

## Using k-pricing for Penalty Calculation in Grid Market

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### Abstract

To distribute risk in Grid, the design of service level agreements (SLAs) plays an important role, since these contracts determine the price for a service at an agreed quality level as well as the penalties in case of SLA violation. This paper proposes a price function over the quality of service (QoS) on the basis of the agreements negotiated upon price and quality objective. This function defines fair prices for every possible quality of a service, which are in line with the business of the customer and incentivize the provider to supply welfare-maximizing quality. Therewith, penalties can be calculated for every possible quality level as the difference between the agreed price and the output of the price function for the effectively met quality. A price function according to the k-pricing scheme is presented for a single service scenario and for a scenario with multiple interdependent services.

### 1. Introduction

In order to drive sustainable business growth, organizations are nowadays constantly improving their information technology systems to produce business results sooner and make them available to a wider audience within the organization. To meet increasing demands on IT services without bearing enormous costs, IT services are sourced out more and more. To be able to fulfill high requirements even on peak demand, services are purchased on demand.

Unfortunately, all organizations purchasing and selling computational services in a Service Oriented Computing (SOC) environment such as Grid have to

bear risks. Each outsourced service used by a company could provide high risks since they cannot be controlled adequate anymore. But also service providers bear risks, since their reputation will shrink and a penalty fee will have to be paid, if it fails to fulfil the Service Level Agreement (SLA).

A SLA defines the responsibilities of the service provider and the consumer of the relevant service by describing the service provided and the quality standards of the service, measurement criteria, reporting criteria as well as penalties in case of SLA violation. A SLA can be seen as contract that clearly outlines the rights and obligations of the parties in order to reduce the scope for disagreement to arise in the course of the parties' business relationship. The source of the largest number of disputes is likely to be the gap between the actual performance by a service and the performance expected by the service consumer. Thus, well-negotiated SLAs that are comprehensive and well drafted are fundamental to minimizing the risk of disputes. Thereby SLAs should be negotiated in a way that both, the service provider and consumer will profit by successful executed SLAs. *"The best SLAs are setup to allow both you and your service provider to share in the success and failure of an agreement"* [3].

Besides the establishment of transparency, the creation of SLAs aims to arrange risk sharing in order to provide a fair market, which equates financially weak and strong enterprises as well as service providers and consumers. To establish a fair agreement it is important that agreed service level objective, price and penalties are accomplishable and in line with the business of the contractual partners. To incentivize providers to fulfill the agreement, penalty design plays an important role. Thereby, penalties have to be designed according to the business value of the service for the customer and in a way that incentivizes the provider to fulfill the agreement satisfactory.

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The main contribution of this paper is to define balanced penalties that induce the right incentives for service providers to meet the SLAs. This paper introduces a price function on the basis of the results of the negotiation (price and service quality). This function provides fair prices for every possible quality of a service, which are in line with the business of the customer and incentivize the provider to supply welfare-maximizing quality. On this basis, penalty fees can easily be calculated for as the difference between the agreed price (for the agreed service level objective) and the output of the price function (for the effectively met quality).

Section 2 discusses related work. Section 3 defines the price function for penalty calculation referring to a k-pricing scheme for a single service scenario and for a scenario with multiple interdependent services. Section 5 concludes with a summary and an outlook on future research.

## 2. Related work

In the field of match making and pricing of services at a certain quality level, considerable research provides several pricing schemes in the Grid context (e.g. [2], [8] for auction and commodity market models and [6] for an overview of state-of-the-art bargaining models).

A review of current research of penalty calculation in the context of Grid Computing showed that there is no satisfactory comprehensive research in this area. In their paper, Yeo and Buyya [9] presented a proportional share allocation technique called “LibraSLA” that takes into account the utility of accepting new jobs into the cluster based on their SLA. Thereby a linear penalty function that reduces the profit of a job over time after the lapse of its deadline is introduced. Equally, a linear penalty function that reduces profit after a jobs run time has been presented by [1]. Multiple interdependent services are not considered in related work.

Since linear penalty functions are not sufficient to determine penalties according to the business valuations of service provider and consumer in practice, this work focuses specifically on presenting a framework for penalty calculation according to the business value of the agreed services for the customer, which also incentivizes the provider to perform at the best. In doing so, the k-pricing scheme, well-known as a method for pricing in double-auctions [4] and already applied for pricing in Grid context [5] is used to come up with a fair and

incentive compatible price function for all possible qualities of multiple, interdependent services. In the following we employ the intuition of k-pricing to the calculation of penalties.

## 3. Penalty calculation

### 3.1. SLA violation

This section explains what SLA violation means in the context of Grid and how SLA violation could be measured. For the purpose of this paper, following general definition of SLA violation is sufficient:

*“A service level agreement is violated, as soon as it is impossible to meet the agreed service level objective(s).”*

The determination of the degree of the breach is important for penalty calculation to enable penalty design according to the occurring loss of the customer. For the purpose of this paper, the quality of service  $q$  is arbitrary normalized on a percentage rate basis, i.e.  $q \in [0,100]$ , in order to provide comparability of the degree of SLA breaches. For example, availability measured in percent could be considered as normalized service quality.

