Supply Chain Coordination Using Returns Policy with Sales Rebate and Penalty under Effort and Price Dependent Demand

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Abstract

A supply chain is two or more parties linked by a flow of goods, information, and funds. When one or more parties of the supply chain try to optimize their own profits, system performance may be hurt. In the standard newsvendor setting, both the returns policy and the sales rebate and penalty (SRP) are coordination contracts that provide incentives to all of the supply chain's members so that the decentralized supply chain behaves nearly or exactly the same as the integrated one. When demand is influenced by the retail price and retailer sales effort, each of the returns policy and the SRP contract no longer coordinates on its own. To solve this problem, we employ a new model combined the returns policy with the SRP contract. By analyzing we find that a properly designed returns policy with SRP achieves coordination and a win-win outcome.

1. Introduction

Coordination among suppliers and retailers is a very important strategic issue in supply chain management. One of the important instruments for supply chain coordination is returns policy. Returns policy is also called buy back contract, it is widely used in the short life-cycle products such as fashion apparel, books, personal computers, toys, and CDs. With returns policy the retailer returns the unsold products to the supplier or the supplier offers a credit on all unsold products to the retailer.

Returns policy has been applied in the real world for a long time. The main objective of the returns policy is to mitigate the risk of overstocking, caused by uncertain demand that retailers face [1]. It is know that the returns policy can eliminate the problem of "double marginalization" and improve supply chain's efficiency [2]. In the literature, the first quantitative analysis of returns policy with a stochastic demand appears in the marketing science literature, within the framework of a single-period inventory model of a manufacturer-retailer channel; this is the benchmark paper by Pasternack [3]. He proves that channel coordination can be achieved when the manufacturer offers partial credit for all unsold goods. After that, many extensions and variations have been attempted (see, e.g., reference lists in [4-7]).

Since the retail price is the main factor to affect the market demand, recently some scholars have extended the returns policy model by allowing the retailer to choose his retail price in addition to his stocking quantity. Cachon [8] demonstrates that a returns policy does not coordinate the newsvendor with price dependent demand. While, Emmons and Gilbert [9] recognize that the returns policy does not coordinate this model, they nevertheless demonstrate a returns policy may still perform better than a wholesale price contract.

Recently, Yao et al. [1] consider a single-period product supply chain with stochastic and price dependent demand to study returns policy for coordinating the supply chain. They also prove that a returns policy can improve supply chain performance. While, considering price dependent demand, they show that the total supply chain's expected profit with returns policy is always lower than the expected profit of the integrated supply chain. The reason is that in their models the supply chain members are primarily concerned with optimizing their own objectives. They also do not put forward a coordination contract to coordinate the supply chain, so that each member's objective becomes aligned with the supply chain's objective.

Besides the retail price, in many settings, retailer sales effort is also important in influencing demand. A retailer can increase a product's demand by hiring more sales people, improving their training, increasing advertising, providing attractive shelf space, and guiding consumer purchases with sales personnel. All of those activities are costly. Considering the supply chain with effort dependent demand, Cachon [8] demonstrate that the returns policy cannot coordinate this supply chain in this setting, and the retailer's optimal effort is lower than the integrated supply chain's. In order to coordinate the supply chain when the sales effort influences the market demand, Taylor [10] designs a returns policy combined with channel rebates to coordinate the supply chain and achieves a win-win outcome.

From above literatures we can see that the returns policy no longer coordinate under effort or price dependent demand because the incentives they provide to coordinate the retailer's quantity decision distort the retailer's effort or price decision. On the other hand, in the real world, the retail price and sales effort often influence the market demand simultaneously. But few articles have studied the supply chain coordination in this setting.

Our model is similar in vein to the one studied in Taylor [10]. However, in Taylor's model he considers that the retail price is exogenous. In this study, we extend the Taylor's model by allowing the price to become a key decision variable for the retailer. The major contributions of this paper include the simultaneous determination of price, sales effort and quantity for the retailer with a unique approach to modeling price and effort sensitive random demand. By analyzing the impact of a returns policy on supply chain coordination under the assumption that the demand is influenced by both the retail price and sales effort, we will provide a new contract, i.e., returns policy with sales rebate and penalty, to coordinate this supply chain, in which the independent retailer can make decisions as the optimal integrated supply chain.

