Monitoring of Power System Dynamic Behavior Using Characteristic Ellipsoid Method

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Abstract

The potential uses of the sub-second GPS-synchronized phasor data collected from various locations within an electric power system promise endless benefits for the applications targeting reliable operation of electric power systems. Despite the undisputable progress achieved in developing visualization tools, alarming tools, modal analysis tools, and statistical analysis tools based on synchrophasor data, there is an emerging need to develop more real-time phasor-based applications. This paper discusses an initial idea of the characteristic ellipsoid approach to monitor the dynamic behavior of an interconnected power system using phasor measurements. This method can be a useful tool for providing wide-area situational awareness for grid operators, identification of system disturbances and detection of system stresses and their locations.

1. Introduction

Synchronized phasor measurement units (PMUs) provide unique capability to monitoring dynamic behavior of power systems in real-time and enhancing control and protection for power grids. Some applications of PMUs in power systems include tracking system dynamic events to study short and long term dynamic phenomena [1], [2], and monitoring inter-area oscillation mode of a power system [3], [4], and so on. PMUs provide the opportunity for real-time monitoring of voltage and current phasors and frequency. Despite the progress achieved in developing visualization tools, alarming tools, modal analysis tools, and statistical analysis tools, there is an evident need in developing more real-time phasor-based applications.

For example, the real-time modal analysis of power system oscillations requires further effort to develop suitable methods to actually transform the modal analysis results into actionable information. There is also a need to develop relatively simpler, easier-to-implement and easier-to-use, but at the same time more informative and actionable approaches.

This paper proposes a completely new initial idea of the characteristic ellipsoid approach (CELL) with the following objectives:

- Develop a new efficient power system monitoring and express-analysis approach allowing increased wide area visibility of the system as well as the situation awareness of the power system's operators;
- Examine the opportunities of using the new idea of CELL to monitor the dynamic behavior of interconnected power systems;
- Develop the mathematical apparatus, algorithms, and prototype MATLAB code to implement the method;
- Provide rules to interpret CELL characteristics and link them with the behavior of the system;
- Examine and demonstrate the method using Western Interconnection phasor measurement data.

2. The idea of CELL

The CELL is a multi-dimensional minimum volume second-order closed surface ("an egg") that contains a certain limited part of the system trajectory, for example, 1-second set of subsequent phasor data. The system trajectory and the ellipsoid are represented in the phasor data space. The shape, volume, orientation, and rate of change of CELL parameters in time provide a new look on the essential information about the system status and dynamic behavior, including

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such characteristics as system stress, increasing or decreasing system motions, the magnitude of disturbances and the mode of motion of some parts of the system against the other parts during the disturbance (mode shape), and so on.

3. CELL approach

Extensive research efforts have focused on computing the Minimum Volume Enclosing Ellipsoid (MVEE) in n-dimensional space R^n containing m given points $a_1, a_2, \ldots, a_m \in R^n$, and several algorithms have been developed for solving the MVEE problem. Generally, these algorithms can be categorized as three categories: first-order algorithms based on gradient-descent techniques [6], second-order algorithms based on interior-point techniques [7], and the algorithms combined first-order and second-order algorithms [8].

Khachiyan [8] proposed a method with currently the best known complexity result for a given point set S of m points in R^n in $O(m^{3.5} log(1/6))$ operations in the real number model of computation. Kumar and Yildirim [9] proposed an algorithm based on revised Khachiyan's algorithm and a column generation strategy to compute a $(1+\varepsilon)$-approximation to the minimum enclosing ball of a given set of points based on the existence of a core set of size $O(1/\varepsilon)$. Ahipasaoglu et al. demonstrate the linear convergence of a simple first-order algorithm for the MVEE problem [10].

The Khachiyan's method [5], [8] to build the MVEE has been selected and implemented during this work.

The CELL equation is formulated as follows [5]:

$$\varepsilon = \left\{ x \in R^n \mid (x-c)^t E(x-c) \le 1 \right\}$$

where x is a point in the n-dimensional space of phasor measurements R^n; c is the ellipsoid center, and E is an $n \times n$ positive definite symmetric matrix of CELL's coefficients, which determines shape and orientation of the CELL. In particular, the axes of ε are eigenvectors of E and the length of the semi-axes is given by $[1/\sqrt{\lambda_1}, \ldots, 1/\sqrt{\lambda_N}]$, where $[\lambda_1, \ldots, \lambda_N]$ are the corresponding eigenvalues of the matrix E.

