Priority Approximation for Batching

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Abstract

We consider here the one-machine serial batching problem under weighted average completion. This problem is known to be \(NP\)-hard and to date no approximation algorithm exists. Batching has wide application in manufacturing, decision management, and scheduling in information technology.

In this paper, we give the first approximation algorithm for the problem. The algorithm has an approximation ratio of 2 and is a priority algorithm, which batches jobs in decreasing order of priority. We also give a lower bound of \(2 + \sqrt{6/4} \approx 1.1124\) on the approximation ratio of any priority algorithm and conjecture that there is a priority algorithm which matches this bound. Adaptive algorithm experiments are used to support the conjecture. An easier problem is the list version of the problem where the order of the jobs is given. We give a new linear time algorithm for the list batching problem.

1 Motivation and Background

We consider the batching problem where a set of jobs \(J = \{J_i\}\) with processing times \(p_i > 0\) and weights \(w_i \geq 0, i = 1, \ldots, n\), must be scheduled on a single machine, and where \(J\) must be partitioned into batches \(B_1, \ldots, B_r\). All jobs in the same batch are run jointly and each job's completion time is defined to be the completion time of its batch. We assume that when a batch is scheduled it requires a setup time \(s = 1\). The goal is to find a schedule that minimizes the sum of completion times \(\sum w_i C_i\), where \(C_i\) denotes the completion time of \(J_i\) in a given schedule. Given a sequence of jobs, a batching algorithm must assign every job \(J_i\) to a batch. More formally, a feasible solution is an assignment of each job \(J_i\) to the \(m_i^{th}\) batch, \(i \in \{1, \ldots, n\}\).

For example, Figure 1 shows two schedules for a 5-job problem where processing times are \(p_1 = 3, p_2 = 1, p_3 = 4, p_4 = 2, p_5 = 1\) and the weights are \(w_1 = w_4 = w_5 = 1\) and \(w_2 = w_3 = 2\). (We note that the encircled values give the average weighted completion time of the depicted schedules.)

The problem considered in this paper has the jobs executed sequentially, thus the problem is more precisely referred to as the \(s\)-batch problem. We note that there is a different version of the problem not studied here, where the jobs of a batch are executed...
in parallel, known as the \( p \)-batch problem. In that case, the length of a batch is the maximum of the processing times of its jobs. The \( s \)-batch is also denoted in \( \alpha \beta \gamma \) notation as the \( 1|s\text{-batch}| \sum w_i C_i \) problem. Also, we will sometimes for convenience write the job data in the form \( \{(p_1, w_1), (p_2, w_2), \ldots, (p_n, w_n)\} \). Thus in the example we would have written \( \{(3, 1), (1, 2), (4, 2), (2, 1), (1, 1)\} \).

Brucker and Albers [1] showed that the \( 1|s\text{-batch}| \sum w_i C_i \) problem is \( \mathcal{NP} \)-hard in the strong sense by giving a reduction from 3-PARTITION. There is a large body of work on batching problems (see e.g. [2, 3, 6, 7, 9, 12]) and batching has wide application in manufacturing (see e.g. [8, 16, 20]), decision management (see e.g. [14]), and scheduling in information technology (see e.g. [10]). More recent work on online batching is related to the TCP (Transmission Control Protocol) acknowledgment problem (see [4, 11, 13]).

In this paper, we give the first approximation algorithm for the \( 1|s\text{-batch}| \sum w_i C_i \) problem. In fact, we give two algorithms, one called \textsc{PseudoBatch}, according to the order of information algorithms, it is natural to consider the jobs \textsc{CanonicalBest} the other one called \textsc{CanonicalBest}. For approximation algorithms, it is natural to consider the jobs according to the order of priorities \( q_i = \frac{w_i}{p_i} \). If the jobs are renumbered such that \( \frac{w_i}{p_i} \geq \frac{w_j}{p_j} \ldots \geq \frac{w_n}{p_n} \), we say that the jobs are in canonical order. An algorithm that schedules the jobs in this order is called a priority algorithm. Both of our approximation algorithms are priority algorithms.

