Agent Based Model for Cardinality Constrained Portfolio Selection Problem: Preliminary Results

Ritesh Kumar  
Indian Institute of Management Calcutta, India  
fp132004@iimcal.ac.in

Subir Bhattacharya  
Indian Institute of Management Calcutta, India  
subir@iimcal.ac.in

Abstract

This paper presents a multi-agent model for the portfolio selection problem in presence of cardinality restriction on the number of stocks to be held in the portfolio. A system of agents divides the initial wealth and follows individual investment strategies starting with pseudo-random portfolios. Periodically, the agents share information about their performances, and can switch portfolios. A final cardinality constrained portfolio is constructed by consolidating individual portfolios formed by the agents based on the past performance of the stocks. The paper proposes the multi-agent model as an alternative to other popular decision models in finance in solving the cardinality constrained portfolio selection problem. The portfolio suggested by the agent based model has been found to perform better than the portfolios suggested by mean-variance models when tried out in real market.

1. Introduction

An investor wants to invest some amount of money in the stock market. Since there are too many stocks in the market, she has decided to invest only in a fixed number of stocks. She now wants to decide (a) what stocks to invest in, and (b) how to apportion the available money among the stocks chosen. To help the decision making process, she has access to the daily movements of all the stocks in the market for the last few years. Obviously, her goal is to choose a portfolio that would maximize her expected return – subject to a ‘reasonable’ risk – if she holds on to the portfolio for some period of time in future. The problem faced by the investor is what is known as the cardinality constrained portfolio selection problem. This is a well researched (see[1], [2], [6], [15]) problem in the finance arena, and can be solved using the widely studied mean-variance model that, in the instant problem, results in a quadratic mixed integer programming problem [6]. This model-based approach assumes a statistical model of stock price dynamics [4], and is as good as the accuracy of the underlying stock market model. This paper presents preliminary results of an alternate way of tackling the problem using an agent-based scheme. The proposed scheme is a model free approach that does not make any assumption about the underlying stock prices. Agents learn an optimal portfolio-selection strategy directly, without forming any explicit model of stock price dynamics.

The proposed agent based model is related to the model of social learning, or learning by imitation; suggested by game theorists. The agents use a first order approximation to a worst case optimal portfolio selection rule, which is similar to the approach to bounded rationality in game theory. In contrast to the bounded-rationality learning in games, we assume that in the portfolio selection problem agent’s opponent (the market) plays the same strategy for all the agent strategies. For a detailed discussion, see [11].

The agent based scheme was tried out on data drawn from FTSE 100 dataset. FTSE 100 is a share index of 100 most highly capitalized stocks listed on London stock exchange. The portfolio suggested by the proposed scheme was found to outperform the portfolio suggested by the mean-variance approach when applied on same out-of-sample data. We consider our findings to be preliminary in nature because we believe the performance of the agent based system can be further improved upon and quite a few different strategies are currently on the anvil.

The rest of the paper is structured as follows. In section 2, we briefly review the literature for solving the cardinality constrained portfolio selection problem and set out the motivations for applying the multi-agent model to this problem. Section 3 gives a few definitions used in our multi-agent model. Section 4 presents the general scheme and working of the multi-agent model. In section 5, we present the results of our experimental investigation. Section 6 concludes the paper and gives directions for future research.

2. Literature Survey and Motivation
In this paper we concentrate on long-only portfolios. That is, short selling – wherein an investor can borrow a stock and sell it with the expectation that the price of the stock will go down in future – is not allowed.

The return of a portfolio is a random variable which depends upon the choice of the portfolio. Let us consider \( n \) assets and let \( w_i \) be the proportion of capital invested in asset \( i \) and \( w = (w_1, w_2, \ldots w_n) \) be the portfolio resulting from this choice. To represent a portfolio, the weights \((w_1, w_2, \ldots, w_n)\) must satisfy a set of constraints that form a feasible set \( P \) of decision vectors. The simplest way to define a feasible set is by the requirement that weights must sum to one. For this basic version of the problem, the set of feasible decision vectors is

\[
P = \{(w_1, \ldots, w_n) : \sum_{i=1}^{n} w_i = 1, w_i \geq 0, \forall i \in \{1, \ldots, n\}\}.
\]

The portfolio selection problem requires models to choose between random variables (portfolios). The purpose of such models is to specify a preference relation among random variables and to identify random variables that are non-dominated with respect to that preference relation.

