Information Personalization in a Two Dimensional Product Differentiation Model: Impact of Market Structure and the Quality-Fit Ratio

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Abstract

In this research, we use a game-theoretic model to examine personalization of information in a two-dimensional product differentiation model, when consumers attach importance to ‘preference fit’ as well as ‘product quality’. We report the equilibrium in terms of two factors: one, the extent of firms’ ex-ante horizontal differentiation; and two, the ‘Quality-Fit’ (Q-F) Ratio, which measures the relative strength of consumer preferences on each dimension of product differentiation. Under different conditions, personalization adoption by one firm in a duopoly can be profitable for neither firm, one firm or both firms. We also highlight conditions under which investments in personalization and product quality can be complements or substitutes to each other. Finally, we show that different values of the Q-F Ratio can lead a competitor to respond to a firm’s personalization by either increasing investments in its own quality (aggressive response) or by reducing investments in its own quality (defensive response).

1. Introduction

Personalization, also known as ‘One-to-One Marketing’ ([18]), refers to the practice of using information technology to treat customers on an individual basis by tailoring products, customer service and other interactions uniquely for each customer. Firms routinely personalize products (such as vitamins, cosmetics, and computers) or the information-related attributes of products (when information is auxiliary to the product - Amazon.com which sells books, but also uses consumer information to create a personalized experience for shoppers through one-click checkout and online recommendations). In this research, we examine personalization of the latter type, which we label as information personalization. Information personalization also applies to the personalization of information goods (i.e., when the information itself is the product such as news and online content).

There are numerous examples of firms which use information personalization. Credit cards are one such example. Credit cards have many attributes such as annual fees, membership points and APR; and different customers may value these attributes differently. For example, some customer might value a low APR more than a low annual fee. Firms may differentiate horizontally by positioning themselves on select attributes (such as Capital One for rewards and Discover Card for cash back), or vertically (such as the Citi Simplicity card which offers a higher quality level in form of faster access to live representatives). Firms use information about consumer preferences to target the right credit card to the right consumer. Similarly, wsj.com (a business news website) and washingtonpost.com (a general news website) offer personalized content to their consumers.

While prior research on personalization ([8], [21]) mainly focuses on horizontally differentiated products, in reality, firms which offer similar products also personalize (for example, Netflix and Blockbuster). Another interesting market structure is when vertically differentiated firms adopt personalization. For example, Amazon.com offers personalization and at the same time also offers higher quality levels than its competitors such as Bestprices.com. The dilemma that managers in such firms face is whether to invest in personalization or in quality improvements or both.

Prior research ([9], [21]) assumes that personalization technology is always perfect and that technology can always help a firm in predicting consumers’ preferences with complete accuracy. However, in reality, personalization technologies are not perfect and many firms are making investments to improve the accuracy of their recommendations. For example, in October 2006, Netflix announced a prize of $1 million to anyone who can improve the efficiency of its movie recommendation software by 10% (netflixprize.com). As of August 2008, no one has been able to claim the prize.

In short, there are many unanswered questions regarding personalization under different market structures, or when firms offer different levels of quality. There is also very little research on how the equilibrium changes when the effectiveness of personalization technologies improves. The research questions that we consider are: (i) under what market conditions does a firm find it profitable to personalize? (ii) are personalization and quality substitutes in
equilibrium? (iii). how does personalization by one firm in a duopoly impact the competitors investments in quality? (iv) how does the equilibrium change with improvements in firms’ ability to personalize?

2. Literature Review

Despite the growing popularity of personalization, there has been scarce research in this area (21). Previous attempts to model personalization have analyzed product personalization, targeted promotions, and personalized pricing ([9], [5], [7]). Other studies have shown that personalization does not lead to higher profits for a firm, even if competitors do not personalize (e.g., [22], [20], [6]). [1] model intelligent agent based personalized pricing in a monopolist setting. [15] summarize the current research in the management science area on personalization. [8] and [9] were the first to introduce the notion of a personalizing firm offering a continuum of products rather than a single product. [21] extend this model to more general settings.

All prior models of personalization consider a one-dimensional product differentiation model. The concept of product differentiation - horizontal ([11], [2]) and vertical ([16], [19]) has been analyzed extensively. In reality, firms are differentiated both horizontally (consumer tastes, physical distance) and vertically (quality); so a two-dimensional model captures the reality and provides additional insights which would be overlooked in a one dimensional setting. [4] were among the first to prove the existence and uniqueness of equilibrium in a two-dimensional setting. [17] solve two dimensional model for equilibrium price and product differentiation (both horizontal & vertical) and concluded that it is optimal for a firm to differentiate fully only on one dimension i.e., follow a MaxMin strategy. [26] model the versioning policies of a monopolist when it can extend the product both horizontally and vertically. [10] uses a two dimensional model to analyze the adoption of remote access in the banking industry.

Compared to prior literature on personalization of physical goods ([9], [21]), there are three implications of using information personalization as a context for our study: one, firms which personalize information (for example, Netflix) usually offer the option of personalization to all customers (as opposed to physical goods, where a firm offers both a standard product and a personalized product – [21]). Two, as opposed to prior work ([9]), we do not consider price personalization. There are numerous examples of firms which personalize information (e.g. netflix.com, wsj.com, emusic.com, movielink.com, vongo.com), but charge a standard price for all customers, irrespective of what level of personalization is served to the customers. Also, the practice of price personalization at a mass level has invited negative publicity (e.g. Amazon.com1). Three, the marginal cost of personalization for information goods is negligible.

