

Simulation of Online Selling with Posted-price and Auctions: Comparison of Dual Channel's Performance under Different Auction Mechanisms

Hila Etzion and Scott Moore
 Ross School of Business, University of Michigan
 {etzionh, samoore}@umich.edu

Abstract

In this paper we use a simulation model to extend previous analytical research on a firm selling consumer goods online using posted price and auction at the same time. With the simulation we investigate three selling regimes: dual channel with sealed-bid auction; dual channel with open-bid auction, and a single channel with only posted price. We examine how the auction format affects the optimal design of the dual-channel, its performance relative to the single channel, and the sales in each channel. We find that in 81% of the cases we examine, the design that maximizes the average revenue from a dual channel with open-bid auction is the same as the design that maximizes the average revenue from a dual channel with sealed-bid auction. Also, the dual/open regime usually outperforms the dual/sealed regime, and both dual channel regimes outperform the single channel.

1. Introduction

In the business-to-consumer market, many firms are selling the same or almost identical products online using auctions and fixed prices simultaneously. The Internet enables firms to operate the two venues in the same space (the Web) and time, and allows consumers to observe and compare the two selling channels with no additional costs. The practice of operating auctions and fixed prices in parallel, on the Internet, raises many important questions. Clearly the two selling channels cannot be treated independently. Optimizing each channel separately results in a suboptimal global design, as the two channels compete in the same market. Though the problem of optimally selecting and designing a single selling mechanism (auction or posted price) has been well addressed in the literature ([12], [4], [6], and [3]), there is little research on how to operate and design such selling mechanisms in parallel.

The first stream of research on consumer goods sold via auction and posted price at the same time did not consider how consumers choose between the two channels. Vakrat and Seidmann (1999) study the

simultaneous sales of identical products via online auctions and a fixed price catalog. They model a one-unit English auction with a deterministic number of bidders, who have identical delay costs, and examine how an exogenous posted-price option affects the expected auction price. They do not model consumers' choice between the two channels, nor do they show what incentives the seller has to conduct such an auction. Van Ryzin and Vulcano (2004) examine the optimal pricing-replenishment policy when the firm sells in two markets, one fixed price market and one auction market, and decides how to split the inventory between the two markets. In this model demand comes from two different and independent streams of customers, and thus there is no need to model consumers' choice.

Etzion *et al.* (2002, 2006) were the first to model a monopolist, with unlimited product supply, offering a sequence of sealed-bid second price auctions and a posted price in parallel. They model the facts that consumers can choose between purchasing the item and bidding, that different consumers arrive at different stages of the online auction and that their expected utility from bidding is a function of their arrival time. They find dominant bidding strategies for consumers, and identify the optimal design of such dual-channels. Motivated by the work of Etzion *et al.*, Caldentey & Vulcano (2007) develop two related models: in the first, a seller with fixed inventory is running an auction parallel to an outside posted price option with unlimited supply; in the second, a seller with a fixed inventory is offering an auction and posted price in parallel. Caldentey and Vulcano's second setting is a model of an auction with a buy-now price, because sales for the posted price reduce the auctioned quantity. This is not the case in Etzion *et al.* (2002, 2006), where auctioned units are dedicated to the auction. In both papers (Etzion *et al.* 2006 and C & V 2007) consumers with single unit demand arrive according to a Poisson process and decide only once, upon arrival, whether to buy the item for the posted price or bid in the **sealed-bid** auction. If a consumer decides to bid, she has to wait until the conclusion of the auction to receive the item. Both papers find the channel design that maximizes the seller's profit

as a function of consumers' arrival rate and delay sensitivity.

These analytical models explore consumers' optimal choice of channel (auction or posted price), optimal bidding strategies, and the optimal design (from the seller's perspective) of the dual channel, when the auctions are conducted in the sealed-bid format. However, most online auctions are conducted using the open, ascending-bid auction format, where bidders can observe the lowest bid needed to win at every moment of the auction. This suggests that consumers have more information available to them when choosing between the posted price and auction participation than in the sealed-bid auction previously modeled. In addition, when participating in an open ascending-bid auction, a consumer might be outbid before the auction ends, in which case she has to reevaluate whether to raise her bid or quit the auction and buy the item for the posted price. Hence, a consumer might face the "buy-bid" decision several times during the auction, with new information available for her each time she needs to make this decision, and not only upon her arrival to the website as modeled in [2], [7] and [8].

It is not clear whether the analytical results regarding the optimal design and the performance of the dual-channel derived when the seller uses the sealed-bid auction format, hold under an open ascending-bid auction. Intuition predicts that it would be more difficult for the seller to segment the market and prevent cannibalization of the posted-price channel when more information is available to consumers. On the other hand, if high valuation consumers often face the option of quitting the auction before its conclusion and buying the item for the posted price (consumers can quit the auction every time their bid is no longer in the *list of winning bids*¹), perhaps most auction winners would be consumers who can not afford the posted price. It is thus unclear whether a dual channel with open ascending-bid auctions yields, on average, higher or lower profit than a dual channel with sealed-bid auctions.²

It is not feasible to analytically derive the optimal design of dual-channels with open ascending-bid auctions. Even when studying dual-channels with sealed-bid auctions (a somewhat simpler problem), Etzion *et al.* had to investigate the optimal designs, for different arrival rates and delay costs, numerically, because the seller's profit equation could not be used to derive close-form solutions for the optimal values of the posted price, the

auction length and the auction quantity. Caldentey and Vulcano (2007) had to solve their model for the limiting (asymptotic) case. In addition, they had to assume that consumers do not know how many units are left in the auction when sales for posted price reduce the auction quantity. Those assumptions can be relaxed when using a simulation to find the average daily profit for various channel designs in order to identify the most profitable designs. A simulation allows us to compare dual-channels with different auction formats, and understand how changing assumptions required for analytical work affect related outcomes.