We recall that the quality of an approximation is measure in terms of its approximation ratio \( \rho \). Given an optimization problem \( \mathcal{P} \) we say that algorithm \( A_\mathcal{P} \) has approximation ratio \( \rho \) if for every instance \( \pi \in \mathcal{P} \),

\[
\rho \leq \frac{\text{cost of the solution given by } A_\mathcal{P} \text{ for instance } \pi}{\text{cost of OPTIMUM for instance } \pi},
\]

where OPTIMUM is value of an optimal solution. We show that \textsc{PseudoBatch} and \textsc{CanonicalBest} both have approximation ratio \( \rho = 2 \). We also give a lower bound of \( 2 + \sqrt{2} \approx 1.1124 \) on the approximation ratio of any priority algorithm and conjecture that \textsc{CanonicalBest} matches this bound. Adaptive algorithm experiments are used to support the conjecture.

A much easier version of the problem is the list version of the problem where the order of the jobs is given, i.e., \( m_i \leq m_j \) if \( i < j \). For example, Figure 2 shows three schedules for a 5-job problem \( \{(3, 1)(1, 1)(4, 1)(2, 1)(1, 1)\} \). The circled values give the average weighted completion time of the schedules to the left.

Brucker and Albers [1] gave a linear time algorithm for the list batching problem. (Thus, to solve the 1|s\text{-batch}| \sum w_i C_i \) problem, it is sufficient to know the order of jobs in the optimal solution.) We give an alternative algorithm in this paper. Our algorithm exploits the fact that the problem can be reduced to a shortest path problem, where the underlying cost matrix is a totally monotone matrix and thus can use the matrix searching algorithm of Larmore and Schieber [15] as a subroutine. A matrix \( A \) is called totally monotone if for all \( i < i' \) and \( j < j' \), \( A[i, j] > A[i, j'] \) implies \( A[i', j'] > A[i', j] \); matrix \( A \) is called Monge if \( A[i, j] + A[i', j'] \leq A[i', j] + A[i, j'] \). Clearly, every Monge matrix is totally monotone. We note that the linear time list batching algorithm is used to implement \textsc{CanonicalBest} in run time \( O(n \log n) \).

Our paper is organized as follows: In Section 2 we give our priority approximation algorithms. Section 3 gives our alternate linear time algorithm for the list batching problem. Section 4 presents the lower bound on the approximation ratio of any priority algorithm. Section 5 describes genetic algorithm experiments. Specifically, we give an adaptive algorithm experiment which supports the conjecture that the approximation ratio of \textsc{CanonicalBest} matches the lower bound. This section also contains the description of a genetic algorithm for the 1|s\text{-batch}| \sum w_i C_i \) problem implemented under GAlib, the object-oriented library of Matthew Wall [19] developed at MIT. We conclude with open problems in Section 6.

### 2 Approximation Algorithms

We give the following technical lemma, which is also known as the “Smith Rule” in the area of scheduling.

**Lemma 1** Given \( p_1, \ldots, p_n > 0, w_1, \ldots, w_n \geq 0 \) with \( \frac{w_1}{p_1} \geq \frac{w_2}{p_2} \ldots \geq \frac{w_n}{p_n} \) and permutation \( \pi \). For \( i = 1, \ldots, n \) let \( P_{\pi(i)} = \sum_{j=1}^{i} p_{\pi(j)} \). Then \( f_\pi = \sum_{i=1}^{n} P_{\pi(i)} w_{\pi(i)} \) is minimized when \( \pi \) is the identity.
Figure 3. PseudoBatch for $p_1 = 0.2, p_2 = 0.6, p_3 = 0.2, p_4 = 0.3, p_5 = 0.1, p_6 = 1.1$.