### 3.2. Requirements on penalty scheme

The value of the penalty fee of a SLA has deep impact on the SLA negotiation and the decision of accepting or rejecting an offer. Along with the price for a service and the reputation and reliability of the prospective associate partner the penalty in case of violation constitute the basis of decision-making whether to accept an offer or not. Hence following objectives for penalty calculation can be identified:

**Requirement 1: Penalties should be fair.** The penalty agreed in a SLA should be proportionable to the loss triggered by violation. Penalty calculation has to be specific to customer’s (and provider’s) business and should truly establish risk sharing [3].

**Requirement 2: Penalties should incentivize contractual partners to perform best for each other.** The penalty agreed in the SLA should incentivize the provider to fulfill the agreement satisfactory to the customer concerning the delivered Quality of service (QoS). This effect can be achieved by establishing a penalty proportional to the degree of SLA violation and by implementing a reward, which will be paid, if the agreed QoS is

outperformed. Although the amount of this penalty and reward respectively has to be greater than the cost the provider may save by not performing at its best. Furthermore, the business value of the provided service level for the customer has to be taken into account, since the consumer should not be worse off, if the provider increases service quality (suppose the business value of a service will diminish as from a certain value).

According to this, the business value of the service for the consumer as well as the production costs expressed by the reservation price for the provider have to be taken into account when calculating the penalty and reward respectively in SLAs.

### 3.3. Required information and assumptions

This section describes the needed information for proper penalty calculation. Thereby a single service scenario, which is about one single SLA between one service consumer and one provider that includes exactly one service, as well as a multiple service scenario is considered. The multiple service scenario is about multiple, interdependent SLAs between one service consumer and one service provider, whereas every service is represented by exactly one SLA.

**Information 1: Business value of service qualities for the consumer.** The business value  $BV$  of a service for the consumer can be expressed as a function of the quality  $q$  of a service. Thereby, the business value represents the amount of currency units  $CU$  the service is worth for the consumer at each possible quality of a service. For example, the consumer is willing to pay \$1.5 for every additional percentage point of availability of a service. Then its business value of this service can be expressed as a straight line  $BV(q) = 1.5q$  [\$] in the domain  $q \in [0,100]$ . The cardinal definition based on the measurement in  $CU$  is essential for penalty calculation, since a same base for the functions  $BV$  and the provider's valuation function  $R$  (introduced below) is indispensable for penalty calculation. Furthermore, the strength of preferences (slope of function  $BV$ ) has magnificent influence on the penalty calculation. The quality is defined as the provided quality relative to the determined maximum quality (e.g. availability expressed in percent). Thus, the business value of a single service can be expressed as follows:

$$BV: q \rightarrow IR^+ \forall q \in [0,100]$$

for single services and

$$BV: (q_1, \dots, q_n) \rightarrow IR^+ \\ \forall q_i \in [0,100]$$

for the multiple service scenario respectively, whereas  $q_i$  represents the quality of service  $i$ . Thereby, the business value represents the amount of currency units  $CU$  each possible combination of qualities of the  $n$  services is worth for the consumer. This graph mirrors the consumer's preferences about the quality of service and could vary for every consumer as well as for every service considered.

To simplify the design of incentive constraints, it is assumed that the function  $BV(q_i)$  is completely differentiable in every direction in the domain  $q_i \in [0,100]$  for all possible  $i = 1, \dots, n$ . Since the business value of a service for the consumer has to be determined in advance of the negotiation and match making process, it is assumed that consumers have already disclosed their valuations to the market directory when it comes to penalty calculation. Furthermore it is assumed that consumers truly disclose business values and do not distort their preferences in order to manipulate penalty calculation. This assumption will be discussed in section 5.

**Information 2: Reservation price of the service provider.** Equally, the reservation price  $R$  (measured in  $CU$ ), which represents the value of the service for the provider can be expressed as a function of the quality  $q$  of service

$$R: q \rightarrow IR^+ \forall q \in [0,100]$$

in the single service scenario and as following graph

$$R: (q_1, \dots, q_n) \rightarrow IR^+ \\ \forall q_i \in [0,100]$$

in the multiple service scenario. Thereby the same assumptions as for the Business Value  $BV(q_i)$  apply. Opportunity costs occurring during service execution because of arising business alternatives are not taken into account, since they are very hard to measure and the service provider will have no incentive to disclose his true estimation.