A very popular decision model used in finance for this purpose is mean-risk model. In a mean-risk model, portfolio decisions are based on two statistics of the portfolio return distribution: the expected return and the value of a risk measure. Markowitz [9] used variance as the risk measure and his work laid the foundation of the present day Modern Portfolio theory (MPT). The mean-variance formulation without any cardinality constraint results in a quadratic programming (QP) problem and a number of algorithms is available for solving such QP problems (See [10]). In the mean-variance framework, due to modeling requirements, cardinality restriction on the number of stocks to be held in the portfolio warrants threshold restrictions on the weights of stocks in the portfolio. The resulting problem is a quadratic mixed integer programming (QMIP) problem and is computationally challenging (see [1], [6], [7], [10]).

One formulation of the cardinality constrained mean-variance (MV) model is given below.

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \sigma_{ij} w_j
\]

subject to:

\[
\sum_{i=1}^{n} w_i \mu_i \geq d
\]

\[
\alpha \delta_i \leq w_i \leq \beta \delta_i \quad \forall i = 1 \ldots n
\]

\[
\sum_{i=1}^{n} \delta_i = k
\]

\[
\sum_{i=1}^{n} w_i = 1
\]

\[
w_i \geq 0 \quad \forall i = 1 \ldots n
\]

\[
\delta_i \text{ binary } \forall i = 1 \ldots n
\]

\( \sigma_{ij} \) represents the co-variance between returns of stock \( i \) and \( j \). \( \mu_i \) is the expected return of stock \( i \). \( \alpha \) and \( \beta \) represents the lower and upper threshold on stock weight, \( d \) is the desired level of expected return and \( k \) is the number of stocks to be held in the portfolio.

Another formulation of mean-variance model can be maximization of portfolio’s expected return while putting a constraint on portfolio’s risk. Jobst et al. [6] use integer restart and re-optimization heuristics for the QMIP when considering mean-variance models with cardinality, threshold and round lot constraints; they show that the mean-variance efficient frontier, in the presence of these constraints, becomes discontinuous. Bienstock [1] and Lee and Mitchell [7] use QMIP techniques to solve the portfolio selection problem with an upper limit on the size of the portfolios. Chang et al. [2] use heuristic algorithms (genetic algorithm, tabu search and simulated annealing) to solve cardinality constrained problems with specified portfolio sizes.

Since the pioneering work of Markowitz, several alternative risk measures like MAD, lower partial moments, gini, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) have been used in the mean risk models [18]. In addition, several other decision making theories like expected utility maximization, stochastic dominance, Bayesian theory, Fuzzy logic and other heuristic based techniques have been used in investment decision making process (see [12], [20], [4], [3], [18]). More recently, many agent based techniques have also been successfully applied in solving the stock trading and portfolio management problem (see [8], [11], [16], [17], [19], [13]) Sycara et al. ([16], [17]) propose design of a portfolio management system that focuses on portfolio monitoring rather than on portfolio selection. Portfolio monitoring involves many concurrent goals like monitoring an asset currently held or monitoring buying and selling of an asset. However, the present work is concerned with portfolio selection, and draws inspiration from Parkes and Huberman [11] work on use of multi-agent models with co-operative search for solving the general multi-period portfolio selection problem.
Parkes and Huberman [11] propose multi-agent models for the portfolio selection problem following the model free approach. Their model assumes a system of agents that share their initial wealth and make individual investment decisions before sharing profit and losses at the end of the final period. In their experiments, carried out across many trials, they tried out three types of agents; agents following non-adaptive search, agents following adaptive search and agents following co-operative search. The agents in non-adaptive search stick to their initial portfolio, buying and selling the stocks as needed to maintain the investment in any stock equal to the initial proportion, throughout the investment period.

The second type of agents follows adaptive search and build on a computationally efficient portfolio updating rule which they call the $\chi^2$ rule. Each agent in the group follows the $\chi^2$ rule. The $\chi^2$ rule updates the weight of a stock in the agent’s portfolio according to the stock’s performance in the most recent time period relative to the portfolio’s performance and its learning rate $\eta$. The learning rates of different agents in the group are different.

The third type of agents follows co-operative search technique. The agents in this group are identical to the agents in the group following adaptive search so far as learning is concerned. In addition, co-operative agents also exchange hints with other agents about its portfolio’s recent performance. It can also switch probabilistically to the best performing portfolio among the agents.

The overall portfolio $w'_i$ of a system of agents in round $t$ is given as

$$w'_i = \frac{\sum_{j=1}^{M} (\text{wealth}^t_j)w'_i}{\sum_{j=1}^{M} \text{wealth}^t_j}$$

where, $\text{wealth}^t_i$ is the wealth of agent $i$ at the start of period $t$, and $M$ is the number of agents.