Proof: Consider permutation $\tau$, which is not the identity. Then $\tau$ has an inversion $j > i$ with $i$ immediately before $j$ in $\tau$. Let $\tau'$ be the permutation with $i$ and $j$ interchanged. We have

$$f_{\tau'} - f_\tau = p_i w_i + (p_i + p_j) w_j - (p_i + p_j) w_i = p_i w_j - p_j w_i \leq 0,$$

since $w_i \leq w_j$. It follows that $f_{\tau'} \leq f_\tau$ and we are done. □

Lemma 2 Let $C_i$ be the completion times of an optimal schedule for the 1|s-batch|\(\sum w_i C_i\) problem. Then we have

$$\sum_{i=1}^{n} w_i C_i \geq \sum_{i=1}^{n} w_i (P_i + 1)$$

Proof: Let permutation $\sigma$ be the order of the optimal schedule. Then

$$P_\sigma^* + 1 \leq C_i.$$

Due to Lemma 1 we have

$$\sum_{i=1}^{n} w_i C_i \geq \sum_{i=1}^{n} w_i (P_\sigma^* + 1) \geq \sum_{i=1}^{n} w_i (P_i + 1).$$

□

We now present a simple, parameterized algorithm, PseudoBatch, for the 1|s-batch|\(\sum w_i C_i\) problem. PseudoBatch first reorders the jobs so that they are in canonical order. Then jobs are assigned to batches in that order. After receiving $J_i$, our algorithm has only two choices, namely whether to assign $J_i$ to the same batch as $J_{i-1}$ or not. We use the phrase “$A$ batches at step $r$” to mean that algorithm $A$ decides that $J_i$ is the first job of a new batch, i.e. $m_i = m_{i-1} + 1$. We use the phrase “current batch” to denote the batch to which the last job was assigned. Then, when $J_i$ is received, $A$ must decide whether to add $J_i$ to the current batch, or “close” the current batch and assign $J_i$ to a new batch. PseudoBatch maintains a variable $P$ which will be the sum of the processing times of a set of recent jobs: we call this set the current pseudobatch.

When $J_1$ is received, $P$ is set to 0. After receiving each subsequent $J_i$, PseudoBatch first adds $p_i$ to $P$. If $P > 1$, PseudoBatch batches and also sets $P$ to zero. Thus, the $i^{th}$ pseudo-batch contains all but the first member of the $i^{th}$ batch, together with the first member of the $(i + 1)^{st}$ batch, unless $i = r$. Every job except $J_1$ belongs to just one pseudo-batch.

Figure 3 gives an example for $p_1 = 0.2, p_2 = 0.6, p_3 = 0.2, p_4 = 0.3, p_5 = 0.1$ and $p_6 = 1.1$.

Theorem 1 PseudoBatch has an approximation ratio of 2.

Proof: As before let $C_i$ be the completion times of an optimal schedule for the 1|s-batch|\(\sum w_i C_i\) problem. Let $\hat{C}_i$ denote the completion times of the jobs when algorithm PseudoBatch is run on the instance and let $m$ be the number of batches created by the algorithm. Clearly we have

$$\hat{C}_i \leq P_i + m + 1$$

and

$$(m - 1) \leq P_i.$$

Thus,

$$\hat{C}_i \leq 2P_i + 2.$$

By Lemma 2 we have

$$\sum_{i=1}^{n} w_i \hat{C}_i \leq 2 \sum_{i=1}^{n} w_i (P_i^* + 1) \leq 2 \sum_{i=1}^{n} w_i C_i.$$

□

Let CanonicalBest be the algorithm which puts the jobs in canonical order and the linear time algorithm of the next section (or the algorithm of [1]) to get the optimal list batching schedule under the canonical order. Clearly we have:

Theorem 2 CanonicalBest has an approximation ratio of 2.

Proof: Algorithm PseudoBatch has approximation ratio of 2 and is a priority algorithm. Given an instance of the problem, algorithm CanonicalBest produces a schedule with weighted average completion no worse than algorithm PseudoBatch. □

3 An Alternate Linear Algorithm for List Batching

We now turn to the run time of Algorithm CanonicalBest. As mentioned earlier, Brucker and Albers
Lemma 3 The matrix $C = (c_{i,j})$ defined in (1) is Monge for all choices of $p_i, w_i \geq 0$. Furthermore values can be queried in $O(1)$ time after linear preprocessing.