A key feature of our model is that we evaluate personalization in a more general two-dimensional model of product differentiation. Here firms are not only horizontally differentiated; they also choose quality endogenously. Before presenting our model, we highlight two key features of a two-dimensional model: Q-F Ratio. In a two dimensional model, consumers have a preference for product fit as well as a different willingness to pay for quality. These dimensions are orthogonal to each other and the total consumer utility is the sum of the consumer utility on each dimension. We introduce a term ‘Q-F Ratio’ to denote which dimension is more preferred by the consumer. E.g., a low Q-F Ratio indicates that consumers have a strong preference for product fit (such as, business news vs. political news; high risk investments vs. low risk investments) and would be reluctant to buy something that does not fit their tastes even if the other firm is offering a superior quality. On the other hand, a high Q-F Ratio indicates that quality levels are more important to consumers than product attributes. E.g., in case of credit cards, consumers may care more about quality, as indicated by fraud protection services and level of customer service, than whether the card gives reward points or cash-back. Mathematically, $Q-F \text{ Ratio} = \frac{q_1 - q_2}{t}$ where $q_1$ and $q_2$ are the levels of quality offered by the firms and $t$ is the transportation cost.

Market Structure: In our model, firms’ location on the horizontal dimension is exogenously determined before the start of the game. Prior studies have mainly considered the case where firms are maximally differentiated on the Hotelling line ([9], [12]). [2] show that firms find it optimal to locate at the opposite ends of the Hotelling line. However, competition between direct marketers can also be represented by firms occupying the same position in the horizontal dimension (e.g. [3]). For example, Amazon.com and Bestprices.com both sell similar books and are largely undifferentiated on product type. However, Amazon offers a much higher quality in terms of better user interface, online reviews, and additional features such as A9 search capability. We consider both market structures in our model. One, we consider the case where firms are located at the opposite ends of the Hotelling line (Locationally Differentiated, or LD); two, when both firms are situated in the center of the market (Locationally Identical or LI).

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2 Location choice represents long term consumer perceptions about a firm and changing location requires significant investments by firms in product re-positioning.
The rest of the paper is organized as follows: in section 3, we present our mathematical model. In section 4, we consider the case where firms are ex-ante locationally differentiated (LD) and solve for equilibrium both with and without personalization. Likewise, we solve for equilibrium when firms are ex-ante locationally identical (LI) in section 5. In section 6, we present the overall equilibrium using a specific cost function. Finally, in section 7, we discuss the conclusions and the limitations of our research.

3. Model

Before we introduce personalization, we present an outline of a general two-dimensional product differentiation model and highlight the equilibrium conditions [19]. We consider a market which has a broad product variety (horizontal differentiation) and various possible quality levels (vertical differentiation). An example of such a product is credit cards. Different customers have different preferences for various credit card attributes. For example, co-branded credit cards offer a huge product variety to customers (e.g. Chase has a variety of co-branded cards such as Chase AAA, Chase AARP, Chase MIT and Harvard and so on). On the other hand, customers also care about the quality of credit cards. Fraud detection services and faster access to live customer service representatives (e.g. the Citi Simplicity card) are some examples of how credit card companies are investing in providing a better quality to customers.

To model such a product where consumers value both the ‘product fit’ (choosing a product which closely matches their preferences) as well as ‘product quality’ (higher quality is always preferable to low quality), we consider a two-dimensional model of consumer preferences - a horizontal component \( x \) representing product preference (e.g. [11]) and a vertical component \( \theta \) representing willingness to pay per unit of quality (e.g. [19]). Thus the coordinates \((x, \theta)\) represent the position of any consumer in the X-Y plane. We make standard assumptions regarding consumer utility in horizontal and vertical differentiation models ([9], [19]). Further, we assume:

- **A1**: Consumer preferences are distributed uniformly over a square of unit area on the X-Y plane such that \( x \in [0, 1] \) and \( \theta \in [0, 1] \)
- **A2**: We assume that the cost of quality is a general convex function \( c(q) \) such that \( c'(q) > 0 \) and \( c''(q) > 0 \). Further, we normalize the cost function such that \( c(q) = 0 \).

Consider a duopoly model where firms locate at \( z_1 \) and \( z_2 \) and offer products of quality \( q_1 \) and \( q_2 \) at price \( p_1 \) and \( p_2 \) respectively. Thus \((z_1, q_1)\) and \((z_2, q_2)\) represent the position of firms 1 and 2 respectively in the two dimensional space. Without loss of generality, we assume that \( q_1 \geq q_2 \).

Consumer utility for a product \((z, q)\) offered by firm 1 is given by \( U_1 = R + \theta \cdot q_1 - t \cdot |x- z_1| - p_1 \). Similarly, for firm 2, \( U_2 = R + \theta \cdot q_2 - t \cdot |x- z_2| - p_2 \). The set of consumers who are indifferent between buying a product from either firm is a line intersecting the X-Y plane. The equation for this line is given by solving \( U_1 = U_2 \).

Mathematically,

\[
\theta(x) = \frac{p_2 - p_1 - t(\|x - z_2\| - \|x - z_1\|)}{q_1 - q_2},
\]

(1)

**Modeling Personalization**: We use a modified version of the framework provided by [9] to operationalize the impact of personalization technology from an economic standpoint. Consumer utility from a personalized product is independent of the consumers’ horizontal location and is given as \( U = R + \theta \cdot q - p \). However, technological limitations prevent a firm from accurately predicting customer preferences at all times. Therefore, we introduce a variable \( \delta \) to represent how accurately the firm can predict consumer preferences and tailor the information content to match with the consumers’ preferences. Therefore, we represent the utility function with personalization as \( U = R + \theta \cdot q - p - x \cdot (1- \delta) \). Clearly, \( 0 < \delta < 1 \) (\( \delta = 0 \) represents a non-personalized (or standard) product and \( \delta = 1 \) represents an accurately personalized product as given in [9]).