**Information 3: Quality of service according to SLA.** The quality of service  $q^*$  defined in the service level objectives (SLO) of the SLA is necessary to determine whether a penalty has to be paid by the provider or a reward has to be paid by the service consumer. For multiple services several SLO have been negotiated:

$$q_i^* \in [0,100] \forall i = 1, \dots, n$$

**Information 4: Price of service according to SLA.** In the negotiation process both parties have agreed upon one certain price  $p^*$  for the services provided at the quality  $q_i^*$ . Thereby it is assumed, that both parties have agreed upon one certain all round price  $p^*$  for all services  $i$  provided at the agreed qualities  $q_i^*$ . This assumption is maintainable, since all considered services, each represented by one SLA, usually belong to one contract between the service provider and consumer. All these services included in one contract should be negotiated together, since interdependencies between the services could have significant influence on the valuation for both contractual partners. Therefore, these interdependencies already have been taken into account in the negotiation process. The result of the negotiation is a bundle of service level objectives  $(q_1^*, q_2^*, \dots, q_{n-1}^*, q_n^*)$  and one agreed all round price  $p^* \in IR^+$  in currency units [CU].

$$p^* \in IR^+, \\ \text{measured in currency units [CU]}$$

This price  $p^*$  will be basis for penalty and reward calculation respectively via a price function described subsequently.

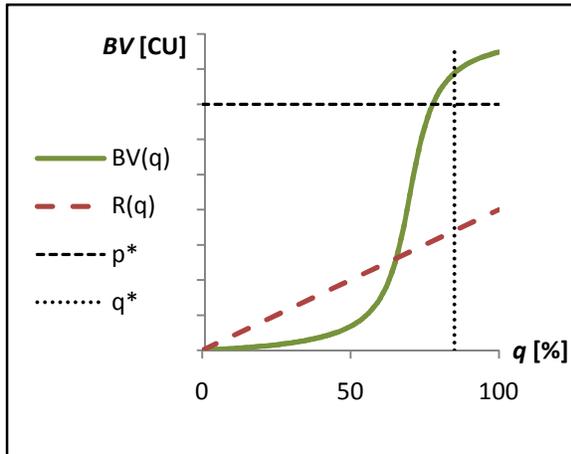


Figure 1. Initial situation for single service

Figure 1. exemplary shows the initial information in the single service scenario: business value for the consumer  $BV(q)$ , reservation price of the provider  $R(q)$ , agreed price  $p^*$  and quality of service  $q^*$  according to SLA.

The exemplary function  $BV$  may be typical for IT services: It increases slowly at low qualities, because very low service quality (e.g. availability) usually will have no useful effect to the consumer. From a certain quality level on,  $BV$  may increase progressive. At high quality levels marginal utility could diminish caused by saturation.

### 3.4. Incentive and fairness constraints

In the following several constraints for designing a price function  $P(q_i)$  are described, which secure to meet the objective of establishing fairness (fairness constraints) as well as incentivizing the provider to act satisfactory to its customer (incentive constraints). This function  $P(q_i)$  represents the total amount of currency units (CU) the consumer will have to pay for the services. The penalty and the reward respectively can be calculated as the difference between the agreed price  $p^*$  and the output  $\bar{P}(\bar{q}_i)$  of effectively met qualities  $\bar{q}_i$ .  $P(q_i)$  can be defined as follows:

$$P: (q_1, \dots, q_n) \rightarrow IR^+ \\ \forall q_i \in [0,100]$$

whereas following constraint has to be fulfilled:

$$P(q_1^*, \dots, q_n^*) = p^*$$

This constraint guarantees the agreed price  $p^*$  for met service level objectives  $q_i^*$ , i.e. no penalty or reward has to be paid, if the agreed qualities are met exactly.

To implement a SLA pricing design which maximizes overall welfare, several constraints have to be added. In this context, the welfare which has to be distributed among the contractual partners is defined by the difference  $W(q) = BV(q) - R(q)$  for every possible service quality  $q$ . To clarify the explanations of the following constraints, only one service is regarded and a case differentiation is introduced in order to reduce complexity:

$$a. \quad \forall q \in \{q \mid \frac{\partial R(q)}{\partial q} \leq \frac{\partial BV(q)}{\partial q}, R(q) \leq BV(q)\}:$$

$$\frac{\partial R(q)}{\partial q} \leq \frac{\partial P(q)}{\partial p} \leq \frac{\partial BV(q)}{\partial q} \quad (IC)$$

$$R(q) \leq P(q) \leq BV(q) \quad (FC)$$

Case a. represents the situation that the given quality of service is rated higher by the consumer  $BV(q)$  than by the provider  $R(q)$ . Furthermore the slope of function  $BV(q)$  is greater than the slope of reservation price  $R(q)$ . In this case welfare  $W(q)$  is positive and increases with rising service quality  $q$ . To incentivize the service provider to deliver higher quality in order to maximize overall welfare, the price  $P(q)$  has to increase faster than the reservation price  $R(q)$ . Although, the price  $P(q)$  has to increase slower than the business value  $BV(q)$  to secure that the consumer will not be worse off, if the provider delivers higher quality. These requirements are expressed by the incentive constraint (IC). The fairness constraint (FC) guarantees that no party will have to bear a loss as long as overall welfare is positive.