We developed a simulation model to investigate three selling regimes: dual channel with sealed-bid auction; dual channel with open-bid auction, and a single channel with only posted price. We examine how the auction format affects the optimal design of the dual-channel, its performance relative to the single channel, and the sales in each channel.

Our results show that when bidders update their expectations regarding the auction price based on the observable information, i.e. the current lowest winning bid, every time they face the bid-buy decision, but use a myopic policy to choose between buying and bidding (not taking into consideration the positive probability that they will quit the auction at a later time), then the optimal design of the dual channel with open-bid auctions is surprisingly similar to the optimal design of the dual channel with sealed-bid auctions. We find that in 81% of the cases we examine, the design that maximizes the average daily revenue from a dual channel with open-bid auction is the same as the design that maximizes the average daily revenue from a dual channel with sealed-bid auction. In addition, we find that the dual/open regime usually outperforms the dual/sealed regime even when the two are set at the design that maximizes the expected dual/sealed revenue (equation for the expected dual/sealed revenue was derived analytically in Etzion *et al.* 2006), and both dual channel regimes outperform the single channel. Only when both arrival rate and delay cost are high, the dual/sealed regime dominates the dual/open.

The paper structure is as follows. In Section 2 we describe the underlying models used in the simulation. In Section 3 we describe the experimental design, i.e. how the analytical models are implemented in the simulation and how we gathered the statistics from the simulation. Section 4 compares the simulation results for the sealed-bid case with the simulation results for the open-ascending bid setting. We conclude in Section 5.

2. Models

In this section we briefly describe the model from Etzion *et al.* (2006), the results of which will be used to simulate a dual-channel with sealed bid auctions. Then,

¹ At each moment of the auction, the "*list of winning bids*" includes the q highest bids, where q is the auctioned quantity.

² We can not use the famous revenue-equivalence theorem due to the existence of the posted price option .

we develop the model that will be used to simulate a dual-channel with open ascending-bid auctions. The main purpose of the simulation is to compare and contrast the results of the two settings.

We model an online seller with unlimited supply, who offers identical items using two selling mechanisms, posted price and auctions, simultaneously. The basic model description holds for both auction formats: sealed and open. The auctions have a fixed duration and are then repeated, and auction units are dedicated to the auction. The seller's objective is to maximize his revenue per unit time. The seller chooses the auction duration T , the quantity to auction q , and the posted price p . Without loss of generality, we assume that the marginal cost of each unit is zero (if this is not so, consumers' valuations of the product can be taken net of the marginal cost). The seller's publicly declared reserve price is R (a public reserve price is equivalent to setting a minimum initial bid.) Risk neutral consumers visit the web site according to a Poisson process with rate λ , and each consumer is interested in purchasing one unit of the good. Consumers have independent private values for the good. We assume that each consumer's valuation, V , is independently drawn from the uniform distribution with support set $[v_l, v_h]$, where $v_l \geq R$. Table 1 Summarizes the notation used in the analytical models.

Low-valuation consumers, those with $V < p$, cannot buy the item for its posted price because the value they get from doing so is negative, so they choose between bidding and staying out of the market. In the dual channel with sealed-bid auction low-valuation consumers choose between bidding and leaving the market only once, upon arrival, while in a dual channel with open-bid auction, they might face this decision multiple times before the conclusion of the auction. Since low-valuation consumers have no other option for obtaining the item, we assume that the delay cost (opportunity cost) per unit time perceived by these consumers is significantly lower than the delay cost per unit time perceived by consumers who can obtain the item instantly by paying the posted price. To simplify the problem, we therefore assume that the delay cost per unit time is zero, i.e. $w = 0$, for consumers with $V < p$. This assumption was used in Etzion *et al.* (2006) but relaxed in Caldentey and Vulcano (2007).

High-valuation consumers, those with $V \geq p$, would buy the item for its posted price if auctions were not offered. High valuation consumers choose between buying the item for its posted price and participating in the auction. It is never optimal for these consumers to do nothing, because their utility from buying the item for the posted price is nonnegative. We assume that when high valuation consumers purchase the item for its posted price they obtain the item instantly. When they choose to bid, they are choosing to experience a delay in obtaining and using the item: bidders must wait until the end of the

Decision variables:

p	posted price.
q	auctioned quantity (per auction).
T	auction length.

Model parameters:

λ	arrival rate of consumers at the website.
$F(v)$	cumulative density function of consumers' valuation distribution with support set $[v_l, v_h]$.
w	delay cost incurred by high-valuation consumers per unit time
R	seller's public reserve price, $R \geq 0$

Other notation:

p_a	auction closing price
S	time remaining in a sealed-bid auction beyond which high-valuation consumers will not participate for the (symmetric) participation-strategy equilibrium.
T_r	remaining time of the auction
B_t	value of the lowest bid on the "winning-bid list" when t time units of the open-auction have passed
h	minimum bidding increment in an open ascending-bid auction.

Table 1: Notation

auction to obtain the item in the dual channel with sealed-bid auctions and must wait at least until the first time they are out-bid in the dual channel with open-bid auctions. Hence, when choosing to bid, these consumers incur a delay cost (opportunity cost) that is an increasing function of the time remaining until they obtain the item.