4. Locationally Differentiated Market Structure

In this case firms are located at the two ends such that \( z_1 = 0 \) and \( z_2 = 1 \). In this market, customers who are indifferent between buying from either firm (i.e. \( U_1 = U_2 \)) lie along the indifference line given by

\[
\theta(x) = \frac{p_2 - p_1 - t - 2 \cdot q_2 \cdot x}{q_1 - q_2}.
\]

(2)

The slope of the indifference line \( = \frac{2 \cdot t \cdot q_1}{q_1 - q_2} \) depends on the ‘Quality-Fit’ \((Q-F)\) Ratio, \( \gamma = \frac{q_1 - q_2}{t} \).

The \( Q-F \) Ratio is a measure of which type of product differentiation is relatively more important to the customers.

Based on the position of the indifference line on the X-Y plane, the demand for firm 1 and firm 2 can be characterized in various ways, as shown in Figure 1. For example, if the indifference line intersects both \( \theta = 0 \) and \( \theta = 1 \) lines on the X-Y plane, then each firm captures customers located close to it for all values of \( \theta \) as shown in Figure 1a. We can show that the condition for this is \( Q-F \) Ratio \( \gamma \leq 2 \). In other words, the consumers’ preference for product fit (given by the transportation cost, \( t \)) dominates their preference for quality; this is known as ‘Horizontal Dominance’.
(see [17]). For example, **Horizontal Dominance** is a characteristic of markets (business news vs. political news; high risk investments vs. low risk investments) where products are differentiated and consumers experience a huge disutility if they do not buy a product of their choice (high values of \( \delta \)).

On the other hand, if the indifference line intersects both the \( x=0 \) and \( x=1 \) lines on the X-Y plane, firm 1 captures all customers who have a higher preference for quality and firm 2 captures all customers who have a lower preference for quality, as shown in Figure 1b. The condition for this is \( Q-F \) Ratio \( \gamma \geq 2 \). In other words, consumers’ preference for quality dominates their preference for variety; this is known as ‘**Vertical Dominance**’ (see [17]). For example, **Vertical Dominance** is a characteristic of markets (such as online music services – emusic.com) where products are relatively homogenous (low values of \( \delta \)) and firms primarily differentiate on quality.

The indifference line can be somewhere in between, as shown in Figure 1c. We can verify that no Nash equilibrium lies in this region. **Insert Figure 1**

### 4.1 Locationally Differentiated with Horizontal Dominance

We solve for equilibrium in two separate cases: one, when no firm offers a personalized product, and two, when one firm offers a personalized product.

Firm profits can be written as:

\[
\pi_1 = D_1 \cdot p_1 - c(q_1); \quad \pi_2 = D_2 \cdot p_2 - c(q_2). \quad (3)
\]

We consider a two stage game as follows: in the first stage, both firms chose their equilibrium quality levels and in the second stage, both firms choose prices. The results can be summarized as follows:

When firms are maximally differentiated horizontally and none of the firms personalizes, there exists a pure strategy Nash equilibrium such that the equilibrium prices and profits are given as follows:

\[
q_1 = q_2 = q^*; \quad p_1 = p_2 = t; \quad \pi_1 = \pi_2 = c(q^*) - c(q^*). \quad (4)
\]

\( q^* \) is the quality level which solves \( \partial \pi_i / \partial q_i = 0 \) where \( i = 1,2 \). This proposition suggests that if firms are located at end points of a line, they offer the same quality level at the same price. Both firms earn the same profits. This is the case of MaxMin differentiation where firms choose to differentiate only on one dimension of product differentiation.

**Personalization:** In the no personalization case, firms are similar in equilibrium. We consider the case when one firm (say, firm 1 without loss of generality) adopts personalization. Solving the consumer utility functions \( U_1 = U_2 \), the indifference line is now given as:

\[
\theta(x) = \frac{p_1 - p_2 - t}{q_1 - q_2} \cdot \frac{2 - \delta}{x}. \quad (4)
\]

We solve the two stage game for quality and price respectively using backward induction. In the second stage, we calculate the equilibrium prices by solving the first order conditions of the firms’ profit w.r.t. price. The equilibrium profits and prices in the second stage are given as:

\[
p_1 = \frac{1}{6}(q_1 - q_2) + t(1 - \delta) \cdot \delta (3 - 2\delta); \quad p_2 = \frac{1}{6}(q_1 - q_2) + t(1 - 2\delta) \cdot \delta (3 - 2\delta)
\]

\[
\pi = \frac{(q_1 - q_2) + 2\delta(3 - 2\delta)}{6\delta(3 - 2\delta)} - c(q_1); \quad \pi = \frac{(q_1 - q_2) + 2\delta(3 - 2\delta)}{6\delta(3 - 2\delta)} - c(q_2).
\]

(5)

The derivation of equation (5) follows from solving the first order conditions with respect to \( p_1 \) and \( p_2 \). In the first stage, let the equilibrium quality levels be \( q_1^{LD,HD} \) and \( q_2^{LD,HD} \). Since we use a general cost function \( c(q) \), we cannot solve for the closed form equilibrium quality levels. However, what is more pertinent to this discussion is how personalization impacts the equilibrium quality levels. The result can be summarized as follows:

**Lemma 1:** Under \( LD \) and \( HD \), an increase in personalization effectiveness leads to an increase in the equilibrium quality level offered by the firm which personalizes, i.e., \( \frac{\partial q_1^{LD,HD}}{\partial \delta} > 0 \). In other words personalization and quality are complements. Also, \( \frac{\partial q_2^{LD,HD}}{\partial \delta} < 0 \) which suggests that personalization by one firm leads to a decrease in quality by the other firm.