The sales rebate and penalty (SRP) is a payment between a supplier and a retailer based on retailer sales to end consumers. Before selling season, the supplier offers a sales target to the retailer, if the final sales quantity is above the target, the supplier gives the retailer a rebate, otherwise, the retailer gives a payment to the supplier as penalty. The sales rebate and penalty is required when the market incentives are insufficient [11]. In this paper, we will employ the complementarity of the SRP and the returns policy to coordinate the supply chain.

The rest of the paper is organized as follows: Section 2 introduces our model assumptions and notation. Section 3 examines a benchmark case of integrated supply chain. In section 4 and 5, we analyze the returns policy and SRP contract respectively when demand is influenced by the retail price and sales effort. In section 6, we develop a model of returns policy with SRP to coordinate the supply chain. Section 7 uses simulation studies to illustrate our results. Section 8 provides concluding remarks and describes future research.

2. Model assumptions and notation

We consider a single period, single product model with a supplier and a retailer. The retailer faces a random, effort and price dependent demand. We consider the case where the selling season of this product is short. At the end of the selling season, the retailer can return the unsold products to the supplier following a returns policy.

The following notation will be used in the formulation:

Let p be the retail price, w the wholesale price, c the supplier's manufacturing cost, v the salvage value, Q the retailer's order quantity. To model retail effort, suppose a single effort level, e, summarizes the retailer's activities and let $g(e)$ be the retailer's cost of exerting effort level e, where $g(0)=0$, $g'(e)>0$ and $g''(e)>0$. Let x be a random variable for customer demand. Let $f(x|(e,p))$ be the probability density function of demand given the effort level e and the price p, and $F(x|(e,p))$ its distribution function, where demand is stochastically increasing in effort and decreasing in price, i.e., $\frac{\partial F(x|(e,p))}{\partial e}<0$ and $\frac{\partial F(x|(e,p))}{\partial p}>0$. Let $S(Q,e,p)$ be expected sales,

$$\begin{aligned}S(Q,e,p) &= E\min(Q,x)\\ &= \int_0^\infty (Q\wedge x) f(x|(e,p))\,dx\\ &= \int_0^\infty \int_{y=0}^{Q\wedge x} dy f(x|(e,p))\,dx\\ &= \int_0^Q \int_y^\infty f(x|(e,p))\,dxdy\\ &= Q-\int_0^Q F(x|(e,p))dx\end{aligned}$$

Let $I(Q,e,p)$ be the excepted left over inventory:

$$I(Q,e,p)=E(Q-x)^+=Q-S(Q,e,p) \tag{1}$$

3. The integrated supply chain

The aim that the supplier offers returns policy to the retailer is to coordinate the supply chain and maximize the whole supply chain's profit. Therefore, at first we should consider the integrated supply chain.

In the integrated supply chain, the supplier acts as his own retailer (i.e., company store). This model will enable us to determine the optimal policy for the system as a whole. The integrated supply chain's profit is:

$$\Pi_T(Q,e,p) = pS(Q,e,p) + vI(Q,e,p) - cQ - g(e) \qquad (2)$$

In the left side of (2), the first term is the expected revenue from sales, the second term is the earnings from salvaging the products remaining from sales, the third term is the production cost for Q products and the last term is the cost of exerting effort level e. Using (1) one can rearrange the integrated supply chain's profit as follows:

$$\Pi_T(Q,e,p) = (p-v)S(Q,e,p) - (c-v)Q - g(e) \qquad (3)$$

The integrated supply chain's profit function need not be concave nor unimodal [8]. Assume there exists a finite (but not necessarily unique) optimal quantity-effort-price, $\{Q^*, e^*, p^*\}$. The following first order conditions are necessary for coordination (but not sufficient),

$$\frac{\partial \Pi_T(Q,e^*,p)}{\partial e} = (p-v)\frac{\partial S(Q,e^*,p)}{\partial e} - g'(e^*) = 0 \qquad (4)$$

$$\frac{\partial \Pi_T(Q,e,p^*)}{\partial p} = S(Q,e,p^*) + (p^*-v)\frac{\partial S(Q,e,p^*)}{\partial p} = 0 \qquad (5)$$

$$\frac{\partial \Pi_T(Q^*,e,p)}{\partial Q} = (p-v)\frac{\partial S(Q^*,e,p)}{\partial Q} - (c-v) = 0 \qquad (6)$$