CELL's volume is expressed as follows [5]:

$$V(\varepsilon) = v_0 \det(E^{-1})^{\frac{1}{2}}$$

where v_0 is the volume of a unit hypersphere in dimension n.

The method consists of solving an optimization problem that minimizes the enclosing ellipsoid volume while obeying inequality constraints, which keep all recent phasor trajectory points inside the ellipsoid.

A natural formulation of the MVEE problem is [5]:

$$\begin{aligned} &\min_{E,c} \det(E^{-1}) \\ &\text{subject to} \\ &(x_i - c)^t E(x_i - c) \le 1, \; i = 1, \ldots, m \\ &E > 0 \end{aligned} \tag{1}$$

where x_i is a vector of phasor measurements at the moment i, and m is the number of phasor vectors for the selected observation interval. Reference [5] provides a computationally efficient modification of problem (1) along with its fast solution algorithm.

Procedure to find solution for problem (1) is automatically repeated for each new data point. The analyzed parameters include voltage magnitudes, local frequencies and power flow. These parameters may be normalized to make parameters of different physical nature and dimension comparable in R^n. The paper describes combinations of different phasor measurements helping to identify and locate such events and physical phenomena as generator trips, inter-area oscillations, static system stress and the others.

4. CELL interpretation rules

Some insights into the behavior of the CELLs can be given to analyze and understand the dynamic behavior of a power system.

- CELL's volume $V(\varepsilon)$ is a measure of system stress reflecting the spatial magnitude of the system trajectory. Relatively small CELL's volume indicates that system motion is not stressed. Large $V(\varepsilon)$ points forward to a disturbed state of the system.
- Specific situation appears when the analyzed part of the system trajectory belongs to a subspace R^M of the measurement space R^N, $R^M \subset R^N, M < N$. In this case some of the eigenvalues (specifically, $N - M$ of them) become very large (tend to infinity), and the ellipsoid's volume $V(\varepsilon)$ tends to zero. To avoid mathematical difficulties wit zero volume ellipsoids and to make these dimension

deficient ellipsoids comparable with the full-dimensional ellipsoids, the following approach can be used. Assume that a CELL's semi-axis cannot be less than certain minimum tolerance value $r_{\min} = 1/\sqrt{\lambda_{\max}}$, so that if the system does not move at all during the observation interval and its trajectory is reduced to a single point, the characteristic ellipsoid becomes an N-dimensional sphere surrounding the system point with the radius equal to $r_{\min} = 1/\sqrt{\lambda_{\max}}$ and the volume equal to $v = v_0 \big/ \sqrt{\lambda_{\max}^N}$. If within the observation interval system trajectory is reduced to a straight line, the CELL becomes an ellipsoid with $N - 1$ semi-axes equal to $r_{\min} = 1/\sqrt{\lambda_{\max}}$ and one semi-axis equal to $r_i = 1/\sqrt{\lambda_i}$, $\lambda_i < \lambda_{\max}$. The volume of this ellipsoid becomes equal to $v = v_0 \big/ \sqrt{\lambda_i \lambda_{\max}^{N-1}}$. Similarly, for a general case of a dimension deficient ellipsoid, The ellipsoid's volume can be evaluated as $v = v_0 \Bigg/ \sqrt{\left(\prod_{\lambda_i < \lambda\max} \lambda_i\right) \prod_{\lambda_i \geq \lambda\max} \lambda_{\max}}$.