Parkes and Huberman tested their models on simulated data under simple market and CAPM market. In both the markets, there were 10 stocks in the asset universe. Under simple market simulations, they assumed that “The dynamics of price changes are independent across stocks and there is no correlation between return and risk across stocks.” In simple market they found that, in general, adaptive agents outperformed non-adaptive agents and communicating agents outperformed both adaptive and non-adaptive agents. In their simulation of the CAPM market, the simple market was augmented with correlations between stock price dynamics. Here the performance difference between cooperative and adaptive agents was not found to be substantial.

As a prelude to the work reported here, we have tried out Parkes and Huberman scheme per se on FTSE 100 data. We found that the final portfolio almost always have all the stocks of the asset universe in the portfolio. Thus the model does not serve the purpose of stock selection well, rather proves useful in deciding the proportion of non-zero investment among these stocks. If the asset universe is large this would mean a very small investment across all the stocks, which is rarely acceptable in practice.

Drawing heavily on Parkes and Huberman’s work, here we propose an alternative cooperative agent based scheme that can serve as an alternative method for solving the cardinality constrained portfolio selection problem. As reported in the Empirical Results section, using the same past real market data of FTSE 100, the agent based model arrives at a portfolio that performs better than the portfolio given by mean-variance model when tried on the same out-of-sample data.

3. Multi-agent model: Some Definitions

Let us consider a market with $t=1,...,T$ discrete investment periods. Let $x'_i$ denote the price relative of stock $i$ in period $t$, the ratio of closing price on day $t$ to closing price on day $t-1$ over the period. The vector $w = (w_1,...,w_n)$, where $w_i \geq 0$ and $\sum_{i=1}^{N} w_i = 1$ defines a portfolio under no short selling; $w_i$ is the fraction of total investment in stock $i$. The return-on-investment in a single period, for an agent with portfolio $w$ and price relatives $x = (x_1,...,x_n)$, is given by the weighted sum over all stocks, $w.x = \sum_{i=1}^{N} w_i x_i$. The return on investment over $T$ periods is given as the product of single period returns $\mathcal{R} = \prod_{t=1}^{T} w'.x'$ . The goal of multi period portfolio selection is to select a sequence of portfolio strategies, $\{w^t\}$ , to maximize a measure of performance over the final return-on-investment. In our case we are more interested in the end-of-investment period portfolio ignoring the intermediate portfolios. The investment period in our case, serves the purpose of learning period based on which an agent takes his portfolio decision.

The multi-agent model is an online decision problem. To guide the design and analysis of an online algorithm, a useful technique is to compare its performance with the performance of an optimal
offline strategy which solves the same decision problem but with information about all the future inputs (This is known as competitive analysis). Let \( \text{Perf}_{\text{comp}}: \mathbb{R} \to \mathbb{R} \) be the performance measure used for comparison of the algorithms. Let \( \text{online}(\{x^t\}) \) be the portfolio’s return on investment using the online algorithm and \( \text{offline}(\{x^t\}) \) be the portfolio’s return on investment from the offline algorithm. The online algorithm is called strongly competitive if its worst case performance is same as that of the offline algorithm in long term, i.e.,
\[
\lim_{T \to \infty} \min \left[ \frac{\text{Perf}_{\text{comp}}(\text{online}(\{x^t\}))}{\text{Perf}_{\text{comp}}(\text{offline}(\{x^t\}))} \right] = 1,
\]
where the minimization is over all feasible input sequences, \( \{x^t\} \) of length \( T \).

The optimal offline strategy used for comparison is the best offline constant rebalanced portfolio (CRP), \( w^*_{\text{CRP}} \), computed with complete information on the sequence of stock prices, \( \{x^t\} = x^1, \ldots, x^T \), which maximizes final return-on-investment, i.e.,
\[
w^*_{\text{CRP}}(\{x^t\}) = \arg \max_u \prod_{t=1}^T w^t x^t
\]
where the maximization is over all constant rebalanced portfolios. A constant rebalanced portfolio maintains the same portfolio across all periods.

In our multi-agent setting we want the agents to follow strategy which is strongly competitive with the best offline constant rebalanced portfolio. Parkes and Huberman in their work used the \( \chi^2 \) rule, which is an approximation to a model free portfolio selection rule EG which is strongly competitive with the best offline constant rebalanced portfolio [5]. Hence we adopt the same rule in our work. Next we revisit the definitions for \( \chi^2 \) rule and performance measure [11] and also give a few other definitions used in our work.