From (6), we get

$$F(Q^* | (e,p)) = \frac{p-c}{p-v} \qquad (7)$$

A contract designed by the supplier is said to coordinate the supply chain if it is able to satisfy the first order conditions at Q^*, e^* and p^*.

4. The independent retailer with returns policy

If both supplier and retailer are independent, they will try to maximize their own expected profits without considering maximizing the total supply chain's expected profit. Wang [12] shows that without supply chain coordination, the independent retailer will always order less than the total supply chain's optimal quantity. The decentralized supply chain's expected profit will be lower than an integrated supply chain's. This phenomenon is well known as "double marginalization" [2]. To encourage the retailer to order more, the supplier will offer a supply chain contract of returns policy so that the supply chain is coordinated. In this way, the supplier shares the risk faced by the retailer.

Unfortunately, when allowing the retailer to exert costly effort and change retail price to increase demand, coordination is challenging. Furthermore, coordination is complicated by the fact that the incentives to align the retailer's order quantity decision may distort the retailer's effort and price decision. In the following sections, we will extend the standard newsvendor model by allowing the retailer to exert costly effort and change retail price to increase demand.

Let the return credit for each unsold unit be given by r where $r \in [v, w)$. Under the returns policy alone, the retailer's profit function is

$$\begin{aligned}&\Pi_R(Q,e,p,r)\\ &= pS(Q,e,p) + rI(Q,e,p) - wQ - g(e)\end{aligned} \qquad (8)$$

where the first term is the expected revenue from the sales, the second term is the earnings from returning the unsold products to the supplier, the third term is the cost of buying Q products from the supplier and the last term is the cost of exerting effort level e. Using (1) we can rewrite the retailer's profit as:

$$\Pi_R(Q,e,p,r) = (p-r)S(Q,e,p) - (w-r)Q - g(e) \qquad (9)$$

Then,

$$\frac{\partial \Pi_R(Q,e,p,r)}{\partial e} = (p-r)\frac{\partial S(Q,e,p)}{\partial e} - g'(e) \qquad (10)$$

$$\frac{\partial \Pi_R(Q,e,p,r)}{\partial p} = S(Q,e,p) + (p-r)\frac{\partial S(Q,e,p)}{\partial p} \qquad (11)$$

Compare (10) with (4) and (11) with (5), thus e^* and p^* cannot be the retailer's optimal effort and price level when $r > v$. But $r > v$ is required to coordinate the retailer's order quantity, so it follows that the returns policy cannot coordinate in this setting.

5. The independent retailer with SRP contract

In a SRP contract, the supplier sets up a product sales target T for the retailer, if the retailer sells the product beyond the target T, the supplier will give the retailer an τ rebate per unit sold above T, otherwise, the retailer will pay the supplier an τ penalty per unit unsold under T. The target rebate and penalty is interesting only if it achieves supply chain coordination for $T < Q^*$ [8].

In this setting, the retailer's profit function is

$$\Pi_R(Q,e,p,\tau) = pS(Q,e,p) + vI(Q,e,p) - wQ + \tau\left[S(Q,e,p) - T\right] - g(e) \quad (12)$$

where the term $\tau\left[S(Q,e,p)-T\right]$ is the earning of rebate if $S(Q,e,p) \geq T$, or else is the cost of penalty. Using (1) we can rearrange the retailer's profit as:

$$\begin{aligned}&\Pi_R(Q,e,p,\tau)\\&= (p - v + \tau)S(Q,e,p) - (w - v)Q - \tau T - g(e)\end{aligned} \quad (13)$$