Proof: Let $W_i = \sum_{\nu=1}^{i} w_{\nu}$ and $P_i = \sum_{\nu=1}^{i} p_{\nu}$ be the partial sum of the $p_i$ and $w_i$ values. Then we have

$$c_{i,j} = c_{i,j} = (W_n - W_i)(s + j) - P_i$$

For $i < i'$ and $j < j'$

$$c[i,j] + c[i',j'] = c[i',j] - c[i,j']$$

$$= (P_{i'} - P_i)(W_{i'} - W_i)$$

$$\geq 0.$$
Furthermore, these values can be queried in sorted first, in summary we have:

$$E_k$$ since these values are of the form $$k$$

This states that an element requires the priorities to be evaluated in $$O(1)$$ time provided that the row minima of the first $$\ell$$ rows are already known.

1. For each row index $$\ell$$ of $$E$$, there is a column index $$\gamma_\ell$$ such that for $$k > \gamma_\ell$$, $$E_{\ell,k} = \infty$$. Furthermore, $$\gamma_\ell \leq \gamma_{\ell+1}$$.

2. If $$k \leq \gamma_\ell$$, then $$E[\ell,k]$$ can be evaluated in $$O(1)$$ time.

3. $$E$$ is a totally monotone matrix.

If these conditions are satisfied, the LARCSH algorithm then calculates all of the row minima of $$E$$ in $$O(n)$$ time. (See also [5]).

Condition 1 is clear from the fact that in matrix $$E$$ all infinities are in the upper triangle of the matrix.

We turn to Condition 2. Condition 2 describes the online protocol underlying the computation of the dynamic programming tableau. It states that an element in column $$k$$ is “knowable” once the row minimum of row $$k$$ is revealed. Figure 6 illustrates this. For example, once the minimum of row 4 (that is, the value for $$E[4]$$) is known then the values of column 4 are available, since these values are of the form $$E[4] + c[\ldots]$$. Furthermore, these values can be queried in $$O(1)$$ time due to Lemma 3.

Condition 3 follows from Lemma 4.

Since CANONICALBEST requires the priorities to be sorted first, in summary we have:

**Theorem 3** Algorithm CANONICALBEST has run time $$O(n \log n)$$.

### 4 A Lower Bound for Priority Algorithms

We now give a lower bound of $$\frac{2+\sqrt{6}}{2} \approx 1.1124$$ on the approximation ratio of any priority algorithm. To show the lower bound, consider a problem which consists of exactly two jobs. There are two orders of the jobs possible. With each order there are two ways to batch the problem; either a single batch consisting of both items or two single batches, each containing one of the items. Since the order of jobs within a batch is irrelevant, two of the possibilities are identical and thus a total of three possibilities exist for **Optimum**, and CANONICALBEST.

Given now a problem $$(\{p_1, w_1\}, \{p_2, w_2\})$$ we assume equal priorities $$\frac{w_1}{p_1} = \frac{w_2}{p_2} = 1$$ it can be shown that $$(\{p_1, w_1\}, \{p_2, w_2\}) = \{(1, 1+\epsilon), (1+\sqrt{6}, 1+\sqrt{6})\}$$ maximizes $$C = \frac{\text{CanonicalBest}}{\text{Optimum}}$$. We note the value $$\epsilon$$ is used to break the tie and force CANONICALBEST to choose an order different than **Optimum**; however, $$\epsilon$$ will tend to 0.