**Definition 1:** The \( \chi^2 \) portfolio updating rule – If \( w^t \) represents the portfolio in investment period \( t \) and \( x^t \) gives the price relatives observed in that investment period, the portfolio, in period \( t+1 \), represented as \( w^{t+1} \) is given as
\[
w^{t+1}_i = w^t_i (\eta (\frac{x^t_i}{w^t . x^t} - 1) + 1)
\]
where \( \eta > 0 \) is the learning rate. This rule increases the fraction of wealth invested in stocks that outperform the portfolio, while decreasing the fraction of wealth invested in the under-performing stocks. Learning parameter \( \eta \) in the \( \chi^2 \) rule differs across agents and determines an agent’s responsiveness to the changes in stock prices (as conveyed by the price relatives). It adjusts weight of stocks moving in a direction that will give a better performance if the price relatives in the current period characterize future periods.

**Definition 2:** \( \text{Perf} \), Performance Measure – The performance, \( \text{Perf} \), of a multi-period portfolio strategy \( \{w^t\} \) for an agent with a logarithmic utility for return on investment is:
\[
\text{Perf} = E_{\{w^t\}} [\log(\prod_{t=1}^T w^t . x^t)]
\]
where the expectation is taken over sequence of price relatives, \( \{x^t\} \), distributed according to market price dynamics, and \( T \) is the number of investment periods.

**Definition 3:** \( \text{Gmeasure} \), Global measure of stock – The measure of stock \( i \)’s overall performance, \( \text{Gmeasure}_i \), is defined as
\[
\text{Gmeasure}_i = \frac{P^t_i}{P^0_i} \times \frac{c_i}{t}
\]
where \( P^t_i \) is the price of stock \( i \) in period \( t \), \( P^0_i \) is the price of stock \( i \) in period 0, \( c_i \) is the count of number of times stock \( i \)’s price has risen relative to last day (or, equivalently, price relative is greater than 1), \( t \) is the current time period.

\( \text{Gmeasure}_i \), measures the goodness of a stock. \( \frac{P^t_i}{P^0_i} \) gives the price relative of the stock on day \( t \) with respect to day 0 which is an indication of the return that would be realized if the stock was purchased at the beginning and held for \( t \) investment periods. \( \frac{c_i}{t} \) is an indication of the pattern stock price has followed in reaching the particular price level on day \( t \). Together they give an indication of the stock’s potential.

**Definition 4:** \( \text{Rmeasure} \), Portfolio measure of stock – The measure of stock \( i \)’s performance in the portfolio of agent \( m \), relative to other stocks in the portfolio, \( \text{Rmeasure}^m_i \) is defined as
\[
\text{Rmeasure}^m_i = w^t_i \times \frac{P^t_i}{P^0_i} \times \frac{c_i}{t} = w^t_i \times \text{Gmeasure}_i
\]
4. Working of the multi-agent model

Let $k$ be the cardinality of the portfolio to be constructed. In our scheme of affairs, each agent forms an independent cardinality constrained portfolio based on movements of stock prices in the past. The portfolios thus formed by the group of agents are then ‘pulled together’ to form the final cardinality constrained portfolio that the investor would use for investments in the near future. At the first investment period of the past under consideration, each agent is handed down a pseudo-random portfolio of $k$ stocks, each having same weight in the portfolio. Agents do not have any idea how the stocks would perform in subsequent time periods. As each agent steps into the next investment period, it adjusts the weights of the stocks based on stock price movements, and can also periodically replace selected poor performing stocks with ‘better’ stocks. Agents also share information among themselves which may result in an agent reinitializing its portfolio with the portfolio of some other agent. Thus the multi-agent model for portfolio construction is a five step process. In step one an initial random portfolio is assigned to each agent. Step two involves updating of weights of the stocks in the portfolio in response to changes in stock prices. In step three a few poor performing stocks in an agent’s portfolio are replaced with better performing stocks. Step four involves exchange of hints among the agents and accordingly a poor performing agent may try to follow the best performer’s strategy hoping to improve its own lot. Step five is the final step of portfolio construction which reports the cardinality constrained portfolio. We now discuss each of these steps in detail.

4.1. Initialization

To start with, every agent is assigned a share of initial wealth $V_0/M$, where $V_0$ is the initial total wealth and $M$ is total number of agents in the group. $M$ initial pseudo-random portfolios are selected and assigned to the agents in the group. An initial portfolio consists of $k$ number of stocks, selected in such a way that each stock is present in approximately equal number of agents’ portfolio across the group.