2.1. Dual Channel with Sealed-bid Auctions

As in [2], [7] and [8], we model the auctions using the sealed-bid $(q+1)$ -price format. In a sealed-bid $(q+1)$ -price auction, bidders submit a bid only once and then wait until the end of the auction to learn the auction price and the set of winners. The winners are the bidders with the q highest bids (q being the auctioned quantity), and each pays a price equal to the $(q+1)$ highest bid (the highest losing bid).

We characterize each consumer by his valuation V , and the time remaining in the auction when he arrives to the website, t' . The optimization problem faced by low-valuation consumers is: $\text{Max} \{U_-^A(V, t'), 0\}$, where

$$U_-^A(V, t') = \text{Max}_{b \in [0, \infty)} \text{Pr}(\text{win}|b)V - E[\text{auction_payment}|b] \quad (1)$$

We define $U_-^A(V, t')$ as the maximum expected value from participating in the auction for a consumer of type (V, t') , when $V < p$, where the expected value is taken over the bids of all other bidders. $\text{Pr}(\text{win}|b)$ is the probability that the consumer wins the item by bidding b , and $E[\text{auction_payment}|b]$ is the expected auction payment by

a bidder who bids b . Notice that $E[\text{auction_payment}|b]$ differs from the expected auction price when bidding b , $E[p_a|b]$, because when the consumer loses in the auction his expected auction payment is zero, but the auction price is not.

A high-valuation consumer arriving with t' time units remaining in the auction solves the following optimization problem: $\text{Max} \{U_+^A(V, t'), U_+^B(V, t')\}$, where

$$U_+^A(V, t') = \text{Max}_{b \in (0, \infty)} \Pr(\text{win}|b)V - E[\text{auction_payment}|b] + \Pr(\text{lose}|b)(V - p) - wt' \quad (2)$$

$$U_+^B(V, t') = V - p$$

$U_+^A(V, t')$ denotes the maximum expected value from participating in the auction for a consumer of type (V, t') with valuation $V \geq p$, and we define $U_+^B(V, t')$ as the value a consumer of type (V, t') derives from purchasing the item for the fixed price on arrival. Finally, $\Pr(\text{lose}|b) = 1 - \Pr(\text{win}|b)$. The consumer evaluates the expected payoff from bidding, using an optimal bidding strategy, and compares it with the payoff from purchasing the item for the posted price.

2.1.1 Bidding Strategies. In the dual-channel with sealed-bid ($q+1$)-price auctions described above, the bidding strategy is given by Lemma 1.

Lemma 1. A weakly dominant bidding strategy for risk-neutral bidders with independent private values in a sealed-bid ($q+1$)-price auction that is conducted parallel to a posted price, p , is the following:

$$b(V) = \begin{cases} V & \text{for } V < p \\ p & \text{for } V \geq p \end{cases} \quad (3)$$

The proof is in Etzion *et al.* (2006). This result is also used in [2]. In our simulation, when running the dual-channel with sealed-bid auction, we use the function given by (3) to determine the agents' bids.

2.1.2 Auction Participation. Low-valuation consumers, those with $V < p$, cannot buy the item for its posted price because the value they get from doing so is negative, so they choose between bidding and staying out of the market. As in Etzion *et al.* (2006) we assume that all low-valuation consumers bid in the sealed-bid auction upon their arrival to the website.

High-valuation consumers, those with $V \geq p$, would buy the item for its posted price if auctions were not offered. High-valuation consumers choose between buying the item for its posted price upon arrival to the website and participating in the auction. Substituting the result of Lemma 1 in Equation 2, we find that a high-valuation consumer participates in the auction if and only

if his expected auction discount, D , exceeds his delay cost, that is if and only if:

$$D \equiv \Pr(\text{win}|p)p - E[\text{auction_payment}|p] \geq wt'. \quad (4)$$

According to [8] there exists a unique symmetric threshold equilibrium, in which all high-valuation consumers choose to bid if and only if $t' \leq S$, where $0 < S < T$ and is given by the solution of the fixed point equation $D(S) = wS$ if $D(T) < wT$, and $S = T$ otherwise. $D(S)$ is a high-valuation consumer's expected auction discount, $\Pr(\text{win}|p)p - E[\text{auction_payment}|p]$, when all **other** high-valuation consumers use the threshold S to choose between bidding and buying for posted-price. This result is stated as Proposition 1 in [8]. Accordingly, consumers with $V \geq p$ choose to participate in the sealed-bid auction rather than to buy the item for its posted price if and only if the remaining time of the auction observed on arrival does not exceed a threshold value, and there is a unique symmetric threshold equilibrium.

Etzion *et al.* (2006) then assume that consumers use a heuristic to find the threshold, suggest one such reasonable heuristic, and show that given the suggested heuristic there exist unique symmetric threshold equilibrium. The heuristic (Heuristic 1 in the Appendix) assumes that consumers use expected values rather than distributions of the number of bidders to estimate their auction discount. In our simulation of a dual channel with sealed-bid auctions, we use the same heuristic as in [8] to find the threshold used by high-valuation consumers to choose between bidding in the auction and buying the item for the posted price. This threshold value is being calculated at the beginning of the simulation, as it depends only on parameters that are observable by all consumers and do not change over time. According to Heuristic 1 consumers make the bid-buy decision as if the number of bidders is deterministic (interpolating on two deterministic values); however, the seller's profit is calculated using the Poisson distributions of arrivals in Etzion *et al.* (2006), and using a realization of arrivals drawn from the Poisson process in the simulation.