From above, we know that

$$\frac{\partial \Pi_R(Q,e,p,\tau)}{\partial e} = (p - v + \tau)\frac{\partial S(Q,e,p)}{\partial e} - g'(e) \quad (14)$$

$$\frac{\partial \Pi_R(Q,e,p,\tau)}{\partial p} = S(Q,e,p) + (p - v + \tau)\frac{\partial S(Q,e,p)}{\partial p} \quad (15)$$

Compare (14) with (4) and (15) with (5), we know that only when $\tau = 0$, the supply chain can be coordinated. But $\tau > 0$ is required to coordinate the retailer's order quantity. Hence, the SRP contract does not coordinate the newsvendor with effort and price dependent demand.

6. Returns policy with SRP

From above sections, we can see that the returns policy or the SRP contract fails to coordinate the retailer's action because it distorts the retailer's price decision and marginal incentive to exert effort. But we find that the returns policy reduces the retailer's incentive, which counteracts the retailer's excessive incentive with a SRP alone. In the following subsections, we will provide a returns policy model combined with a SRP to strengthen incentives for retailer sales effort and retail price so that the supply chain is coordinated.

6.1. Supply chain coordination strategies

Consider the returns policy with SRP, the retailer's profit function under a target rebate and penalty is

$$\begin{aligned}&\Pi_R(Q,e,p,r,\tau)\\&= pS(Q,e,p) + rI(Q,e,p) - wQ\\&\quad + \tau\left[S(Q,e,p) - T\right] - g(e)\\&= (p - r + \tau)S(Q,e,p) - (w - r)Q - \tau T - g(e)\end{aligned} \quad (16)$$

For coordination the first order conditions must hold,

$$\begin{aligned}\frac{\partial \Pi_R(Q,e^*,p,r,\tau)}{\partial e} &= (p - r + \tau)\frac{\partial S(Q,e^*,p)}{\partial e} - g'(e^*)\\&= \frac{\partial \Pi_T(Q,e^*,p)}{\partial e} = 0\end{aligned} \quad (17)$$

$$\begin{aligned}&\frac{\partial \Pi_R(Q,e,p^*,r,\tau)}{\partial p}\\&= S(Q,e,p^*) + (p^* - r + \tau)\frac{\partial S(Q,e,p^*)}{\partial p}\\&= \frac{\partial \Pi_T(Q,e,p^*)}{\partial p} = 0\end{aligned} \quad (18)$$

$$\begin{aligned}\frac{\partial \Pi_R(Q^*,e,p,r,\tau)}{\partial Q} &= (p - r + \tau)\frac{\partial S(Q^*,e,p)}{\partial Q} - (w - r)\\&= \frac{\partial \Pi_T(Q^*,e,p)}{\partial Q} = 0\end{aligned} \quad (19)$$

From (17), (18) and (19), we can get a coordinating returns policy with SRP, $\{r^*, \tau^*\}$,

$$\begin{cases} r^* = w - c + v \\ \tau^* = w - c \end{cases} \quad (20)$$

Therefore, in this setting a returns policy with SRP can coordinate the newsvendor with effort and price dependent demand.

Substituting (20) into (16), we get the retailer's expected profit:

$$\begin{aligned}&\Pi_R(Q^*,e^*,p^*,r^*,\tau^*)\\&= (p^* - v)S(Q^*,e^*,p^*) - (c - v)Q^*\\&\quad - (w - c)T - g(e^*)\\&= \Pi_T(Q^*,e^*,p^*) - (w - c)T\end{aligned} \quad (21)$$

The supplier's expected profit is

$$\begin{aligned}&\Pi_S(Q^*,e^*,p^*,r^*,\tau^*)\\&= \Pi_T(Q^*,e^*,p^*) - \Pi_R(Q^*,e^*,p^*,r^*,\tau^*)\\&= (w - c)T\end{aligned} \quad (22)$$

From (22), we can know that the supplier's expected profit is determinate after the T is set. Since the retailer bears all the demand risk, in order to interest the retailer in agreeing this coordination contract, the supplier should adjust the T to let the retailer get more profit than non-coordination contract, e.g., the wholesale price contract.