Thus under \( LD \) and \( HD \), personalization and quality are complementary strategies because firm 1 offers a higher quality product after adopting personalization. Firm 2 responds to personalization by firm 1 by lowering its own quality. We label this as a ‘defensive response’ when a competitor reduces its investments in quality when a firm adopts personalization. Combining the results in lemma 1 with the firms’ profit function from equation (5), we can calculate the impact of personalization on firm profitability. This is summarized in the following proposition:

**Proposition 1:** If firms are locationally differentiated and \( q_1^{10,10} - q_2^{10,10} < 2 - \delta \), the firm which does not personalize earns lower profits, i.e., \( \partial \pi_t / \partial \delta < 0 \). The firm which personalizes can earn higher or lower profits than the no personalization case depending on the effectiveness of personalization. In other words, \( \partial \pi_t / \partial \delta > 0 \) for \( \delta \) greater than a certain threshold value, \( \delta^* \).

Proofs of Lemma 1 and Proposition 1 are shown in the Appendix. The threshold value, \( \delta^* \) depends on the convexity of the cost of quality \( c(q) \), i.e., higher the

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3 Proofs of remaining lemmas and propositions proceeds similarly, and are not shown here due to space limitations.
convexity of the cost function, higher is the threshold value of $\delta$ at which personalization is profitable for a firm. This result is interesting because it suggests that under LD and HD, personalization can lead to lower profits for both firms (even when only one firm personalizes) than the no-personalization case if the effectiveness of the technology in predicting customers’ preferences is low. In a two-dimensional model, firms can differentiate on quality and since this quality differentiation increases with personalization ($\frac{\partial(q_{1,LD,HD} - q_{2,LD,HD})}{\partial \delta} > 0$), the impact of quality differentiation dominates the price competition effect at higher values of $\delta$. This result is unique to a two-dimensional model because one dimensional models, which do not consider quality differentiation, can overlook the fact that a firm can personalize as well as offer a higher quality level to make more profits than the no-personalization case ([23],[24]).

Both firms personalize: Repeating the above analysis for the case when both firms have the capability to personalize, we can summarize the results in the following proposition:

**Proposition 1a:** Under LD and HD when both firms personalize, profits are decreasing in $\delta$ for both firms. In other words, $\frac{\partial \pi_i}{\partial \delta} < 0$ where both firm $i$ and firm $j$ adopt personalization.

The profits of both firms are

$$\pi_i = \pi_j = \frac{t(1-\delta)}{6} - c(q^*)$$

Comparing the expressions for revenues and cost of quality in propositions 1 and 1a, we can show that profits of firm 2 are lower when both firms personalize than when only firm 1 personalizes. Therefore, in case of LD and HD, no equilibrium exists where both firms personalize. Here we assume that both firms are symmetric in their capacity to personalize, i.e., they offer a personalized product with the same effectiveness, $\delta$.

### 4.2 Locally differentiated with Vertical Dominance

Vertical dominance ($VD$) is a characteristic of markets with a high $Q-F$ Ratio, $\gamma$ i.e., where quality difference $(q_1-q_2)$ between firms is high compared to the horizontal disutility parameter $t$. As in section 4.1, we set up a two stage game and solve for equilibrium quality and prices. Without loss of generalization, let us assume that firm 1 is the high quality firm, i.e., $q_1 > q_2$. When neither of the firms offers a personalized product, the equilibrium is as follows:

$$p_1 = \frac{2(q_1^* - q_2^*)}{3}; p_2 = \frac{q_1^* - q_2^*}{3}$$

$$\pi_1 = \frac{4(q_1^* - q_2^*)}{9} - c(q_1^*), \pi_2 = \frac{q_1^* - q_2^*}{3} - c(q_2^*)$$

(6)

where $q_1^*$ and $q_2^*$ are the equilibrium quality levels. This result is a standard quality differentiation model result where one firm offers a higher quality product at a higher price and the other offers a lower quality product at a lower price.

**Personalization:** Since the no-personalization case has two firms, viz, one which offers a higher quality product and the other which offers a lower quality product, we consider two separate cases: one, when the high quality firm personalizes and two, when the low quality firm personalizes. As in section 4.1, we proceed by backward induction. From the slope of the indifference line, we can infer that Vertical Dominance is the equilibrium for locationally differentiated firms if the $Q-F$ Ratio $\gamma((q_1 - q_2)/t) \geq 2.6$.

If firm 1 and firm 2 choose quality levels $q_1$ and $q_2$ respectively in the first stage and the high quality firm (firm 1) adopts personalization with effectiveness $\delta$, the equilibrium profits and prices in the second stage are given as:

$$p_1 = \frac{4 \cdot (q_1 - q_2) + t \cdot \delta}{6}; p_2 = \frac{2 \cdot (q_1 - q_2) - t \cdot \delta}{6}$$

$$\pi_1 = \frac{\delta \cdot (\gamma \cdot q_1 - q_2) - c(q_1)}{36(q_1 - q_2)}, \pi_2 = \frac{(\gamma \cdot q_1 - q_2) - c(q_1)}{36(q_1 - q_2)}$$

(7a)