A global list of all the stocks is maintained and the first agent is free to pick up $k$ number of stocks randomly from this global list. The stocks already selected from the global list get omitted from the global list. The second agent then randomly picks up $k$ number of stocks from the remaining stocks in the global list which again get omitted from the list reducing the choices available to the next agent. This procedure is continued till there are less than or equal to $k$ number of stocks left in the global list of stocks. The stocks left in the global list are then included in the next agent’s portfolio. Once the global list is fully exhausted the list is fully replenished with all the stocks, and any shortfall in the last assigned agent’s portfolio is made up randomly from the fully replenished global list (and the selected stocks get omitted from the global list for next agent). In this way the procedure of formation of initial random portfolio is continued for all the agents. All stocks in an agent’s portfolio are assigned equal weight of $1/k$, i.e. has equal proportion of wealth to begin with.

The reason for assigning the stocks to the portfolio in the way discussed above warrants some discussion. As known from other models for portfolio selection in finance like the popular mean-variance model, the risk and return of the portfolio is not just a function of individual stock’s performance but also depends upon the way stocks behave together. For example, the covariance of stocks in the portfolio is also a contributing factor to the risk in the mean-variance model. By assigning the stocks in the way discussed above we try to capture this effect. While a stock is present in approximately same number of agents across the group, the other stocks it exists with in different agents are different. We hence get an indication of the way a portfolio performs with different combination of stocks, and try to make the set of choices more balanced and heterogeneous.

4.2. Updating

Every agent in the system updates the weight allocated to stocks in its portfolio at the end of each investment period based on the most recent price relatives by using the $\chi$ “portfolio updating rule” as proposed in [11]. The agents have heterogeneous learning rate uniformly distributed between lower bound $\eta_l$ and upper bound $\eta_u$. This learning rate determines the degree of risk aversion of agent. An agent who is risk loving is quite proactive in adjusting the weight based on a stock’s recent performance relative to other stocks in the same portfolio. Such an agent is characterized by a high learning rate. On the other hand, a risk adverse agent has a relatively low learning rate, and, hence is conservative in adjusting the relative weights of stocks in the portfolio. This step ensures continuous adjustment of agent’s portfolio normally desired for good performance.
4.3. Replacement

The updating process as described above, adjusts relative weights of the \( k \) stocks in an agent’s portfolio. But some stocks in the portfolio may turn out to be consistent poor performers relative to others, and hence, its weight will slide at a rate that would depend on the learning rate of the agent. The replacement step allows an agent to periodically get rid of poor performing stocks while ensuring that only \( k \) number of stocks with non-zero investment is present in the portfolio of any agent at any time.

Every agent in the system of agents keeps a count of the time periods for which investment in the same set of stocks was maintained (time since last change in portfolio’s stocks). When this count reaches a pre-specified replacement window size of \( \lambda \) time periods, the agent identifies relatively poor performers and replaces them. The Portfolio measure of stock, \( R_{\text{measure}}^i \), gives an intuitive measure of stock \( i \)’s goodness vis-à-vis other stocks in the portfolio of agent \( m \). After every replacement window size, \( y \) stocks in the agent’s current portfolio with lowest value of the \( R_{\text{measure}}^i \) get replaced with other better performing stocks from the asset universe.

The stock that would replace an outgoing stock of the portfolio is selected as follows. A global list of all the stocks, sorted in descending order of Global measure of stock, \( G_{\text{measure}} \), is maintained in each time period. For any outgoing stock from an agent’s portfolio, all the stocks not already present in that agent’s portfolio and having \( G_{\text{measure}} \) values greater than that of the outgoing stock form a set of eligible stocks. The replacing stock is then picked up randomly from the set of these eligible stocks. If the set of eligible stocks is empty, the current outgoing stock is not replaced. The replacing stock is introduced in the current portfolio with the same weight as the outgoing stock.

At the end of every \( \lambda \) periods, every agent first carries out replacement of stocks in its portfolio, and then follows the \( \chi \) portfolio updating rule to update the weights of stock in the new portfolio.

4.4. Co-operation

In the previous steps agents have been taking the portfolio decisions in isolation. However agents can possibly improve their performances by sharing information with other agents. In our proposed multi-agent model, the agents exchange hints among themselves to indicate the performance of their portfolios. An agent performing poorly may switch to the portfolio of the agent performing best at that time. Both agents have the same portfolio immediately after switching. But, they follow separate strategies as represented by their learning rates \( \eta \) and their subsequent stock replacements, which are likely to be different. Additional parameters, like switching probability \( p \), and exchange window size \( \tau \), decides the extent of communication among the agents.