2.2. Dual Channel with Open-Bid Auctions

Most online auctions are conducted using the open ascending-bid auction format, in which bidders can observe the lowest bid needed to win at every moment of the auction. This suggests that consumers have more information available to them when choosing between the posted price and auction participation than in the sealed-bid auction previously modeled. In addition, when participating in an open ascending-bid auction, a consumer might be outbid before the auction ends, in which case she has to reevaluate whether to raise her bid, exit the market, or quit the auction and buy the item for the posted price. Hence, a consumer might face the “*buy-bid*” decision several times during the auction, with new

information available for her each time she needs to make this decision.

We assume that the current *lowest-winning-bid*, B_t , is common knowledge, and that there is a minimum bid increment, h . Consumers that do not have a bid in the *winning-bid-list* can bid any amount larger than or equal to $nB(t)$, the lowest bid required to enter the auction at time t , defined as:

$$nB(t) = \begin{cases} B_t + h & \text{if the number of "winning" bids} \geq q \\ \text{Max}\{R, h\} & \text{else} \end{cases}$$

All auction winners pay a price equals to the *lowest-winning-bid* at the conclusion of the auction. We also assume that in the case of a tie (several bids of the same amount in the *winning-bid-list*, and one of them needs to be dropped from the *winning-bid-list* due to a new bid), priority is given based on the time of the bid. Consumers that have not obtained the item yet and don't have a bid in the *winning-bid-list* face a two stage decision problem: 1) If I choose to bid in the auction, what should I bid? 2) Should I bid in the auction? The alternative is to buy the item for the posted price for a high-valuation consumer and to exit the market for a low-valuation consumer.

3.2.1 Bidding Strategies. First we define the bidding strategies used by consumers who choose to enter the ascending-bid auction, which will be implemented in the simulation. The purpose of this paper is to examine how changing the auction format affects the results that were derived analytically for dual-channels with sealed-bid auctions. Thus we try to make all other conditions as equal as possible. When bidding is costless (as was assumed in [8]) and there is an option of obtaining the item for a fixed price, a likely bidding strategy is one in which consumers bid the minimum amount necessary to enter the *winning-bids-list*. Thus, for the open ascending-bid auction we assume that all consumers who choose to bid use the following bidding strategy:

$$b(t) = nB(t) \quad (5)$$

Choosing to bid, the consumer makes the minimum bid required to enter the *winning-bids-list* because he does not want to be "locked" in the auction. We do not argue that the strategy given by Equation (5) is the optimal or even the dominant bidding strategy. However, for the sake of this paper, and to keep everything as simple as possible and as similar as possible to the model of the dual channel with sealed-bid auctions, we assume all bidders employ the bidding strategy given by (5), and we use it in the simulation. When bidders bid according to the strategy given in Equation (5), they face the buy-bid decision more often than if they were to bid in larger increments, that is, bidders would be given an opportunity to "quit the auction" more often than with any other bidding strategy. It seems that being able to re-examine the buy-bid decision as often as possible, utilizing updated

information regarding arrivals and bids, can only benefit the bidders when bidding is costless.

In an extension to this paper we plan to evaluate the performance of the dual channel when consumers have heterogeneous bidding strategies. This can be done with our simulation, but would be too complicated to be done analytically.

3.2.2 The Buy-Bid Decision. A high-valuation consumer ($V \geq p$) who does not have an item and does not have a bid in the *winning-bids-list* at time t needs to decide whether to buy the item for the posted price p or submit a bid that equals nB_t . The main difficulty in modeling a dual channel with open-bid auctions is to model how high-valuation consumers make this decision.

As in the case of a dual-channel with sealed-bid auctions, we assume that a high-valuation consumer forms some beliefs regarding the competition he would face in the auction and thus regarding the expected auction discount over the posted price. Then, the high-valuation consumer trades off the expected auction discount and the expected delay cost. However, there are two main differences in the information the consumer needs to consider when facing the buy-bid decision in a dual channel with an open-bid auction. First, when making the buy-bid decision facing an open-bid auction, the buyer observes the current lowest bid, B_t , which provides information regarding the **bidding history** up to that moment. This information is not available in a sealed-bid auction. Second, when making the buy-bid decision facing an open-bid auction, by choosing to bid the consumer commits to the auction only until the next time he faces the buy-bid decision (being outbid) and not necessarily until the end of the auction.

It seems reasonable to assume that consumers incorporate the information about the current lowest bid, B_t , in their decision process; they simply need to consider only competition from consumers with valuation higher than the minimum required bid, nB_t . However, incorporating the information regarding the positive probability of quitting the auction before it ends is significantly more complicated. $\text{Pr}(\text{stay2end} | B_t, t)$ is the probability of being an active bidder at the end of the auction, conditional on observing a *lowest winning bid* of B_t at time t (the time of making the buy-bid decision). Thus, $\text{Pr}(\text{stay2end} | B_t, t)$ is actually the probability that the buyer would **find it optimal** to keep increasing his bid **every time** he faces the buy-bid decision, until the auction ends. To evaluate this probability the consumer needs to know: 1) the distribution of the times at which he will face the buy-bid decision, and 2) the distribution of the value of the lowest-winning-bid, B_t , at each point of time. Solving this problem requires complicated dynamic programming skills! Thus, we assume that high-valuation consumers have bounded rationality and use a **myopic**

policy to choose between buying the item for the posted price and bidding, as stated as Heuristic 2.

Heuristic 2. High-valuation consumers evaluate their *auction discount* and *delay cost* assuming that if they submit a bid then the probability that later they will find it optimal to stop participating in the auction is zero.