On the other hand, if the lower quality firm (firm 2) adopts personalization with effectiveness $\delta$, the equilibrium profits and prices in the second stage are given as:

$$p_1 = \frac{4 \cdot (q_1 - q_2) - t \cdot \delta}{6}; p_2 = \frac{2 \cdot (q_1 - q_2) + t \cdot \delta}{6}$$

$$\pi_1 = \frac{\delta \cdot (\gamma \cdot q_1 - q_2) - c(q_1)}{36(q_1 - q_2)}, \pi_2 = \frac{(\gamma \cdot q_1 - q_2) - c(q_1)}{36(q_1 - q_2)}$$

(7b)

From equations (7a) and (7b), we can show that $\frac{\partial \pi_1}{\partial \delta} < 0$ always holds irrespective of the value of $q_1$, $c(q_2)$ and $\delta$. Thus firm 2 (which is the lower quality firm) always offers the lowest quality level $q_2$, even when it offers a personalized product. The equilibrium quality level of firm 1 can be obtained by solving the first order condition. $\frac{\partial \pi_1}{\partial \delta} = 0$. The results are given in Lemma 2.

**Lemma 2:** The low quality firm always offers the lowest possible quality level, i.e. $q_2^{VD,LD} = q_1$. The quality level offered by the high quality firm decreases with $\delta$ i.e. $\frac{\partial q_1^{VD,LD}}{\partial \delta} < 0$. These results hold irrespective of whether the high quality firm or the low quality firm personalizes.

Thus under LD and VD, if the high quality firm personalizes, personalization and quality are
substitutes, i.e., the higher quality firm can reduce its investments in quality after adopting personalization. If the low quality firm personalizes, the high quality firm responds by lowering its own quality level ("defensive response"). Also, under LD and VD, firms are differentiated on both the horizontal and vertical dimensions. Thus, this is a case of MaxMax equilibrium. Combining the results in Lemmas 2 and equations 7a and 7b, we get the effect of personalization on firm profitability.

**Proposition 2a:** Under LD and VD when both firms personalize, firm profits are given as

\[
\pi_1 = \frac{4(q_1^*-q_1^*)}{3} - c(q_1^*), \quad \pi_2 = \frac{q_1^*-q_2^*}{9} - c(q_2^*)
\]

where \(q_1^*\) and \(q_2^*\) are the equilibrium quality levels obtained by solving \(\frac{\partial \pi_i}{\partial q_i} = 0\) where \(i=1,2\).

Thus, when firms are symmetric in their capability to personalize, the equilibrium reduces to the no-personalization case (under LD and VD). Firm profits and quality levels are independent of the personalization parameter, \(\delta\).

5. **Locationally Identical (LI) Market Structure**

In this case, firms are minimally differentiated horizontally and both firms locate at the center of the X-axis. Many firms, which essentially sell undifferentiated goods such as books and CDs, fall in this category where these firms try to differentiate by offering a higher or lower quality levels than their competitors. Online portals such as Yahoo and MSN are another example where firms offer products (mainly, information) which are largely horizontally undifferentiated. In the absence of personalization, the equilibrium, when firms offer horizontally undifferentiated products, is a simple vertical differentiation model where one firm offers a higher quality product than the other firm ([14]). We incorporate this market structure in our two dimensional model by considering both firms located at the center of the market i.e. \(z_1 = z_2 = \frac{\lambda}{2}\). In the no-personalization case, since the firms are located at the same point on the X-axis, customers experience the same disutility if they buy from either firm. In other words, consider a customer located at a distance \(x\) from the center. Her utility is given as \(U_1 = R - \theta q_1 - p_1 - tx\) and \(U_2 = R - \theta q_2 - p_2 - tx\). The indifference line is given as \(\theta = \frac{p_1 - p_2}{q_1 - q_2}\). Clearly, the slope of this line is zero and hence the indifference line is horizontal because no locational difference is created. Only vertical dominance equilibrium is possible in this case. The equilibrium profits and prices can be summarized as: if firms are ex-ante similar on the horizontal dimension and no firm personalizes, the equilibrium prices and profits are given as:

\[
p_1 = \frac{2(q_1-q_2)}{3}, \quad p_2 = \frac{2(q_2-q_1)}{3}, \quad \pi_1 = \frac{4}{9}(q_1-q_2) - c(q_1), \quad \pi_2 = \frac{q_1-q_2}{9} - c(q_2)
\]

This result follows a standard vertical differentiation model ([14]). The high quality firm prices higher and earns more profits than the low quality firm. Horizontal dominance is not possible in the no-personalization case because the indifference line is always vertical.

The interesting observation about locationally identical models is that personalization by one firm introduces a measure of horizontal differentiation, i.e., it creates differential locational disutility. For example, consumers experience lower locational disutility if they buy a personalized product than if they buy a standard product. If firm 1 personalizes, the consumer utility now becomes \(U_1 = R - \theta q_1 - p_1 - tx(1-\delta)\) and \(U_2 = R - \theta q_2 - p_2 - tx\), where \(x\) is the distance between the customer’s ideal product and the center of the market (\(x = \frac{\lambda}{2}\)). The indifference line is given as

\[
\theta(x) = \frac{p_1 - p_2 - tx}{q_1 - q_2} = \frac{x}{\frac{\lambda}{2} - x}(8).
\]

This indifference line clearly depends on \(x\) and hence locational differentiation is created.

5.1 **Locationally Identical with Horizontal Dominance**

In a locationally identical market and \(\gamma < \frac{\lambda}{2}\), when firm 1 personalizes, the equilibrium prices and quality are given as follows:

\[
p_1 = \frac{1}{6}(q_1 - q_2 + 4 \cdot t \cdot \delta), \quad p_2 = \frac{1}{6}(q_1 + q_2 + 2 \cdot t \cdot \delta)
\]
\[ \pi_i = \frac{1}{36t\delta} [q_i - q - 4t\delta]^2 - c(q_i); \pi_2 = \frac{1}{36t\delta} [q_2 - q - 2t\delta]^2 - c(q_2) \]

From equation 9, we can show that \( \frac{\partial \pi_i}{\partial q_i} > 0 \) for \( i=1,2 \). Solving for first stage equilibrium, we can infer the impact of personalization on product quality.