6.2. Optimal solutions

In this subsection we are going to derive the optimal solutions of Q^*, e^* and p^*. From above subsection, we know that if the supplier offers a coordinating returns policy with SRP, the optimal values of Q, e and p decided by the independent retailer equal the total supply chain's optimal values. So in this subsection we will use the integrated supply chain to derive the optimal solutions.

Consider the setting in which demand is stochastic and an additive function of retailer sales effort and price. Specifically, let demand be given by $x = y(e,p)+\xi$, where $p \in [c,\overline{p}]$, $\overline{p}$ is the maximum admissible price, i.e., $x\big|_{p=\overline{p}} = y(e,\overline{p})+\xi = 0$. It is natural to assume that $y(e,p)$ decreases in price, i.e., $\frac{\partial y(e,p)}{\partial p} < 0$, and $y(e,p)$ is a concave, increasing function in effort, i.e., $\frac{\partial y(e,p)}{\partial e} > 0$ and $\frac{\partial^2 y(e,p)}{\partial e^2} \le 0$, that is to say, the marginal effectiveness of effort is decreasing. ξ is a random variable. It has a continuous distribution $\Phi(\bullet)$ with density $\phi(\bullet)$. It is also reasonable to assume that $\Phi(\bullet)$ is invertible and that has a continuous $\phi(\bullet)$ derivative $\phi'(\bullet)$. So the density and distribution of demand x can be expressed as $f(x|(e,p)) = \phi(x - y(e,p))$ and $F(x|(e,p)) = \Phi(x - y(e,p))$. Under these assumptions, (3) can be rewrite as follows:

$$\Pi_T(Q,e,p) = (p-c)Q - (p-v)\int_{y(e,p)}^{Q} \Phi(x - y(e,p))dx - g(e) \tag{23}$$

It is easy to verify that $\Pi_T(Q,e,p)$ is concave in Q and e. So from (23), with given effort e and price p, the optimal order quantity for the retailer, Q^*, is

$$Q^* = \Phi^{-1}(\frac{p-c}{p-v}) + y(e,p) \tag{24}$$

Now substitute (24) into (23):

$$\Pi_T(Q,e,p) = (p-c)y(e,p) + (p-v)\int_{\underline{\xi}}^{\Phi^{-1}(\frac{p-c}{p-v})} \xi\phi(\xi)d\xi - g(e) \tag{25}$$

From (25), we have

$$\frac{\partial \Pi_T(Q,e,p)}{\partial e} = \frac{\partial y(e,p)}{\partial e}(p-c) - g'(e) \tag{26}$$

and

$$\frac{\partial \Pi_T(Q,e,p)}{\partial p} = (p-c)\frac{\partial y(e,p)}{\partial p} + y(e,p) + \int_{\underline{\xi}}^{\Phi^{-1}(\frac{p-c}{p-v})} \xi\phi(\xi)d\xi + \frac{c-v}{p-v}\Phi^{-1}(\frac{p-c}{p-v}) \tag{27}$$

Because $g''(e) > 0$ and $\frac{\partial^2 y(e,p)}{\partial e^2} \le 0$, we find

$$\frac{\partial^2 \Pi_T(Q,e,p)}{\partial e^2} = \frac{\partial^2 y(e,p)}{\partial e^2}(p-c) - g''(e) < 0 \tag{28}$$

So $\Pi_T(Q,e,p)$ is strictly concave in e. Let's fix p, then the optimal effort level e^* satisfy:

$$\frac{\partial y(e^*,p)}{\partial e}(p-c) - g'(e^*) = 0 \tag{29}$$

Therefore, from (29), e^* can be expressed as a function of p.

Before deriving the optimal price p^*, we present the following mild assumptions about $y(e,p)$ and ξ.