The policy described in Heuristic 2 is myopic because the consumer does not take into account the positive probability that he might choose to quit the auction when facing the buy-bid decision again in the future. Instead, the consumer makes the buy-bid decision assuming that if he joins the auction he would keep increasing his bid until the conclusion of the auction (if necessary up to the posted price), and employs the same rationale regarding the behavior of other high-valuation consumers. In other words, the high-valuation consumer assumes that if he chooses to bid at time t , then he is making the right decision. Notice that if the consumer chooses to bid at time t , but at a later time chooses to quit the auction and buy the item for the posted price (i.e. chooses to stop raising his bid), then he made the wrong decision at time t , and would have been better off just buying the item for the posted price at time t .

Given Heuristic 2, a high-valuation consumer making the buy-bid decision at time t behaves as if he is facing a sealed-bid auction with reserve price nB_t . Accordingly, we assume that the high-valuation consumer forms expectation regarding the competition from other high-valuation consumers using Heuristic 3.

Heuristic 3. A high-valuation consumer making the buy-bid decision at time t estimates the competition from other high-valuation consumers using the *symmetric-equilibrium threshold* value of a sealed-bid auction with reserve price nB_t .

As in the sealed-bid case, we assume that consumers use expected number of bidders to evaluate their auction discount rather than the distributions of number of bidders. That is we assume that consumers use Heuristic 1. Observing the minimum bid he can submit, nB_t , the high-valuation consumer forms expectations regarding the number of additional high-valuation consumers that will remain bidders until the end of the auction, N_{HV}^O , and the number of low-valuation consumers that participate in the auction **after** he submits his bid, N_{LV}^O .

The expected number of low-valuation consumers that can affect the auction price after the consumer submits his bid is thus given by:

$$N_{LV}^O = \lambda \Pr(nB_t < V < p)T. \quad (6)$$

According to Heuristic 3, when t time units of the auction have passed, the consumer making the buy-bid decision estimates the competition from other high-valuation consumers by

$$N_{HV}^O = \lambda \Pr(V > p)S(nB_t), \quad (7)$$

where $S(nB_t)$ is the *unique symmetric threshold* used by high-valuation consumers facing a sealed-bid auction with reserve price nB_t , as found in Etzion *et al.* (2006). Thus, a high-valuation consumer facing the buy-bid decision in a dual channel with open ascending-bid auction chooses to bid if and only if the remaining time of the auction does not exceeds $S(nB_t)$.

The threshold used by the high-valuation consumer in the open-bid auction differs from the threshold used in the sealed-bid auction for two reasons. First, the suggested threshold is a function of B_t , the *lowest winning bid* at time t , and second its value keeps changing during the auction. Everything else begin equal, as B_t increases, the threshold S decreases. Thus, clearly S decreases over time (because B_t is non-decreasing in time). A sharp decrease in S (due to a sharp increase in B_t) might make high-valuation bidders drop out of the auction if given the opportunity; however, this was not what they expected upon entering the auction.

Our suggested model of consumers' behavior captures the following two important features that differentiate the dual channel with open-bid auctions from the dual channel with sealed-bid auctions: 1) Consumers use the information regarding the bid history, summarized by B_t , when making the buy-bid decision, and 2) Consumers face the buy-bid decision multiple times during the auction.

Even though consumers make the buy-bid decision assuming they will not quit the auction at a later time (Heuristic 2), they do face the bid-buy decision multiple times, use a different threshold each time (Heuristic 3), and quit the auction at some cases. Thus it is not clear if the results for the optimal design of the dual channel with open-bid auctions, under the proposed Heuristics 2 and 3, would be similar to the results for the dual channel with sealed-bid auction. We acknowledge that the current model does not capture the fact that consumers might estimate the distribution of the time until they quit, and use this information to make the buy-bid decision.

3.2.3 Bid-Quit Decision. A consumer with low valuation needs to decide if to bid upon arrival to the website and then every time he is being outbid from the winning-bids-list. We assume that the low-valuation consumer bids iff $V \geq nB_t$. Notice that the consumer might bid even if he expects an auction price such that $p_a > V$. This will not be true if bidding is costly.

3. The Simulation

We created a model in NetLogo [13] that simulates the processes described above. The values of the arrival rate, λ , the delay cost per unit time, w , and the values of the design parameters q , p , and T are set before running the simulation. Then inter-arrival times are drawn from

Exponential distribution with parameter $1/\lambda$, and each arrival (agent) is assigned a valuation drawn from the uniform distribution, with support set [*min-valuation*, *max-valuation*].

With the same set of agents (same realization of arrival times and valuations) generated by the simulation, we run the market-simulation under three separate selling regimes: 1) dual channel with open ascending-bid auction, 2) dual channel with sealed-bid ($q+1$)-price auction and 3) only posted price, chosen optimally; each for duration of T time units. For each of these three regimes we capture many pieces of data, including revenue and ending auction price. After this set of three market-simulations is completed, a new set of agents with different arrival times and valuations is generated and another set of three market sessions, one for each of the three regimes, is run. We repeated this process 600 times for each combination of parameters values (λ, w, q, p, T) in order to come up with the statistics (such as average daily revenue and average auction price) for a given channel design. We ran the market 600 times for each design because 10,000 numerical sequential sampling of 1000 values from a $N(750,375)$ distribution showed that after 600 samples the cumulative average changes by less than 0.3% over the last 20 draws and by no more than 3% over the last 60 draws. In all of the runs we set $R=0$ and $h=1$.