- The derivative $V' = \frac{\Delta V}{\Delta t}$ (calculated numerically for a certain number of subsequent measurements) provide information on the trend in the system behavior. Positive V' signals increasing spatial magnitude of the system trajectory; negative V' implies system trajectory stabilization.
- Sudden increase of $V(\varepsilon)$ signifies a disturbance. CELLs are able to determine such disturbances as voltage sags and swells caused by power system faults, equipment failure and control malfunctions; momentary interruptions, which are the results of momentary loss of voltage in a power system; oscillatory transient disturbances, which occur when a sudden, non-power frequency change happens in positive and negative polarity values in the steady state condition of voltage, current, or both.
- The shape and orientation of CELLs are also informative. The orientation of the ellipsoid's axes is specified by the eigenvectors S_i, $i = 1,\ldots,n$, of E. The lengths of the semi-axes are given by the eigenvalues λ_i, $i = 1,\ldots,n$, of E.
- The eigenvector $S_{\max} = S_i$ corresponding to the largest λ_i indicates the dominating direction of the system motion. The angles between $S_{\max}$ and the coordinates of R^n help to identify phasors (and system locations) involved in the system's dominating motion.
- The orientation of $S_{\max}$ also helps to understand whether the phasors move in phase or out of phase.

5. Testing the algorithm

In this paper, some initial results of applying the CELL analysis to a real system disturbance are presented. The CELL prototype algorithm has been implemented in MATLAB and tested using real phasor data from the US Western Interconnection system disturbance on July 24, 2006 [11].

During this disturbance, a 500-kV transmission line in the Northwest region faulted and tripped to lock out at 15:28:32.4 PDT. The protective scheme properly operated to drop 1661 MW of the Pacific Northwest generation, inserted 1400-MW breaking resistor at a substation for 0.5 seconds, and inserted 500-kV shunt capacitor at another substation. All generation was restored by 15:46 PDT.

For illustrative purposes, the CELL is represented in three-dimensional space. Disturbed voltage magnitudes and frequencies at three different locations in the western part (subsequently called Sub1, Sub2, and Sub3) are shown in Figures 1 and 2. In the figures, the first spike is caused by the transmission line trip. The second spike is caused by protective actions subsequent to the disturbance.

Each CELL is constructed using a moving data window that includes 2 seconds of the most recent synchrophasor data. Each subsequent CELL is obtained by advancing the previous data window by 1/3 seconds.

The two consecutive CELLs constructed from the normalized voltage magnitudes at the three substations are shown in Figures 3 and 4. Figure 3 shows CELL at 15:28:32.333 PDT, just before the disturbance occurrence, and Figure 4 shows CELL at 15:28:32.667 PDT, after the disturbance occurrence. From Figures 3 and 4 we can observe that the CELL experiences a dramatic change in both volume and shape during the disturbance occurrence. The sudden

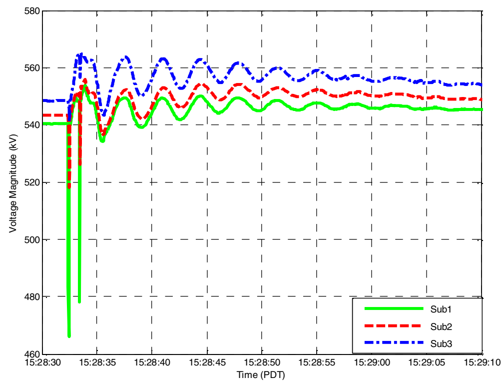

Figure 1. Voltage magnitude at the selected locations

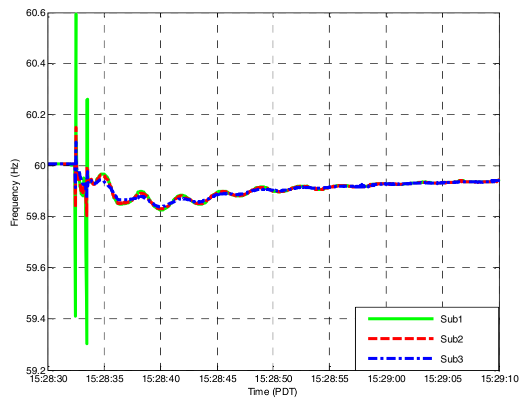

Figure 2. Frequency at the selected locations

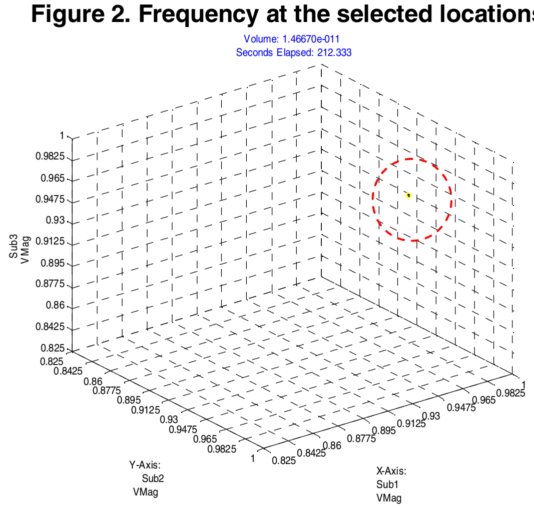

Figure 3. A CELL in normalized voltage magnitude space at 15:28:32.333 PDT (just before disturbance)