**Lemma 3:** The firm which personalizes also offers a product of higher quality than the firm which does not personalize. Also, the quality level of the higher (lower) quality firm decreases (increases) with \( \delta \), i.e. \( \frac{\partial q_{1,HD}^{LI}}{\partial \delta} < 0 \) and \( \frac{\partial q_{2,HD}^{LI}}{\partial \delta} > 0 \). Moreover, personalization leads to a decrease in vertical differentiation in the market, i.e.,

\[ \frac{\partial (q_{1,HD}^{LI} - q_{2,HD}^{LI})}{\partial \delta} < 0. \]

Thus under \( LI \) and \( HD \), personalization and quality are substitutes, i.e., the firm which personalizes reduces its quality level as the effectiveness of personalization increases. Moreover, firm 2 responds to firm 1’s personalization by increasing its quality levels. We label this as ‘aggressive response’ because the low quality firm increases its investments in quality after the competitor personalizes. Further, we can show that given zero costs of quality, both firms will choose the highest possible quality level, \( \bar{q} \). Thus under \( LI \) and \( HD \), firms are essentially minimally differentiated on both dimensions of product differentiation, thus leading to a \( MinMin \) equilibrium. Proposition 5 gives the impact of personalization on firm profits.

**Proposition 3:** In a vertically differentiated market and \( \frac{q_{1,HD}^{LI} - q_{2,HD}^{LI}}{t} < \frac{\delta}{2} \), when firm 1 personalizes, both firms earn higher profits. In other words, \( \frac{\partial \pi_1}{\partial \delta} > 0 \) for \( i=1,2 \).

This result is interesting because it suggests that personalization adoption by one firm leads to an increase in profits for both firms in the market. The intuition for this result is that the effect of personalization is to create horizontal differentiation in a market where firms are ex-ante similar. Customers located farther away from the firms get a higher increase in utility due to personalization than customers located closer to the firms. This enables firms to price higher and extract a higher surplus. This result is also interesting because it suggests that even though firms are locationally identical, \( HD \) can still be the equilibrium. In other words, personalization can lead consumers to value product fit in an ex-ante purely vertically differentiated market. In such an equilibrium, the firm which personalizes captures customers located away from the center of the market and the firm which does not personalize captures customers located close to the center; and both firms offer quality levels \( q_{1,HD}^{LI} - q_{2,HD}^{LI} \geq \frac{\delta}{2} \). This is the case of \( MinMin \) differentiation, i.e., both firms are minimally differentiated in terms of both location and quality. It is not surprising that the profits in this case depend on the personalization parameter, \( \delta \). If \( \delta=0 \) (no-personalization case), this reduces to the classical Bertrand equilibrium where both firms earn zero profits. Therefore personalization enables undifferentiated firms to earn non-zero profits.

### 5.2 Locationally Identical with Vertical Dominance

We solve this similar to section 4.2 and omit the details due to space limitations. The result can be summarized in the following proposition.

**Proposition 4:** Under \( LI \) and \( VD \) (for \( q_{1,VD}^{LI} - q_{2,VD}^{LI} \leq \frac{\delta}{2} \)), profits are increasing with \( \delta \) for the firm which personalizes and decreasing in \( \delta \) for the firm which does not personalize.

**Both firms personalize**

If both firms have the capacity to personalize, the equilibrium in \( LI,VD \) is the same as given in case of \( LD, VD \). In case of \( LI,HD \), personalization by both firms leads to a Bertrand equilibrium.

**Summary:** We can summarize our results regarding the impact of personalization on firm profits and quality levels in Table 1. —Insert Table 1—

### 6. Equilibrium Conditions

In this section, we use a specific cost function to compare the equilibrium profits in the different cases analyzed above. We characterize the equilibrium in terms of our exogenous parameters \( \bar{q}, \theta, \text{and} \delta \). Given the intractability of results when using higher order cost functions, we highlight our equilibrium with a constant marginal cost of quality (as has been done by prior research with two dimensional product differentiation models [10], [25]). [26] uses a linear cost function. The results are summarized in Table 2. In the future, we plan to analyze equilibrium conditions with a more general cost function. Since \( LD \) and \( LI \) are exogenous decisions to both firms, we characterize the equilibrium for each of these separately. —Insert Table 2—

Table 2 highlights different conditions under which either one or both firms find it profitable to personalize. For example, we find that both firms personalize in equilibrium if the Q-F ratio is greater than a certain threshold (\( 2-\delta \) in case of \( LD \) and \( \delta /2 \) in case of \( LI \)). On the other hand, under \( LI \) and if the Q-F ratio is less than a certain threshold (\( \delta /2 \)), only one firm finds it profitable to personalize and both firms...
earn higher profits. Finally, under LD and if the Q-F ratio is less than a certain threshold (2- δ), neither firm finds it profitable to personalize.