4. The Optimal Design

Using MGRID (Michigan Grid Research and Infrastructure Development) grid technology at the University of Michigan, we tested a wide combination of variable settings with the simulation. Our goal was to find the optimal design strategies for different combinations of arrival rate and delay cost. We iterated over four values of arrival rates and four values of delay cost: λ - 1, 5, 20, 30 [1/day] and w - 0.5, 1, 2, 4 [\$/day]. For each (λ, w) combination we tried all the design-combinations over a specific set of values for q, p , and T as shown in Table 2.

	Tested values
q : quantity	1, 2, 3, 5, 10, 15, 20, 25, 30, 35, 40
p : posted price	50, 53, 57, 60, 65, 68, 73
T : auction length	2, 4, 8, 10, 15, 24, 48, 72, 96, 120, 144, 168

Table 2: Tested values of decision variables

There are 924 possible (q, T, p) combinations in Table 2. We chose these values as a reasonable starting place for our investigation because they spanned the range of the optimal settings for dual channel with sealed-bid auctions as found by [8], but also limited the number of combinations that we would have to investigate (necessary for computational reasons). We draw consumers valuation from the uniform distribution on $[0, 100]$ and assumed $R=0$ and $h=1$.

For each of the 14,784 combinations of (λ, w, q, p, T), we generated 600 sets of arrival times and valuations, and with each such set we ran one dual/open market session (open-bid auction parallel to a posted price p), one dual/sealed market session (sealed-bid auction parallel to a posted price p), and one market session of length T with only posted price of \$50 (the optimal price when valuations are drawn from the uniform distribution on $[0, 100]$) for a total of 26,611,200 runs.

For each combination of (λ, w, q, p, T) and for each sales regime (dual/open, dual/sealed and just posted), we collected a wide variety of outcome measures. The optimal design for each (λ, w) pair, is given by the (q, p, T) combination that leads to the highest average daily revenue (calculated using the relevant 600 market sessions). Table 3 lists terminology we use to refer to the revenue maximizing designs for a given (λ, w) pair.

Table 4 shows for each (λ, w) pair the settings where the *maximum simulated revenues* were achieved by the dual/open channel and by the dual/sealed channel (i.e. the best D/S and best D/O designs). It also shows the *maximum simulated revenues* for each of these regimes, as well as the *best analytic design* and the *maximum expected D/S revenue*.

4.1 Comparison of Best Designs

Correlation between dual/open and dual/sealed revenues. The maximum simulated revenues for dual/open and for dual/sealed were both reached at the same settings for 13 of the 16 (λ, w) pairs (in 81% of the cases). Further, the larger of the two maximum simulated revenues was on average only 0.93% larger than the other value and never more than 3.2% larger. Both of these are part of a broader set of data supporting the contention that the total revenues received through the two dual channel regimes are highly correlated. The mechanism of correlation is probably the common draws from the underlying distributions of valuations and arrival times.

<i>Expected D/S revenue</i>	The expected daily D/S revenue for given values of (q, T, p) according to the revenue equation derived in Etzion <i>et al.</i> 2006
<i>Simulated D/O revenue</i>	The average daily revenue achieved by the simulation from 600 D/O market sessions at a given design (i.e. at given values of (q, T, p))
<i>Simulated D/S revenue</i>	The average daily revenue achieved by the simulation from 600 D/S market sessions at a given design
<i>Best analytic design (BAD)</i>	The (q, T, p) combination from Table 2 that maximizes the <i>expected D/S revenue</i> . The equation for the expected D/S revenue was derived in [8]
<i>Best D/S design</i>	The (q, T, p) combination that in the simulation maximized the average daily revenue from a D/S channel (i.e. maximized the <i>simulated D/S revenue</i>)
<i>Best D/O design</i>	The (q, T, p) combination that in the simulation maximized the average daily revenue from a D/O channel (i.e. maximized the <i>simulated D/O revenue</i>)
<i>Maximum expected D/S revenue</i>	The expected daily revenue from a D/S channel at the <i>best analytic design</i>
<i>Maximum simulated D/S revenue</i>	The average daily revenue achieved by the simulation from the 600 D/S market sessions at the <i>best D/S design</i> .
<i>Maximum simulated D/O revenue</i>	The average daily revenue achieved by the simulation from the 600 D/O market sessions at the <i>best D/O design</i> .