$$b. \quad \forall q \in \{q \mid \frac{\partial R(q)}{\partial q} \leq \frac{\partial BV(q)}{\partial q}, R(q) \geq BV(q)\}:$$

$$\frac{\partial R(q)}{\partial q} \leq \frac{\partial P(q)}{\partial p} \leq \frac{\partial BV(q)}{\partial q} \quad (IC)$$

$$BV(q) \leq P(q) \leq R(q) \quad (FC)$$

On the other hand case b. represents the situation that reservation prices of the provider are higher than the monetary utility for the consumer  $R(q) > BV(q)$ . Furthermore the slope of function  $BV(q)$  is greater than the slope of the reservation price function  $R(q)$ . This means that overall welfare  $W(q)$  is negative but increasing with rising service quality  $q$ . The incentive constraint (IC) implements the incentive for the service provider to supply higher quality in order to maximize overall welfare without making the consumer being worse off, i.e. the price  $P(q)$  has to increase faster than its reservation price  $R(q)$  and slower than  $BV(q)$ . The fairness constraint (FC) guarantees that no party will make a profit as long as overall welfare is negative.

$$c. \quad \forall q \in \{q \mid \frac{\partial R(q)}{\partial q} > \frac{\partial BV(q)}{\partial q}, R(q) \leq BV(q)\}:$$

$$\frac{\partial BV(q)}{\partial q} \leq \frac{\partial P(q)}{\partial p} \leq \frac{\partial R(q)}{\partial q} \quad (IC)$$

$$R(q) \leq P(q) \leq BV(q) \quad (FC)$$

Case c. represents the situation of positive overall welfare  $W(q)$ , which is decreasing with rising service quality  $q$ . To incentivize the service provider to deliver lower quality in order to maximize overall welfare, the price  $P(q)$  has to increase slower than  $R(q)$ , but faster than the business value  $BV(q)$  to secure that the consumer will not be worse off if the provider decreases quality. Again, the fairness constraint secures that no party will have to bear a loss as long as overall welfare is positive.

$$d. \quad \forall q \in \{q \mid \frac{\partial R(q)}{\partial q} > \frac{\partial BV(q)}{\partial q}, R(q) \geq BV(q)\}:$$

$$\frac{\partial BV(q)}{\partial q} \leq \frac{\partial P(q)}{\partial p} \leq \frac{\partial R(q)}{\partial q} \quad (IC)$$

$$BV(q) \leq P(q) \leq R(q) \quad (FC)$$

Finally, in case d. overall welfare  $W(q)$  is negative and decreasing with rising service quality  $q$ . To incentivize the service provider to deliver lower quality in order to maximize overall welfare, the price  $P(q)$  has to increase slower than the reservation price  $R(q)$  of the provider but faster as the business value to the consumer again. The fairness constraint guarantees that no contractual partner will make a profit as long as overall welfare is negative.

Note that following assumption forms the basis of these constraints: If the marginal reservation price of the service provider equals the marginal revenue of the service provider, it will act according the preferences of the service consumer.

In conclusion, these constraints can be extended for the multiple service scenario as follows:

$$P: (q_1, \dots, q_n) \rightarrow IR^+$$

$$\forall q_i \in [0, 100]$$

s.t.

$$P(q_1^*, \dots, q_n^*) = p^*$$

and the incentive constraint (IC):

$$\min \left\{ \frac{\partial R(q_i)}{\partial q_i}, \frac{\partial BV(q_i)}{\partial q_i} \right\} \leq \frac{\partial P(q_i)}{\partial q_i} \\ \leq \max \left\{ \frac{\partial R(q_i)}{\partial q_i}, \frac{\partial BV(q_i)}{\partial q_i} \right\} \quad \forall q_i \in [0, 100]$$

and the fairness constraint (*FC*):

$$\begin{aligned} \min\{R(q_i), BV(q_i)\} &\leq P(q_i) \\ &\leq \max\{R(q_i), BV(q_i)\} \\ \forall i &= 1, \dots, n \end{aligned}$$

Every function  $P(q_i)$  which fulfills these constraints incentivizes the service provider to act in a way that maximizes overall welfare and guarantees that no party will bear a loss, if overall welfare is positive (and no party will make a profit, if overall welfare is negative respectively), i.e. the objective “service provider and consumer win and lose together” is implemented.

Even for not completely differentiable valuation functions  $BV(q_i)$  and  $R(q_i)$ , a price function  $P(q_i)$ , which fulfills the requirements can be designed by introducing steps according the considerations made above.

The introduced constraints do not determine the distribution of welfare to service provider and customer. This profit distribution is depending on the design of the price function  $P(q_i)$  according to these constraints. An exemplary price function which guarantees fair distribution of the overall welfare is introduced in the following section.

### 3.5. Using k-pricing for penalty calculation for single services

This section aims to come-up with a price function that fulfills the incentive constraints introduced above for a single service scenario, i.e. only one SLA between one service provider and one consumer is considered. The resulting penalty of a SLA can easily be computed as the difference of the agreed price  $p^*$  for met service level objective and the output of the price function for effectively met service quality  $\bar{q}$ :

$$Penalty(\bar{q}) = p^* - P(\bar{q})$$

Thereby a negative penalty is called a reward, since it has to be paid from the consumer to the provider for outperforming services, i.e. if the effectively met service quality is higher than the agreed service level objective.