change in the CELL volume explicitly signifies that a disturbance has just occurred in the system. Furthermore, the CELL has changed significantly in all three dimensions. It means

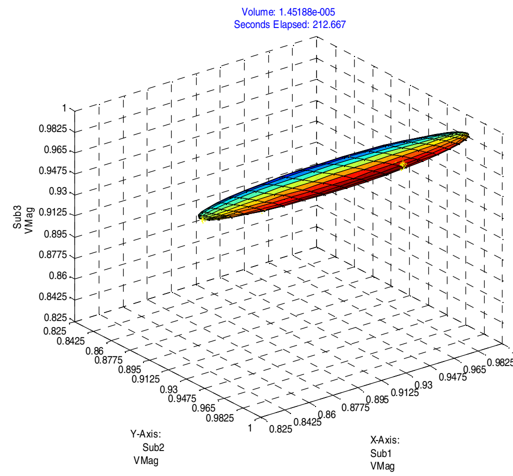

Figure 4. A CELL in normalized voltage magnitude space at 15:28:32.667 PDT (after disturbance)

that the disturbance has a significant effect on all the three locations.

Two consecutive CELLs constructed from the normalized frequencies at the selected three locations are shown in Figures 5 and 6. Similar significant changes in both volume and shape of the CELL during the period of disturbance are observed, which means the disturbance also has an effect on the frequency at these locations. It can be seen from Figure 6 that the largest change occurs in the X-axis, and smallest change occurs in the Z-axis. It suggests that the disturbance has a bigger impact on frequency at Sub1 than at Sub2. This is consistent with frequency plot in Figure 2. Figures 5 and 6 suggest a sudden active power mismatch near the same substation.

Finally, the two consecutive CELLs constructed from the normalized real power flows on three 500-kV lines, called Line1, Line2, and Line3, are shown in Figures 7 and 8. From Figure 8, one can tell that the disturbance has a bigger impact on Line3 real power flow compared to the other two lines.

Figure 9 show the derivative of the CELL's volume constructed from normalized voltage magnitudes. As can be seen from Figure 9, the volume changes significantly only in a few seconds after the disturbance occurs. In the other intervals, the change in volume is negligible. This indicates that the disturbance lasted only about 3 seconds, and that the system finally returned to its normal operation.

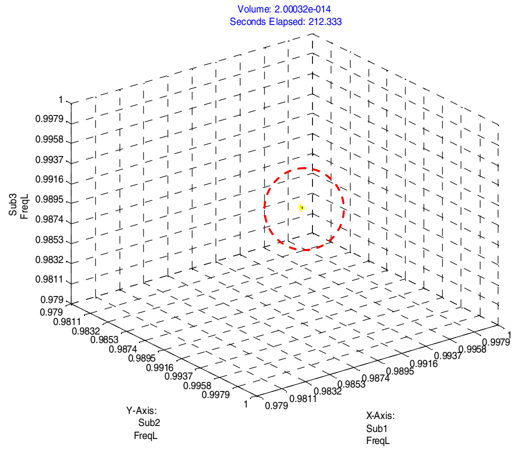

Figure 5. A CELL in normalized frequency space at 15:28:32.333 PDT (just before disturbance)

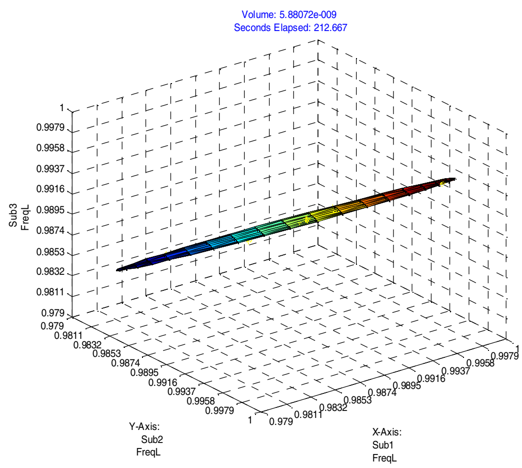

Figure 6. A CELL in normalized frequency space at 15:28:32.667 PDT (after disturbance)