Table 3: Terminology

λ	w	0.5			1			2			4		
		Loc (q, T, p)	Max	AtExp	Loc (q, T, p)	Max	AtExp	Loc (q, T, p)	Max	AtExp	Loc (q, T, p)	Max	AtExp
1	D/O	1-144-53 25.47	27.18 14.63	26.33 13.70	1-168-57 25.65	26.80 14.22	25.12 13.23	1-96-57 24.53	27.33 19.41	27.04 13.41	1-168-50 26.25	27.92 13.07	26.37 13.68
	D/S	1-144-53 25.47	26.69 14.39	25.92 13.57	1-168-57 25.65	26.24 14.06	24.53 13.13	1-144-53 24.80	26.53 14.53	26.35 13.21	1-168-50 26.25	27.09 12.69	25.54 13.37
	JP		25.05			25.05			25.05			25.05	
	Exp	1-168-53	25.69		1-168-53	25.81		1-168-53	26.03			1-168-53	26.40
5	D/O	2-72-57 127.80	132.26 45.46	128.35 37.14	1-72-53 128.95	132.90 45.81	128.77 27.83	3-168-53 130.15	134.26 30.34	132.55 30.23	10-168-53 134.44	137.18 26.71	133.97 27.90
	D/S	2-72-57 127.80	131.35 45.38	127.55 37.11	1-72-53 128.95	131.77 45.28	128.00 27.45	3-168-53 130.15	133.02 30.10	131.60 29.70	10-168-53 134.44	137.21 27.29	134.10 27.67
	JP		125.06			125.06			125.06			125.06	
	Exp	2-96-50	129.02		5-168-53	129.79		5-168-53	131.38			10-168-57	135.42
20	D/O	1-10-57 509.63	529.94 254.45	509.93 190.42	2-24-50 515.56	537.56 159.92	513.19 53.35	2-15-50 513.48	528.87 189.66	525.01 52.30	35-168-57 550.23	549.63 57.10	547.76 55.80
	D/S	1-10-53 514.33	526.73 232.28	507.68 190.31	2-24-50 515.56	534.64 158.45	512.80 52.06	40-168-57 526.93	530.15 51.89	526.48 51.37	35-168-57 550.23	551.69 56.81	550.11 55.64
	JP		499.80			499.80			499.80			499.80	
	Exp	1-15-50	515.67		30-168-53	516.53		35-168-53	527.92			40-168-57	550.68
30	D/O	1-8-57 765.79	809.59 348.91	764.61 318.19	1-15-50 770.50	794.04 255.17	764.82 314.40	2-10-50 769.71	796.03 291.82	789.31 64.62	40-168-53 817.44	818.54 66.92	815.41 66.99
	D/S	1-8-57 765.79	805.11 348.49	759.62 316.79	1-15-50 770.50	790.99 254.70	760.26 312.81	2-10-50 769.71	791.71 288.34	790.38 63.37	40-168-53 817.44	820.87 66.84	817.45 66.90
	JP		749.76			749.76			749.76			749.76	
	Exp	1-8-53	773.50		1-10-53	773.64		40-168-53	790.19			40-168-57	818.55

Table 4: Best D/O designs, best D/S designs, best analytic designs and the corresponding revenues, as well as the simulated D/S and D/O revenues at the best analytic design

For each λ/w pair, going from left to right within a row and then top-down, the data displayed is (with numbers from the 5/1 pair as an example)

- D/O row** 1. *Best D/O design* (1-72-53) and the *Expected D/S revenue* at the *best D/O design* (128.95).
2. *Maximum Simulated D/O revenue* (132.9) and the related *standard deviation* (45.81)
3. *Simulated D/O revenue* at the *BAD* (128.77), and the *standard deviation* (27.83).
- D/S row:** 1. *Best D/S design* (1-72-53) and the *Expected D/S revenue* at the *best D/S design* (128.95)
2. *Maximum Simulated D/S revenue* (131.77), and the related *standard deviation* (45.28)
3. *Simulated D/S revenue* at the *BAD*, and the *standard deviation*
- JP row:** 1. *Average daily revenue* from all market simulations with the given arrival rate and $p = \$50$.
- Exp row:** 1. *best analytic design*. (5-168-53)
2. *Maximum expected D/S revenue*. (129.79)

Variability between the simulation and the analytic outcomes. The best D/S and best D/O designs are generally only slightly different than the best analytic design (*BAD*). However, in some cases the *BAD* differs significantly from the best designs found by the simulation (for example for the pairs [5,1] and [20, 1]). For the [5,1] pair, considering that the *simulated maximums* are less than \$2 greater than the *maximum expected D/S revenue* while the standard deviations of the simulated revenue values are about 45, it is not too surprising that this might occur. This invalidates neither the analytical model nor the simulation. It simply highlights both that the simulation makes apparent the wide variability of the outcomes that an auction can have, and that we would not expect to achieve the “expected” value of this auction in the simulation (or, by implication) for a very long time.

4.2 Comparison of Revenues

Now let us consider the next research question, which is concerned with determining which dual channel setup is more profitable for the seller: dual-sealed or dual-open. First, we note that the average daily revenues achieved by just posted price sales in the simulation differ from the analytically expected revenues from only posted price sales by an average of 0.08% and by no more than 0.21%.

Just posted price sales extracts less daily revenue than dual/channel regimes. The maximum simulated revenues achieved by both dual channel regimes (at their relative best designs) are greater than the average revenue achieved by the single channel’s sales. In addition, the simulated revenues for both D/S and D/O achieved at the *best analytic designs* are greater than the average revenue achieved by the single channel.

D/O often performs better than D/S. For markets with either low arrival rate or low delay cost, the dual channel with an open-bid auction results in slightly higher average daily revenues. For markets with both a high customer arrival rate and a high delay cost, the dual channel with a sealed-bid auction generally gives better results. On average, at the *best analytic setting* the dual-open regime returns 4.7% higher average daily revenue than just posted revenue while dual-sealed returns 4.1% higher. At the settings that maximize the simulated revenue for each regime, dual-open returns on average 7.7% more revenue while dual-sealed returns 6.8% more revenue than just posted price. In either case, dual-open revenues are generally quite close to dual-sealed revenues. The largest gap in average daily revenues is for auctions with very low customer arrival rates or very low waiting costs.

Next we address the third research question which is concerned with determining the performance difference between the two types of dual channel regimes if both are

run with the decision variables (q, t, p) set at the *best analytic design*. Since the best analytic design of the dual/sealed channel is more easily determined, this question hopes to get at how much revenue would be lost if a dual/open regime is run using the *best analytic design* of the dual/sealed regime.