Sattherthwaite and Williams [4] presented a k-pricing scheme to distribute the welfare of trades in double auction markets. The underlying idea of the k-

pricing scheme is to determine prices for a buyer and a seller on the basis of the difference between their valuations. For instance, suppose that a buyer wants to purchase a computation service for \$5 and a seller wants to sell for at least \$4. The difference between these bids is  $W = 5 - 4 = 1$ , where  $W$  is the surplus of this transaction that can be distributed among the participants. For a single service exchange, the k-pricing scheme can be formalized as follows [5]. According to chapter 0, let  $BV(\bar{q}) = \bar{b}\bar{v}$  be the valuation of the consumer and  $R(\bar{q}) = \bar{r}$  be the valuation of the provider of the service at a given quality level  $\bar{q}$ . The price for the service of quality  $\bar{q}$  can be calculated by  $P(\bar{q}) = k\bar{b}\bar{v} + (1 - k)\bar{r}$  with  $k \in [0,1]$ . In regard to the above-mentioned valuations for the service with the quality  $\bar{q}$ , the resulting price is  $P(\bar{q}) = 0.5 * 4 + (1 - 0.5) * 5 = 4.5$  [\$] for  $k = 0.5$ .

Sattherthwaite and Williams assumed that  $BV(\bar{q}) \geq R(\bar{q})$ , i.e. the consumer has a valuation for the commodity which is at least as high as the seller's reservation price. Otherwise no trade would occur. This assumption is not applying in the context of penalty calculation, since (unwished) situations could occur in which the business value for the consumer is lower than the reservation price of the provider. E.g., suppose the provider has served 100% availability until a certain point of time and suddenly it is unable to provide the service due to an electrical power outage until the agreed time is past. Then the met quality of service could be unexpected low and the situation mentioned above could occur.

The k-pricing scheme can also be applied to penalty calculation in SLAs in order to provide a price function that guarantees fair distribution of the welfare according to the constraints introduced in the previous chapter for every possible valuation functions  $BV(q)$  and  $R(q)$ :

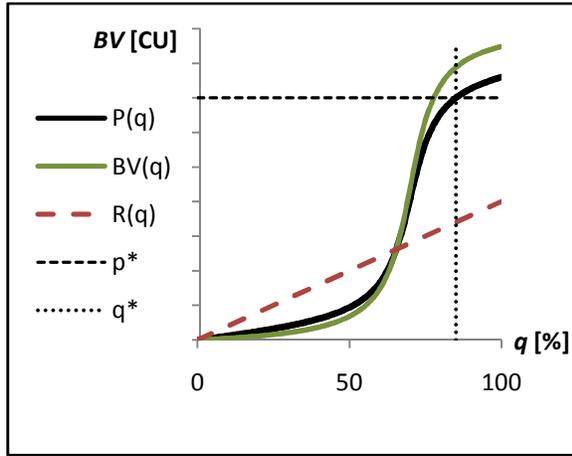
$$\begin{aligned} P(q) &:= k * BV(q) + (1 - k) * R(q) \\ \forall q &\in [0,100] \end{aligned}$$

whereas following equation determines parameter  $k$ :

$$k := \frac{p^* - R(q^*)}{BV(q^*) - R(q^*)}$$

This equation guarantees the fulfilment of the constraint  $P(q^*) = p^*$  and secures the distribution of positive and negative overall welfare according to the welfare distribution agreed in the SLA negotiation process (for the price  $p^*$  of the service at the quality

level  $q^*$ ). Figure 2. shows the price function  $P(q)$  for the exemplary initial situation showed in Figure 1.



**Figure 2. Price function calculation using k-pricing (k=0.8)**

### 3.6. Using k-pricing for penalty calculation for multiple services

Provided that only one overall price has been negotiated (see section 3.3) to meet exactly one service level objective for every service, k-pricing could be used to calculate fair, incentive compatible penalties for multiple services as follows:

$$P(q_1, \dots, q_n) := k * BV(q_1, \dots, q_n) + (1 - k) * R(q_1, \dots, q_n) \\ \forall q_i \in [0, 100]; i = 1, \dots, n$$

with

$$k := \frac{p^* - R(q_1^*, \dots, q_n^*)}{BV(q_1^*, \dots, q_n^*) - R(q_1^*, \dots, q_n^*)}$$

Again, this equation guarantees the fulfilment of the constraint  $P(q_1^*, \dots, q_n^*) = p^*$  and secures the distribution of positive and negative overall welfare according to the welfare distribution agreed in negotiation process. The incentive constraints determined in section 3.4 are met by this price function.

### 3.7. Application in Grid Markets

Since the mathematical formulation of valuation functions, especially for n-dimensional valuations in the multiple service scenario, would be too complex in practice, a simplified framework with tiered valuation functions is introduced in this section.