We use square brackets $$[\ldots]$$ to denote a batch. Now consider:

$$\{(1, 1+\epsilon), (1+\sqrt{6}, 1+\sqrt{6})\}$$ (4)

$$C = (s + p_1 + p_2)(w_1 + w_2)$$

$$= (1 + 1 + \epsilon + 1 + \sqrt{6})(1 + 1 + \sqrt{6})$$

$$= (3 + \sqrt{6} + \epsilon)(2 + \sqrt{6})$$

$$\approx 24.25$$

$$\{(1 + \sqrt{6}, 1 + \sqrt{6})\}$$ (5)

$$C = (s + p_1)w_1 + (2s + p_1 + p_2)w_2$$

$$= (1 + 1 + \epsilon)1 + (2 + 1 + 1 + \sqrt{6})(1 + \sqrt{6})$$

$$= (2 + \epsilon) + (4 + \sqrt{6})(1 + \sqrt{6})$$

$$\approx 24.25$$

$$\{(1 + \sqrt{6}, 1 + \sqrt{6})\}$$ (6)

$$C = (s + p_2)w_2 + (2s + p_2 + p_1)w_1$$

$$= (1 + 1 + \sqrt{6})(1 + \sqrt{6}) + (2 + 1 + \sqrt{6} + 1)(1 + \epsilon)$$

$$= (2 + \sqrt{6})(1 + \sqrt{6}) + (4 + \sqrt{6})(1 + \epsilon)$$

$$\approx 21.80$$

**Optimum** chooses order (6) as it gives lowest cost possible. CANONICALBEST is forced to choose between (4) or (5) due to the slight increase of the priorities of $$(1, 1+\epsilon)$$, caused by $$\epsilon$$, ordering that job first. Both
choices have the same cost. Thus:

\[
C \geq \frac{C_{can}}{C_{opt}} \\
\geq \frac{(3 + \sqrt{6} + \epsilon)(2 + \sqrt{6})}{(2 + \sqrt{6})(1 + \sqrt{6}) + (4 + \sqrt{6})(1 + \epsilon)}
\]

As \( \epsilon \) tends to 0, we have

\[
C \geq \frac{2 + \sqrt{6}}{4} \\
\approx 1.1124.
\]

5 Adaptive Algorithm Experiments

In the previous section we have exhibited a solution which gives the worst approximation ratio for CANONICALBEST when there are two jobs. The question remains whether a more difficult problem exists considering a larger number of jobs. We conjecture that this is not the case. In this section we will give results of computer experimentation that support this conjecture.

To explore problem spaces consisting of more than two jobs an evolutionary algorithm was developed. Individuals consisted of a single problem with an arbitrary number of jobs \( i \), where \( 2 \leq i \leq 6 \). Jobs were limited to six because the optimal solution was obtained by exhaustive search. For an individual, the number of jobs and the job data represent the genetic makeup of that individual. The fitness function \( f(x) \) used to evaluate the suitability of individual \( x \) for inclusion in successive evolutions is simply the CANONICALBEST competitive ratio of the individual’s problem.

The algorithm is seeded with a single individual consisting of at least one job. Any individual with a single job will always have \( f(x) = 1.0 \): mutation is used to quickly produce individuals with two or more genes with \( f(x) > 1.0 \). The evolutionary environment sustains a total of \( \mu = 50 \) parents. Using a simple deterministic selection process, all pairs of parents were mated to produce \( \lambda = \mu^2 = 2500 \) offspring. Of the resulting population of \( \mu + \lambda = 2550 \), the strongest 50 were retained for the next generation.

In evolutionary algorithms, offspring are typically the product of mutation and crossover of their parents; however in our search we only relied on mutations. Mutations of a parent consisted of processing the parent’s jobs (genes); any single job had a probability of \( \phi_{mut} = 0.1 \) of producing a mutation; otherwise it was simply copied to the child. Several types of mutations were applied. Mutations affected either the weight or processing time of a job.

- With probability 0.25 values could be scaled by a random amount with the intention of introducing random variability thereby landing on new neighborhoods of the landscape.
- Values could double (probability 0.25) or values could be halved (probability 0.25) in order to quickly converge on instances that relied on small or large values of a particular job in relation to others.
- With probability 0.25 values could also change by small amounts for the case that a job could be made harder by very small tweaks to the current design. (This is a bit similar to use of \( \epsilon \) in Section 4).

Finally, with probability 0.1 we applied an additional change to the child, where the number of jobs was changed with probability 0.5 to increase and with probability 0.5 to decrease by one job. Decreases were performed by the deletion of a random job with equal probability. Addition of a job was introduced by either adding a random job, or by duplicating one of the already present jobs with equal probability. Change was avoided if it would produce job sizes outside of the allowable range.