We examine the *simulated daily revenue* for the two dual channels (D/S and D/O) at two different settings: 1) the *best analytic design* and 2) the *best D/S design*, as well as the simulated average daily profit from only posted price of \$50. What we find is that that dual/open does reasonably well in both settings and doesn’t have to be designed under its own restrictions (or assumptions). We can make several observations concerning these results:

Dual/open channel generally results in the highest revenue. For the λ/w combinations with either a low λ or a low w , for both the *BAD* and the *best D/S design*, the revenue from a dual/open channel is greater than both the revenue from a dual/sealed channel and the average revenue from just posted price sales. If both λ and w have high values, then it is better to use a dual/sealed channel instead of either of the other two.

Simulated D/O revenue at the best D/S design is higher than the Simulated D/O revenue at the BAD. Of course it is the case that *simulated D/S revenue* at the *best D/S design* is at least as good as the *Simulated D/S revenue* at the *best analytic design*; by definition this must be the case. However, we find that the dual/open channel is generating exceptional revenue at a setting that is chosen because it generates the *maximum simulated dual/sealed revenue*. This is another indication that the two dual channel regimes are highly correlated: if the dual/sealed regime is able to get superior revenues, then the dual/open regime is able to get superior revenues.

5. Summary

In this paper we use a simulation model to extend previous analytical research on a firm selling consumer goods online using posted price and auction at the same time. Previous analytical work studied consumers’ choice of channel, dominant bidding strategies, and the optimal design of the dual channel when auctions are conducted in the *sealed-bid* format. The simulation model is necessary because it more easily lends itself to an investigation of dual-channels with different auction formats, such as the commonly used open *ascending-bid* format.

With our simulation we investigate three selling regimes: dual channel with sealed-bid auction; dual channel with open-bid auction, and a single channel with only posted price. Our suggested model of consumers’ behavior captures the following two important features that differentiate the dual channel with open-bid auctions from the dual channel with sealed-bid auctions: 1)

Consumers use the information regarding the bid history, summarized by B_t , when making the buy-bid decision, and 2) Consumers face the buy-bid decision multiple times during the auction. Our results support previous work showing that dual channels, in which identical items are sold via auctions and posted price at the same time, can increase sellers' profit if designed correctly. We found that the maximum simulated revenues achieved by both dual channel regimes (at their relative best designs) are greater than the average revenue achieved by the single channel's sales. In addition, the simulated revenues for both D/S and D/O achieved at the *best analytic designs* are greater than the average revenue achieved by the single channel.

The dual/sealed and dual/open simulations yield similar results for the best dual channel designs. Recall that in our model of the dual/open channel, consumers make the bid-buy decision ignoring the positive probability that they will quit the auction before it ends (Heuristic 2). In addition, high-valuation consumers use a sealed-bid threshold value, which changes over time according to the lowest winning bid, to estimate the competition they would face in the auction and thus their expected auction discount. Nonetheless, we believe that the similarity of the optimal designs is surprising because in the dual/open channel consumers face the bid-buy decision multiple times during the auction, and because they change the threshold they use every time they face the bid-buy decision.

In the future we plan on varying the simulation in several ways to see how these variations affect both the seller's average daily revenue and the best auction designs. We plan on investigating the effects of variable agent waiting costs, letting low valuation consumers have non-zero waiting costs, allowing bidding to be costly, and allowing consumers to have variable bidding strategies. Further, we plan on investigating the effects of allowing consumers to have different bidding strategies. Finally, we plan on investigating the effects that limited sniping might have on the results. We hope that these findings will inform further analytical work that might be done in this complicated area.

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7. Appendix

Heuristic 1: A high valuation consumer evaluates his expected auction discount over the posted price as if the number of (other) high valuation bidders arriving in a period of length t is $\lceil \lambda \Pr(V \geq p)t \rceil$ with probability ρ and $\lfloor \lambda \Pr(V \geq p)t \rfloor$ with probability $(1 - \rho)$, where $\rho = \lambda \Pr(V \geq p)t - \lfloor \lambda \Pr(V \geq p)t \rfloor$, and the number of low-valuation bidders arriving in a period of length t is $\lceil \lambda \Pr(V < p)t \rceil$ with probability γ and $\lfloor \lambda \Pr(V < p)t \rfloor$ with probability $(1 - \gamma)$, where $\gamma = \lambda \Pr(V < p)t - \lfloor \lambda \Pr(V < p)t \rfloor$.

When all other high-valuation consumers use a threshold S , a high-valuation consumer estimates his expected auction discount, $D(S)$, assuming the number of low-valuation bidders is $N_{LV} = \lambda \Pr(V < p)T$ and the number of additional high-valuation bidders is $N_{HV} = \lambda \Pr(V \geq p)S$. Then the consumer's expected auction discount is:

$$D_i(s) = \rho(s)\gamma d(\lceil N_{HV}(s)+1 \rceil, \lceil N_{LV} \rceil) + (1-\rho(s))(1-\gamma) d(\lfloor N_{HV}(s)+1 \rfloor, \lfloor N_{LV} \rfloor) + (1-\rho(s))\gamma d(\lfloor N_{HV}(s)+1 \rfloor, \lceil N_{LV} \rceil) + \rho(s)(1-\gamma) d(\lceil N_{HV}(s)+1 \rceil, \lfloor N_{LV} \rfloor)$$

Where, using order-statistics of uniform distribution:

$$d(x,y) = \begin{cases} p-R - (p-R)\frac{y-(q-x)}{y+1} & \text{if } x \leq q \text{ and } y+x > q \\ 0 & \text{if } x > q \\ p-R & \text{if } x+y \leq q \end{cases}$$