To ease the formulation of valuation functions, the quality range of all services  $s_i$  is divided into a certain number  $j_i$  of intervals. The relevant service qualities have to be valued by the provider and the consumer with one value for each interval in the single service scenario and with one value for each possible combination of quality domains in the multiple service scenario (since services are interdependent). Thereby, the number of intervals  $j_i$  does not necessarily have to be equal for all services and the length of intervals does not necessarily be the same within one service. These parameters have to be set according to the usual structure of the valuation functions for every service. The parameter setting stands for a trade-off between convenience and accuracy, i.e. a high number of short valuation intervals will provide better incentives, but will also raise complexity. It must be pointed out, that complexity increases very fast with the number of relevant services. With two services and five quality intervals for each service,  $5^2 = 25$  combinations have to be valued by both contractual partners. With three services and the same classification, already  $5^3 = 75$  combinations have to be valued. This classification of quality levels leads to step functions according to the business value and the reservation price of the services, which could be discontinuous at the interval bounds. In the intervals the slope of the functions is zero. According to this, the valuation functions generally can be defined as follows:

**Information 1 (simplified): Business value  $BV(q_i)$  of service qualities for the consumer for  $n$  services:**

$$BV: \prod_{i=1}^n Q_i \rightarrow IR^+ \forall i = 1, \dots, n$$

whereas  $BV$  is a graph spanned over the Cartesian product

$$\prod_{i=1}^n Q_i = Q_1 \times \dots \times Q_n := \{(q_1, \dots, q_n) | q_i \in Q_i; i = 1, \dots, n\}$$

with

$$Q_i := (q_{1,1}, q_{1,2}, \dots, q_{1,j_1-1}, q_{1,j_1})$$

Thereby  $Q_i$  represents the set of quality intervals  $q_{i,1}, q_{i,2}, \dots, q_{i,j_1-1}, q_{i,j_1}$  of service  $i$ , whereas  $j_i$  is the number of quality intervals of service  $i$ . In other

words, all possible combinations of quality intervals of the different services have to be valued by the consumer by allocation of one positive, monetary value.

**Information 2 (simplified): Reservation price  $R(q_i)$  of the service consumer for  $n$  services.** Equally, the reservation price of the provider can be expressed as a graph:

$$R: \prod_{i=1}^n Q_i \rightarrow IR^+ \forall i = 1, \dots, n$$

whereas  $R$  is spanned over the Cartesian product  $\prod_{i=1}^n Q_i$  again.

With information about the results of the negotiation process, as a bundle of service level objectives  $(q_1^*, q_2^*, \dots, q_{n-1}^*, q_n^*)$  and one agreed all round price  $p^*$  for all the services, the price function applying k-pricing can be written as follows:

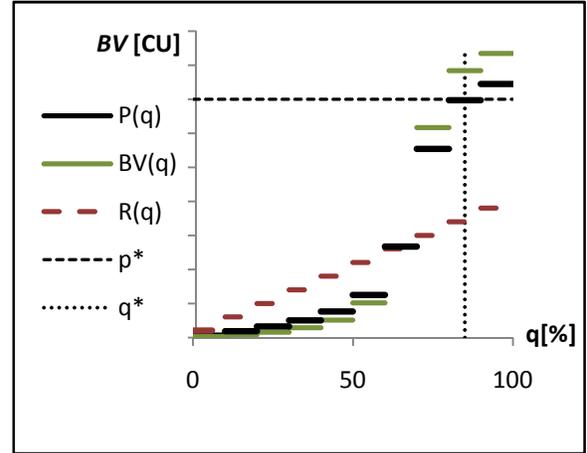
$$P(q_1, \dots, q_n) := k * BV(q_1, \dots, q_n) + (1 - k) * R(q_1, \dots, q_n) \\ \forall q_i \in [0, 100]; i = 1, \dots, n$$

with

$$k := \frac{p^* - R(q_1^*, \dots, q_n^*)}{BV(q_1^*, \dots, q_n^*) - R(q_1^*, \dots, q_n^*)}$$

This equation guarantees the fulfilment of the constraint  $P(q_1^*, \dots, q_n^*) = p^*$  and secures the distribution of positive and negative overall welfare according to the welfare distribution agreed in negotiation process. Of course, fulfilling the incentive compatibility conditions presented in section 3.4 is not feasible with tiered functions. Since these step functions could be considered as approximation and pass into real valuation functions for sufficiently small intervals, incentive compatibility can be achieved in some degree anyway.

Figure 3. visualises k-pricing in case of the simplified scenario by using step functions for service valuation for a single service.



**Figure 3. Simplified step functions  $BV(q)$ ,  $R(q)$  and  $P(q)$  using k-pricing ( $k=0.8$ )**

In the following a numerical example with two interdependent services is drawn to clarify the (simplified) k-pricing scheme in multiple services scenario. For the sake of clarity a graphic account has been left out.

