constraints have the advantage of reducing problem size and thus may allow solution of more realistic problems of larger size and with more practical levels of service. Another promising area for research is to incorporate measures of congestion at hubs (see [2]). This may lead to a greater number of smaller hubs and may also offer opportunities for isolated hubs.

6. References


Figure 4. Optimal networks for expanding demand $p=3-6$ hubs, $q=2-6$ hub arcs.