7. Discussion and Conclusions

In this research, we extend prior work on personalization in several ways: (i) we consider a more general model where firms make decisions about price and quality at the same time; (ii) we model personalization in the context of products and services where information is the key element of personalization; (iii) we use a general personalization parameter to account for the uncertainty in predicting customer preferences with accuracy; and finally, (iv) we model equilibrium conditions under different market structures. We find that such a two-dimensional setting gives us more insights than one-dimensional models. For example, a one-dimensional model predicts that personalization by a firm in a duopoly always leads to lower profits when firms locate at end points of a Hotelling line, even if the competitor does not personalize ([23]). For a two-dimensional model, we introduce the term Quality-Fit (Q-F) Ratio to measure whether consumers value product fit over higher quality product. We show that when firms are locationally differentiated (i.e., are located at opposite ends of the Hotelling line), prior results hold only when the Q-F ratio is low and when the effectiveness of the personalization technology is low. When the Q-F Ratio is high, personalization leads to higher profits for the firm which personalizes in such a model. For a low Q-F Ratio, a firm which personalizes can still earn higher profits than the no-personalization case, provided the effectiveness of the technology is high. For example, our model predicts that in case of competition between differentiated products (such as emusic.com versus countrymusic.com or wsj.com versus washingtonpost.com), personalization is adopted by a firm if consumers have a strong preference for product quality over product fit or if the effectiveness of the technology is high.

We also consider the case when firms are locationally identical on the horizontal axis. We find that for low Q-F ratio, profits of both firms increase with personalization. E.g., consider the competition between Amazon.com and Bestprices.com in the online book stores market where the products are largely undifferentiated and the preference for product fit is high (e.g., a customers who is interested in one type of books (say, computer books) cannot be easily persuaded to buy another type of books (e.g., architecture books)). Our model predicts that in such a scenario, personalization by the high quality firm (in this case, Amazon.com) will result in higher profits for both firms. The intuition is that personalization relaxes the Bertrand competition between firms, and both firms price higher and earn higher profits in the presence of personalization. This result is also unique to a two-dimensional model because it suggests that personalization by one firm can lead to increase in profits for both firms. Finally, we show that depending on the market structure, the Q-F Ratio, and the personalization effectiveness parameter, either both or only one or none of the firms finds it optimal to personalize.

This research also contributes to the literature on two dimensional differentiation models. We show that MaxMin equilibrium is not always dominant in such a model ([17]). Given that location choice is exogenously determined, we show that firms may choose to differentiate on both (MaxMax) or one (MaxMin) or none (MinMin) dimensions depending on the market structure, personalization adoption and the Q-F Ratio. The MinMin equilibrium is especially interesting because it suggests that under some conditions, personalization allows firms choose minimum differentiation on both dimensions. For example, personalization can enable firms such as Mp3.com and E-music, or dealcatcher.com and deals2buy.com to differentiate and make positive profits even though both offer similar services.

In this research, we assume zero marginal cost of personalization, i.e., the cost of personalization is independent of the personalization parameter δ. Assuming a non-zero cost of personalization allows us to solve for an equilibrium personalization level. Our results when a single firm adopts personalization are not likely to change in this scenario. However, when we consider the scenario that both firms personalize, it is likely that firms choose different levels of personalization and we can have additional equilibriums depending on the market structure Q-F ratio and the convexity of the personalization cost function.

Some limitations of our work are as follows: although a two-dimensional model captures reality more closely than a pure horizontal differentiation only or pure vertical differentiation only, firms use a variety of strategic tools which can be too complex to capture in a single model. E.g., our model considers symmetric personalization capabilities for both firms. Future research can capture the case when firms are asymmetric in their personalization capabilities (firms have different values for δ). Also, pricing of information goods can be complex in reality. E.g., some firms charge a per product price while others charge a subscription fee or a mix of both. Some firms give away their products for free in exchange for advertising revenues. Future research can attempt to capture the impact of personalization in each of these scenarios.
To verify how equilibrium quality level changes with \( \delta \) we derive the expression for \( \frac{\partial q}{\partial \delta} \) as follows:

\[
\frac{\partial \pi}{\partial q} = 0 \Rightarrow \left( \frac{\partial^2 \pi}{\partial q^2} \right) \frac{\partial q}{\partial \delta} + \left( \frac{\partial^2 \pi}{\partial q \partial \delta} \right) = 0
\]

Thus, \( \frac{\partial q}{\partial \delta} = -\frac{\partial^2 \pi}{\partial q \partial \delta} \cdot \frac{1}{\partial^2 \pi/\partial q^2} \)

Using the profit expressions, we can show that

\[
\frac{\partial^2 \pi}{\partial q \partial \delta} = \frac{1}{36t - 118t \delta - c''(q_1)} \left( \frac{q_1 - q_2 + 2t}{18t(2 - \delta)^2} - c''(q_1) \right)
\]

The numerator is positive. Denote the revenue of firm 1 by \( R_1 \) such that \( R_1 = D_1 \cdot p_1 \). Therefore, firm 1 profit \( \pi_1 = R_1 - c(q_1) \).

The sign of the denominator depends on the term. Since \( \frac{\partial R_1}{\partial q_1} - c'(q_1) > 0 \) at \( q_1 = q_1^* \) (equilibrium quality level) and also \( \frac{\partial R_1}{\partial q_1} - c'(q_1) \bigg|_{q_1 > q_1^*} < 0 \) and \( \frac{\partial R_1}{\partial q_1} - c'(q_1) \bigg|_{q_1 < q_1^*} > 0 \), therefore \( \frac{\partial R_1}{\partial q_1} - c'(q_1) < 0 \).

Hence the denominator is negative and the overall expression for \( \frac{\partial q}{\partial \delta} \) is positive. Therefore, the quality level of firm 1 increases with an increase in the effectiveness of its personalization. Similarly, we can show that the quality level of firm 2 decreases with an increase in the effectiveness of firm 1’s personalization.