Every agent maintains a count of the period for which it holds the current portfolio since last portfolio switching. When this count is greater than or equal to \( \tau \), the agent posts its average performance and current portfolio on the global blackboard. The average performance is computed as the average per-period return over a finite number of recent periods, \( \tau \). An agent performing poorly can switch to the best performing portfolio on the blackboard with probability \( p \). If the switch takes place, the period count for which the current portfolio is held is reset to zero.

If the time period of switch and replacement coincides, an agent first carries out portfolio switching probabilistically before carrying out replacement of stocks and updating of the portfolio.

A point to note is that step 4.3 and 4.4 ensures periodic selection of good stocks. Continuous selection would not be a good strategy as the stock’s potential could be observed only over a few time periods and not in a single period.

4.5. Portfolio construction

The last step in our multi-agent model is the construction of the final portfolio with non-zero investment in only \( k \) of stocks. The procedure is detailed below.

The agents in the group are arranged in descending order of their end-of-investment period wealth. The portfolios of the top \( q \) percent of the agents are taken for the purpose of final portfolio formation. We then add up the weights in stock \( i \), \( \forall i \in n \) across portfolios of these top \( q \) percent agents to get

\[
w_i = \sum_{m=1}^{M} w_{i}^T \quad (\forall i \in n),\]

where \( M_q \) represents the agents in the top \( q \) percent. This weight \( w_i \), in a sense, represents the importance given to stock \( i \) by best performing agents. The portfolio so formed will in general have more than \( k \) number of stocks and the sum of final weights across all stocks in this portfolio will be more than one. The final cardinality constrained portfolio is formed by picking up \( k \) number of stocks with highest weights from this candidate portfolio and normalizing them so that the weights add up to one.
When the portfolio is formed by averaging across many trials, then we average across trials the weights of stocks in top \( q \) percent agents in every trial and then follow the procedure as described above.

The complete algorithm for multi-agent portfolio selection is given below

**Algorithm:** Multiagent Portfolio ( )

Gobal: 
- \( T \); // Number of investment periods
- \( n \); // Number of stocks
- \( M \); // Number of agents
- \( k \); // Cardinality value
- \( V_o \); // Initial wealth

1. For each agent \( m \in M \)
   a. Allocate initial wealth i.e. \( m\.wealth = V_o / M \)
   b. Allocate an initial random portfolio with non-zero investment in \( k \) stocks, i.e. \( m\.w = \text{Initialize}(k) \)
   c. Assign a random learning rate from the uniform distribution, i.e. \( m\.\eta \sim \mathcal{U}(\eta_1, \eta_2) \)
   d. Initialize replacement window counter, i.e. \( m\.replacement\_window\_count = 0 \)
   e. Initialize exchange window counter, i.e. \( m\.exchange\_window\_count = 0 \)

2. Initialize investment period counter, i.e., \( t = 1 \)

3. While \( (t \leq T) \)
   a. Read the current stock price relatives, i.e. \( (price, x) = \text{price\_relatives}(t) \)
   b. For each agent \( m \in M \)
      i. Trade at the current prices to maintain investment in stocks as per the current stock weights, i.e. \( m\.trade(price) \)
      ii. Update agent’s wealth, i.e. \( m\.wealth = m\.wealth \ast (m\.w \cdot x) \)
      iii. Update agent’s average performance over most recent \( \tau \) investment periods, i.e. \( m\.perf = m\.average(m\.w \cdot x, \tau) \)
      iv. Increment replacement window counter, i.e. \( m\.replacement\_window\_count = m\.replacement\_window\_count + 1 \)
      v. Increment exchange window counter, i.e. \( m\.exchange\_window\_count = m\.exchange\_window\_count + 1 \)
      vi. If agent’s exchange window counter is greater than or equal to the exchange window size \( \tau \), then post agent’s performance and current portfolio on global blackboard, i.e. If \( (m\.exchange\_window\_count \geq \tau) \) then \( m\.post() \)
   c. For each agent \( m \in M \)
      i. If agent’s exchange window counter is greater than or equal to the exchange window size \( \tau \), then probabilistically switch to the portfolio of the best performer on the global blackboard and reset agent’s exchange window counter to zero if switching occurs, i.e. \( (m\.exchange\_window\_count, m\.w) = m\.switch(p) \)
      ii. If agent’s replacement window counter equals the replacement window size \( \lambda \) then
         - Sort stocks in descending order on basis of \( R\text{measure}^* \)
         - For each stock in the bottom \( y \) stocks in this sorted list
            - Form a set of eligible stocks with \( G\text{measure} \) values greater than that of current stock and not already present in agent’s current portfolio;
            - Randomly pick a stock from this set of eligible stocks and replace the current stock in the agent’s portfolio with this new stock. The new stock is assigned the same weight as the outgoing stock and the weight of the outgoing stock is initialized to zero;
            - If the set of eligible stocks is empty do not replace the current stock;
            - Reset agent’s replacement window counter to zero;
      iii. Update agent’s portfolio using the \( \chi^2 \) rule, i.e., \( m\.w = m\.update(x) \)
   d. Increment investment period counter, i.e., \( t = t + 1 \)