In this example, two interdependent services are considered. These services are supposed to be complements for the consumer, i.e. these services should be consumed together. In economics, a good 1 is a complement to good 2, if the demand for good 1 goes down when the price of good 2 goes up [7]. A famous example of complement goods would be hamburgers and hamburger buns. In a Grid market, CPU and storage could be considered as complement goods, since there may be only very limited use for an application to purchase computing power without having the appropriate memory capacity. The same services could be of substitute character to the service provider, for example if the provision of these services is independent from each other. Then the reservation price for both services will simply be the sum of the reservation prices of the two single services. Following tables show exemplary valuations of two services, which are of complement character for the consumer and of substitute character for the provider respectively.

**Table 1. Exemplary business value  $BV(q_1, q_2)$  of complements for the consumer**

|                        | $q_{2,1} \in [0,25)$ | $q_{2,2} \in [25,50)$ | $q_{2,3} \in [50,75)$ | $q_{2,4} \in [75,100]$ |
|------------------------|----------------------|-----------------------|-----------------------|------------------------|
| $q_{1,1} \in [0,25)$   | 0                    | 5                     | 10                    | 20                     |
| $q_{1,2} \in [25,50)$  | 5                    | 50                    | 60                    | 70                     |
| $q_{1,3} \in [50,75)$  | 10                   | 60                    | 100                   | 110                    |
| $q_{1,4} \in [75,100]$ | 20                   | 70                    | <b>110</b>            | 150                    |

**Table 2. Exemplary provider's reservation price  $R(q_1, q_2)$  of substitutes**

|                        | $q_{2,1}$<br>€ [0,25) | $q_{2,2}$<br>€ [25,50) | $q_{2,3}$<br>€ [50,75) | $q_{2,4}$<br>€ [75,100] |
|------------------------|-----------------------|------------------------|------------------------|-------------------------|
| $q_{1,1} \in [0,25)$   | 10                    | 20                     | 30                     | 40                      |
| $q_{1,2} \in [25,50)$  | 20                    | 40                     | 50                     | 60                      |
| $q_{1,3} \in [50,75)$  | 30                    | 50                     | 60                     | 70                      |
| $q_{1,4} \in [75,100]$ | 40                    | 60                     | <b>70</b>              | 80                      |

Suppose the service level objective is to meet a quality  $q_1 \in [75,100]$  for service  $s_1$  and quality  $q_2 \in [50,75)$  for service  $s_2$  for an all round price  $p^* = 80$  [CU], which has been determined for met service level objectives in the negotiation process. So factor  $k$  can be calculated as

$$k = \frac{p^* - R(q_1^*, q_2^*)}{BV(q_1^*, q_2^*) - R(q_1^*, q_2^*)} = \frac{80 - 70}{110 - 70} = 0.25$$

With the equation presented above the price can be calculated for every possible combination of services.

$$P(q_{1,a}, q_{2,b}) = k * BV(q_{1,a}, q_{2,b}) + (1 - k) * R(q_{1,a}, q_{2,b})$$

$\forall a = 1, \dots, 4$  and  $\forall b = 1, \dots, 4$

Table 3. shows the results of a price calculation.

**Table 3. Prices  $P(q_1, q_2)$  calculated via k-pricing ( $k = 0,5$ )**

|                         | $q_{2,1}$<br>€ [0,25) | $q_{2,2}$<br>€ [25,50) | $q_{2,3}^* = q_{2,3}$<br>€ [50,75) | $q_{2,4}$<br>€ [75,100] |
|-------------------------|-----------------------|------------------------|------------------------------------|-------------------------|
| $q_{1,1} \in [0,25)$    | 7.5                   | 16.25                  | 25                                 | 35                      |
| $q_{1,2} \in [25,50)$   | 16.25                 | 42.5                   | 52.5                               | 62.5                    |
| $q_{1,3}^* \in [50,75)$ | 25                    | 52.5                   | 70                                 | 80                      |
| $q_{1,4} \in [75,100]$  | 35                    | 62.5                   | 80 = $p^*$                         | 97.5                    |

The calculated prices are always between the valuations of the relevant service partners of the contractual partners. For the agreed service level objective  $q_1^* \in [75,100]$  and  $q_2^* \in [50,75)$  the calculated price  $P(q_1^*, q_2^*)$  equals the agreed price  $p^*$ .

From these prices again the penalty and reward respectively can be calculated as

$$Penalty(\bar{q}_1, \bar{q}_2) = p^* - P(\bar{q}_1, \bar{q}_2)$$

whereas  $\bar{q}_i$  is the actually met quality of service  $i$ .