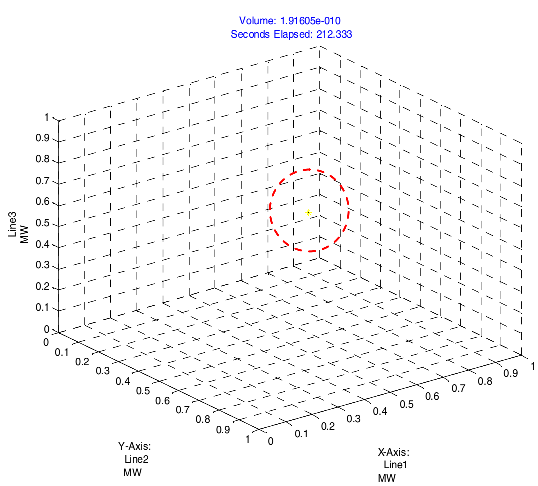

Figure 7. A CELL in normalized power flow space at 15:28:32.333 PDT (just before disturbance)

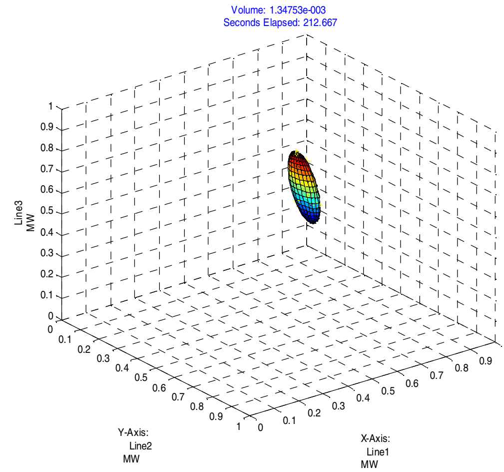

Figure 8. A CELL in normalized power flow space at 15:28:32.667 PDT (after disturbance)

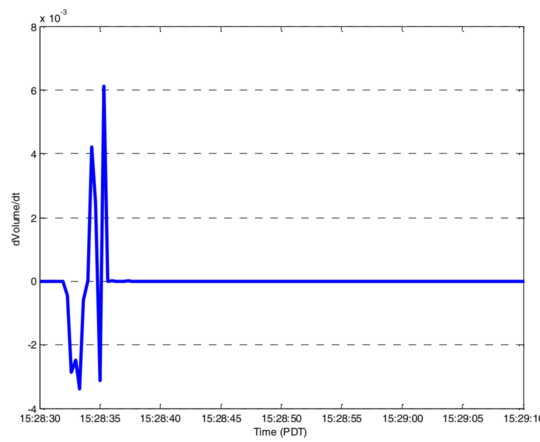

Figure 9. Derivative of the volume for CELL constructed from normalized voltage magnitude

In our MATLAB codes we implemented dynamic display of ellipsoid changes corresponding to PMU measurement sequences. This approach allows to illustrate pulsating ellipsoidal volumes that could happen when a stable oscillation with harmonics or variable speed around the orbit with a relatively longer period than the sampling period. Note that the actual CELL analysis can be automated in real time for a multidimensional space including all phasor measurements. In the future, the CELL's interpretation rules can help to communicate the real-time information to the system dispatchers in verbal (i.e., synthesized voice) or graphical form (i.e., maps, pointers, and text bubbles).

6. Conclusion

This paper discusses a new initial idea of characteristic ellipsoids approach to monitoring the behavior of interconnected power systems using synchronized phasor measurements. This method has the potential to be a very effective and useful tool for providing wide-area situational awareness for grid operators, identification of system disturbances, system stress and oscillations. The most important finding of this work is that the CELL idea is working, and that it can lead to the development of a very innovative and useful power system real-time behavior monitoring tool for the electric power industry.

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