4. Sort agents in decreasing order of their end-of-investment period wealth;
5. Form a candidate portfolio by adding the portfolios of top \( q \)\% of the agents from this sorted list;
6. Sort the candidate portfolio on the basis of stock weights. Pick top \( k \) stocks from this candidate portfolio and normalize them to form the final cardinality constrained portfolio.
5. Empirical Results

In this section we investigate the performance of the multi-agent model which simulates a group of investors. We check the out-of-sample performance of the portfolios constructed using our multi-agent model approach, and compare them to the performance of the portfolios formed using the traditional mean-variance approach.

5.1. Dataset description

Four datasets, drawn from the FTSE 100 index, were used for this investigation. The characteristics of the datasets are given in Table 1 below. As evident from Table 1, for the first dataset daily price relatives of 79 stocks that belong to the index throughout the period Jan 2000 - December 2003 were considered. For each of the remaining 21 stocks’ data, there is at least one missing data item in the specified period. For the out-of-sample analysis, we considered three out-of-sample periods of different time spans: Jan 2004, Jan-Feb 2004 and Jan-March 2004. The other three in-sample periods, and corresponding out-of-sample periods were decided in a similar manner. The parameters in our preliminary investigation were chosen by means of running a number of simulations. A natural question that arises is how sensitive are the results to the choice of these parameters. This is a topic of our future research.

<table>
<thead>
<tr>
<th>Dataset no.</th>
<th>n</th>
<th>In-sample period</th>
<th>Out-of-sample periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79</td>
<td>Jan 2000 - Dec 2003</td>
<td>Jan 04</td>
</tr>
<tr>
<td>2</td>
<td>79</td>
<td>Jan 2001 - Dec 2004</td>
<td>Jan 05</td>
</tr>
<tr>
<td>3</td>
<td>78</td>
<td>Jan 2002 - Dec 2005</td>
<td>Jan 06</td>
</tr>
<tr>
<td>4</td>
<td>78</td>
<td>Jan 2003 - Dec 2006</td>
<td>Jan 07</td>
</tr>
</tbody>
</table>

5.2. Parameter values

\[ M = 300, \quad Trials = 1000, \quad k = 15, \quad V_0 = 1, \quad y = 5, \quad q = 0.1 \ \text{i.e.} \ 10\% . \]

Other parameters’ values are given in Table 2 below.

<table>
<thead>
<tr>
<th>Dataset no.</th>
<th>( T )</th>
<th>( \lambda )</th>
<th>( \tau )</th>
<th>( \eta_l )</th>
<th>( \eta_h )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1007</td>
<td>50</td>
<td>75</td>
<td>0.8</td>
<td>0.9</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>1011</td>
<td>50</td>
<td>75</td>
<td>0.8</td>
<td>0.9</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>1010</td>
<td>50</td>
<td>75</td>
<td>0.8</td>
<td>0.9</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>1011</td>
<td>100</td>
<td>100</td>
<td>0.1</td>
<td>0.2</td>
<td>0.05</td>
</tr>
</tbody>
</table>

5.3. Portfolio performance

We investigate the potential of the portfolios formed using the multi-agent model. As evident from Tables 3, 4, 5 and 6 the portfolios formed using the multi-agent model has higher in-sample expected return and lower risk (standard deviation) when compared to the index (FTSE 100) during the same period. When tested out-of-sample, for different datasets, we notice that in nearly all the cases (except two cases in dataset 4) across different datasets, the multi-agent portfolios achieve higher average returns across one month, two months and three months out-of-sample periods than the index.

We were more interested in comparing the multi-agent portfolios’ performance with performance of the portfolios constructed using cardinality constrained mean-variance models (with threshold restrictions considered as 0.00001 and 1). We considered two mean-variance portfolios on each dataset.

MV1: This portfolio is obtained by minimizing portfolio’s variance with the portfolio’s expected return constrained to be greater than or equal to multi-agent portfolio’s value of in-sample expected return.

MV2: This portfolio is obtained by maximizing portfolio’s expected return with the portfolio’s variance constrained to be less than or equal to multi-agent portfolio’s value of in-sample variance.