**Table 4. Penalties and rewards for different service levels**

|                         | $q_{2,1}$<br>€ [0,25) | $q_{2,2}$<br>€ [25,50) | $q_{2,3}^* = q_{2,3}$<br>€ [50,75) | $q_{2,4}$<br>€ [75,100] |
|-------------------------|-----------------------|------------------------|------------------------------------|-------------------------|
| $q_{1,1} \in [0,25)$    | 72.5                  | 63.75                  | 55                                 | 45                      |
| $q_{1,2} \in [25,50)$   | 63.75                 | 37.5                   | 27.5                               | 17.5                    |
| $q_{1,3}^* \in [50,75)$ | 55                    | 27.5                   | 10                                 | 0                       |
| $q_{1,4} \in [75,100]$  | 45                    | 17.5                   | 0                                  | -17.5                   |

For the agreed service level objective  $q_1^* \in [75,100]$  and  $q_2^* \in [50,75)$  the penalty is zero, since the calculated price  $P(q_1^*, q_2^*)$  equals the agreed price  $p^*$ . The penalty for  $\bar{q}_1 \in [50,75)$  and  $\bar{q}_2 \in [75,100]$  is also zero, because of the symmetry of the valuations in this example. If the SLO is outperformed, i.e.  $\bar{q}_1 \in [75,100]$  and  $\bar{q}_2 \in [75,100]$ , the consumer will pay a reward (negative penalty) to the service provider.

**Table 5. Benefit of service provider  $P(q_1, q_2) - R(q_1, q_2)$** 

|                         | $q_{2,1}$<br>€ [0,25) | $q_{2,2}$<br>€ [25,50) | $q_{2,3}^* = q_{2,3}$<br>€ [50,75) | $q_{2,4}$<br>€ [75,100] |
|-------------------------|-----------------------|------------------------|------------------------------------|-------------------------|
| $q_{1,1} \in [0,25)$    | -2.5                  | -3.75                  | -5                                 | -5                      |
| $q_{1,2} \in [25,50)$   | -3.75                 | 2.5                    | 2.5                                | 2.5                     |
| $q_{1,3}^* \in [50,75)$ | -5                    | 2.5                    | 10                                 | 10                      |
| $q_{1,4} \in [75,100]$  | -5                    | 2.5                    | 10                                 | <u>17.5</u>             |

Table 5. shows the benefit of the service provider at each quality level combination calculated as the difference  $P(q_1, q_2) - R(q_1, q_2)$ . Table 6. presents the associated benefits for the consumer, calculated equivalent as the difference between its valuation and the determined price  $BV(q_1, q_2) - P(q_1, q_2)$ .

**Table 6. Benefit of service consumer  $BV(q_1, q_2) - P(q_1, q_2)$** 

|                         | $q_{2,1}$<br>€ [0,25) | $q_{2,2}$<br>€ [25,50) | $q_{2,3}^* = q_{2,3}$<br>€ [50,75) | $q_{2,4}$<br>€ [75,100] |
|-------------------------|-----------------------|------------------------|------------------------------------|-------------------------|
| $q_{1,1} \in [0,25)$    | -7.5                  | -11.25                 | -15                                | -15                     |
| $q_{1,2} \in [25,50)$   | -11.25                | 7.5                    | 7.5                                | 7.5                     |
| $q_{1,3}^* \in [50,75)$ | -15                   | 7.5                    | 30                                 | 30                      |
| $q_{1,4} \in [75,100]$  | -15                   | 7.5                    | 30                                 | <u>52.5</u>             |

As can be seen from these tables, both parties will bear a loss, if at least one service is delivered at the lowest quality. As soon as both services are provided in a quality greater or equal than 25%, both parties will make a (positive) profit, which is

increasing with rising quality of both services. For every possible change of service qualities making the provider being better off, the service consumer will also be better off. Hence, the provider is incentivized to provide both services in best possible quality to maximize its profit. Due to the fixed ratio of overall welfare distribution provided by the k-pricing scheme, this will maximize the benefit of the consumer too. In this case the SLO is outperformed by the service provider and an overall welfare even greater than the welfare in the originally agreed situation will be distributed fairly.

#### 4. Conclusion

Penalty calculation is crucial for true risk sharing in SLAs, since it determines the distribution of the associated risk of the SLA among the contractual partners. The k-pricing scheme provides fair and economical reasonable prices for every quality level of a single service. Thereby the overall welfare is distributed at a fixed ratio according to the result of a negotiation process for all possible met service qualities. Thus, the party with the bigger share of positive welfare will also bear a higher share of the loss, if the SLA is violated, i.e. the party which gets a bigger share of the positive welfare also bears a higher risk. Due to these characteristics and the fact, that k-pricing fulfils the incentive constraints presented in section 3.4 (proof trivial) and provides fair welfare distribution, k-pricing is the ideal scheme for penalty calculation in automated markets. However, the problem of using k-pricing for penalty calculation is that it provides incentives for market participants to distort their true valuations. Suppose a service consumer is interested in a service at one specific level of quality. Then he is incentivized to understate its valuation for all other service levels in order to maximize penalties in case of violated SLA and to minimize rewards in case of outperformed SLO. This incentive problem cannot be solved in context with k-pricing, since the price function is always directly associated with the valuations and thus directly influence able. Thus, the incentive to report true valuations has to be provided by other components in a Grid Market, maybe the Market Mechanism, which matches supply and demand. Just as well the creditability of the contractual partners regarding the disclosed valuations could indirectly be gauged by observing valuations for similar services in the past. But surely the incentive to distort valuations will remain as a challenge to be solved.

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