**Proof of Proposition 1**

Profits for firm 1 under LD and HD are as follows:

\[ \pi_1 = \left( q_1 - q_2 + 2t(3 - \delta) \right)^2 - c(q_1) \]

where \( q_1 \) and \( q_2 \) are equilibrium quality levels. Taking first order condition with respect to \( \delta \) we can show that \( \frac{\partial \pi_1}{\partial \delta} = \frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_2} + \frac{\partial \pi_1}{\partial \delta} \).

Since \( \frac{\partial \pi_1}{\partial q_1} \) is zero at equilibrium, therefore the expression reduces to

\[ \frac{\partial \pi_1}{\partial \delta} = \left( q_1 - q_2 + 2t(3 - \delta) \right) \left( q_1 - q_2 + 2t(1 - \delta) \right) \]

This is greater than zero only if the term \( q_1 - q_2 > 2t(1 - \delta) \). At \( \delta = 0 \), LHS is zero and RHS is +ve. At \( \delta = 1 \), LHS is -ve and RHS is zero. Since both LHS and RHS are monotonic in \( \delta \), there exists at most one value of one value of \( \delta \) such that \( \frac{\partial \pi_1}{\partial \delta} > 0 \) for \( \delta \rightarrow \delta^* \).

**APPENDIX**

**Proof of Lemma 1**
Table 1: Summary of Equilibrium Under Different Configurations

<table>
<thead>
<tr>
<th>Market Structure</th>
<th>Equilibrium</th>
<th>Who Personalizes</th>
<th>Profits</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD ($x_1 = 0, x_2 = 1$)</td>
<td>HD</td>
<td>Firm 1</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &gt; 0$ for $\delta &gt; \delta^*$, $\frac{\partial \pi_2}{\partial \delta} &lt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &gt; 0$; $\frac{\partial q_2}{\partial \delta} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>HD</td>
<td>Both</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &lt; 0$, $\frac{\partial \pi_2}{\partial \delta} &lt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &gt; 0$; $\frac{\partial q_2}{\partial \delta} &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Firm 1</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &gt; 0$, $\frac{\partial \pi_2}{\partial \delta} &lt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &lt; 0$; $\frac{\partial q_2}{\partial \delta} = 0$</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Firm 2</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &lt; 0$, $\frac{\partial \pi_2}{\partial \delta} &gt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &lt; 0$; $\frac{\partial q_2}{\partial \delta} = 0$</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Both</td>
<td>Does not depend on $\delta$</td>
<td>Does not depend on $\delta$</td>
</tr>
<tr>
<td>LI ($x_1 = x_2 = \frac{1}{2}$)</td>
<td>HD</td>
<td>Firm 1</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &gt; 0$, $\frac{\partial \pi_2}{\partial \delta} &gt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &lt; 0$; $\frac{\partial q_2}{\partial \delta} &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>HD</td>
<td>Both</td>
<td>Betrand Competition; zero profits</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Firm 1</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &gt; 0$, $\frac{\partial \pi_2}{\partial \delta} &lt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &lt; 0$; $\frac{\partial q_2}{\partial \delta} = 0$</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Firm 2</td>
<td>$\frac{\partial \pi_1}{\partial \delta} &lt; 0$, $\frac{\partial \pi_2}{\partial \delta} &gt; 0$</td>
<td>$\frac{\partial q_1}{\partial \delta} &lt; 0$; $\frac{\partial q_2}{\partial \delta} = 0$</td>
</tr>
<tr>
<td></td>
<td>VD</td>
<td>Does not depend on $\delta$</td>
<td>Does not depend on $\delta$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Equilibrium conditions when both firms have the capability to personalize

<table>
<thead>
<tr>
<th>Case</th>
<th>Q-F Ratio</th>
<th>Who Personalizes</th>
<th>Equilibrium</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD</td>
<td>$&lt; 2 \cdot \delta$</td>
<td>None</td>
<td>$\pi_1 = \frac{t}{2}$; $\pi_2 = \frac{t}{2}$</td>
<td>Using a general cost function, we can show that firm 1 can personalize successful for $\delta &gt; \delta^*$</td>
</tr>
<tr>
<td>LD</td>
<td>$&gt; 2 \cdot \delta$</td>
<td>Both</td>
<td>$\pi_1 = \frac{4(\theta - \theta)}{9}$; $\pi_2 = \frac{4(\theta - \theta)}{9}$</td>
<td>Both firms personalize and earn same profits as the no personalization case</td>
</tr>
<tr>
<td>LI</td>
<td>$&lt; \delta / 2$</td>
<td>Firm 1</td>
<td>$\pi_1 = \frac{4t}{9}$; $\pi_2 = \frac{t \delta}{9}$</td>
<td>Both firms earn higher profits after one firm adopts personalization.</td>
</tr>
<tr>
<td>LI</td>
<td>$&gt; \delta / 2$</td>
<td>Both</td>
<td>$\pi_1 = \frac{4(\theta - \theta)}{9}$; $\pi_2 = \frac{(\theta - \theta)}{9}$</td>
<td>Both firms personalize and earn same profits as the no personalization case</td>
</tr>
</tbody>
</table>

The intuition is as follows: when the marginal cost of quality is zero, both firms offer the highest possible quality level. Consequently, there is no vertical differentiation between firms and profits go down after personalization. On the other hand, with a convex cost of quality, firm 1 offers a higher quality than firm 2 and this quality differential increases with personalization parameter $\delta$, leading to higher prices. Therefore, for high enough values of $\delta$, firm 1’s profits increase with $\delta$. 

Figure 1: Possible Configurations for the Indifference Curve and Corresponding Demand for firm 1 and firm 2