When the algorithm is seeded with \( \{(1.0, 1.0)\} \) the resulting evolution was uninteresting. At generation 5 an individual was arrived at consisting of \( \{(1.00, 1.00), (4.00, 4.00)\} \), where \( f(x) = 1.1111 \). From this time up until at least generation 18813, \( \mu \) consisted of 50 clones of this individual and appeared to remain stable despite the existence of a known, more difficult problem (Section 4). Similarly, when seeded with that problem, \( \{(3.45, 3.45), (1.00, 1.00)\} \), \( \mu \) immediately converged on 50 clones of this individual, with no variation at all, up until at least generation 20553. No other, more difficult, problem is found.

When seeded with a random job of \( \{(0.33, 0.54)\} \) things were a bit more interesting. The early genera-

<table>
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<th>Gen</th>
<th>( f(x) )</th>
<th>Problem</th>
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<tbody>
<tr>
<td>49</td>
<td>1.109481</td>
<td>{(1.01,0.40), (3.20,1.26)}</td>
</tr>
<tr>
<td>144</td>
<td>1.109964</td>
<td>{(1.01,0.40), (3.20,1.26)}</td>
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<tr>
<td>284</td>
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<td>26522</td>
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<td>{(1.00,0.40), (3.20,1.26)}</td>
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</table>

Table 1. Improvements During Random Seeded Case.
tions showed a more diverse $\mu$ consisting of six unique individuals; all with $f(x) = 1.0$. In generation 49 the individual of $\{(1.01, 0.40), (3.20, 1.26)\}$ was found with $f(x) = 1.109481$ and $\mu$ consisted of 50 copies. Later generations showed further small improvements. A sampling of that evolution is illustrated in Table 1. All the improvements appear to be the result of the small tweaks mutations, other mutations failing to produce competitive individuals. No further improvement was found as of generation 72233.

We now turn to an unrelated implementation under GAlib, the object-oriented library of Matthew Wall [19] developed at MIT. Because it is possible to give an optimal schedule in linear time if the order is fixed (as described in Section 3) in the $1|\text{s-batch}|\sum w_i C_i$, it is natural to consider a genetic algorithm where the search space is the set of permutations. Then each individual’s fitness – its weighted average completion – can be evaluated in linear time.

We have to define mutation and crossover. A mutation simply swaps two arbitrary elements of the permutation. For the crossover it is important to devise a mechanism that retains some features of the original two individuals in such a meaningful way that results in two new permutations. In the ordered crossover first used by Prins (see [17]), one takes a random subsequence of the first parent’s permutation and insert it directly into the child. As described in Figure 7, the child is then completed by taking material from the second parent’s permutation, where elements are inserted into the child in the order they occur in that parent, starting after the second cut location, and ignoring elements already inserted from the first parent.

All experiments had the following parameters in common:

- Population Size: 1000
- Number of Generations: 5000
- Number of jobs: 100
- Crossover probability: 0.85
- Mutation probability: 0.005

The results were compared with the (conservative) lower bound of Lemma 2 and our results consistently gave solutions within a ratio of $\rho = 1.59$. The detailed results of these experiments are in [18].

6 Conclusions

In this paper we have given the first approximation algorithm for the $1|\text{s-batch}|\sum w_i C_i$ problem. We have also shown that no priority algorithm can have approximation ratio less than $2+\sqrt{6}$. As we have pointed out, we conjecture that algorithm CANONICALBEST matches this lower bound. However, it is an interesting open research problem to prove the correctness of this conjecture.

Note that the lower bound of $\frac{2+\sqrt{6}}{4}$ holds only for priority algorithms. It is current research to investigate if there is a lower bound even if this assumption is dropped, or to give a polynomial approximation scheme in case such a lower bound does not exist.

We note that a version of the algorithm PSEUDOBATCH is useful for online batching problems and we given results for the online case in [4].

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