<table>
<thead>
<tr>
<th>Period</th>
<th>Statistics</th>
<th>Multi-agent</th>
<th>MV-1</th>
<th>MV-2</th>
<th>FTSE 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-sample</td>
<td>Exp Ret</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0014</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.0085</td>
<td>0.0068</td>
<td>0.0085</td>
<td>0.0139</td>
</tr>
<tr>
<td>OS1</td>
<td>Avg Ret</td>
<td>0.0030</td>
<td>0.0019</td>
<td>0.0031</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.0083</td>
<td>0.0059</td>
<td>0.0077</td>
<td>0.0055</td>
</tr>
<tr>
<td>OS2</td>
<td>Avg Ret</td>
<td>0.0029</td>
<td>0.0025</td>
<td>0.0026</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.0070</td>
<td>0.0059</td>
<td>0.0074</td>
<td>0.0057</td>
</tr>
<tr>
<td>OS3</td>
<td>Avg Ret</td>
<td>0.0024</td>
<td>0.0018</td>
<td>0.0018</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td>Std Dev</td>
<td>0.0081</td>
<td>0.0080</td>
<td>0.0097</td>
<td>0.0071</td>
</tr>
</tbody>
</table>
It is interesting to note that, going by in-sample results, the MV models provide better promising portfolios. In all the datasets, the in-sample portfolios of MV1 and MV2 outperform the portfolio of multi-agent model in risk or return. However, when it comes to out-of-sample performance, the portfolio formed by the multi-agent model has always ensured better average return than MV1. Compared to MV2, except in the case of one-month out-of-sample period in dataset 1, the multi-agent portfolio again provides better average return. The daily out-of-sample cumulative returns depicted in Figures 1, 2, 3 and 4 also clearly highlight the better performance of the multi-agent portfolios vis-à-vis mean-variance portfolios. The out-of-sample standard deviations of the multi-agent portfolio are also quite comparable with those of the portfolios of MV models. Thus, our experiments on FTSE 100 demonstrate that, in general, the proposed multi-agent model may have the potential of suggesting portfolios with promises of better performance than those suggested by mean-variance models.

<table>
<thead>
<tr>
<th>Table 4: Performance on dataset 2</th>
<th>Table 5: Performance on dataset 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Period</strong></td>
<td><strong>Statistics</strong></td>
</tr>
<tr>
<td><strong>Multi-agent</strong></td>
<td><strong>MV-1</strong></td>
</tr>
<tr>
<td><strong>MV-2</strong></td>
<td><strong>FTSE 100</strong></td>
</tr>
<tr>
<td><strong>In-sample</strong></td>
<td><strong>Exp Ret</strong></td>
</tr>
<tr>
<td>Exp Ret</td>
<td>0.0011</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0086</td>
</tr>
<tr>
<td><strong>OS1</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0006</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0065</td>
</tr>
<tr>
<td><strong>OS2</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0016</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0060</td>
</tr>
<tr>
<td><strong>OS3</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0006</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0056</td>
</tr>
<tr>
<td><strong>Period</strong></td>
<td><strong>Statistics</strong></td>
</tr>
<tr>
<td><strong>Multi-agent</strong></td>
<td><strong>MV-1</strong></td>
</tr>
<tr>
<td><strong>MV-2</strong></td>
<td><strong>FTSE 100</strong></td>
</tr>
<tr>
<td><strong>In-sample</strong></td>
<td><strong>Exp Ret</strong></td>
</tr>
<tr>
<td>Exp Ret</td>
<td>0.0012</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0089</td>
</tr>
<tr>
<td><strong>OS1</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0013</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0082</td>
</tr>
<tr>
<td><strong>OS2</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0015</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0068</td>
</tr>
<tr>
<td><strong>OS3</strong></td>
<td><strong>Avg Ret</strong></td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.0016</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0080</td>
</tr>
</tbody>
</table>
6. Conclusions and future research

This work addresses a realistic version of the portfolio selection problem, with restriction on the number of stocks to hold in the portfolio. In this paper we propose an agent based model for solving this problem which can serve as an alternative for construction of cardinality constrained portfolio. We tested the performance of the portfolios constructed using this method on live data from FTSE 100. Our experimental results show that the portfolios constructed using multi-agent method may outperform the index and the mean-variance portfolios. However, a lot of investigation is needed before we arrive at a robust scheme. What would be a generic prescription for choosing the parameter values? How sensitive is the performance of the portfolio to the choice of parameters? The current scheme of co-operation is a broadcast mechanism. Would it be better to devise a peer-to-peer dialogue and exchange? Work is currently on to answer some of these questions.

References