Assumption 1. The demand function $y(e^*,p)$ is positive and strictly decreasing for $p \in [c,\overline{p}]$. The elasticity $\eta = -p\frac{dy(e^*,p)/dp}{y(e^*,p)}$ of the demand function is increasing for $p \in [c,\overline{p}]$, i.e., $\frac{d\eta}{dp} \ge 0$.

The Assumption 1 is intuitive: as the price increase, the demand decreases by a large percentage, which makes it less desirable to raise the price further. It is common in the literature ([13], [14]).

We define $h(\xi) = \frac{\phi(\xi)}{1-\Phi(\xi)}$ as the failure rate function of the random distribution for ξ, and assume that:

Assumption 2. The distribution of the random variable ξ has an increasing failure rate (IFR), i.e. $h'(\xi) \ge 0$ for all ξ.

The above IFR property of ξ is indeed satisfied by most of the theoretical distributions, and the detailed description on the property of IFR can be found in [15].

Using the similar proof in [14], we can get:

Property 1. Under assumptions 1 and 2, the supply chain's expected profit $\Pi_T(Q,e,p)$ is quasi-concave in p on $[c,\overline{p}]$. Hence there exists a unique retail price p that maximizes $\Pi_T(Q,e,p)$.

Using the case in section 7, we can plot the relation between the retail price and the total supply chain's profit as Figure 1. It is an example of Property 1.

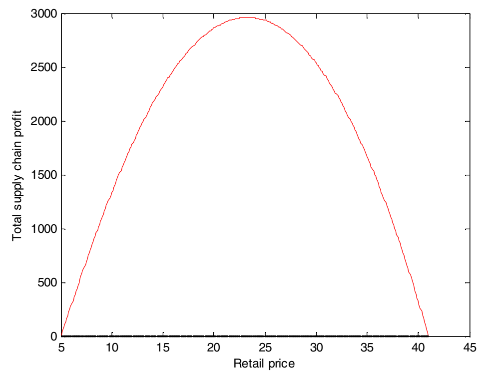

Figure 1. The relation between the retail price and the total supply chain's profit

So the optimal retailer price p^* can be obtained from the solution of $\frac{\partial \Pi_T(Q,e,p)}{\partial p} = 0$. Specifically:

$$(p^* - c)\frac{\partial y(e,p^*)}{\partial p} + y(e,p^*) + \int_{\underline{\xi}}^{\Phi^{-1}(\frac{p^*-c}{p^*-v})} \xi\phi(\xi)d\xi + \frac{c-v}{p^*-v}\Phi^{-1}(\frac{p^*-c}{p^*-v}) = 0 \tag{30}$$

Here, we can use a one-dimensional search to find the optimal p^*.

6.3. Effect of coordination

At this stage, a valid question is if our model can address the case in which the supplier and retailer coordinate their decisions. Such coordination can be interpreted as the result of a contract that achieves coordination in a decentralized supply chain. In this subsection, we analyze the expected profits of the supplier, the retailer and the supply chain as a whole. Our focus is on the comparison of coordination and non-coordination.

We first introduce the coordination contract, i.e., the returns policy with SRP. To illustrate the models and gain additional insight from the analysis above, in the rest of this subsection, we assume that $y(e,p) = a + be - kp$ and $g(e) = \mu e^2/2$, where $a,b,k,\mu > 0$. We will arbitrarily choose the PDF of ξ as uniformly distributed within the range of $[A,B]$ where $B > A$. These assumptions have been commonly used in supply chain models in order to facilitate analysis (see e.g., [9] and [10]).

Under above assumptions, from (29), we get the optimal level e^* is

$$e^* = \frac{b(p-c)}{\mu} \tag{31}$$

From (31), we can know that e^* is increasing for p, it is natural in real life. The more profit margin will stimulate the retailer to exert more effort. Substituting (31) into $y(e,p)$, we obtain

$$y(e^*,p) = a + \frac{b^2(p-c)}{\mu} - kp = a - \frac{b^2 c}{\mu} - (k - \frac{b^2}{\mu})p \tag{32}$$

Since $y(e^*,c) \geq 0$, we can get $k \leq \frac{a}{c}$. In order to satisfy assumption 1, we have $\frac{b^2}{\mu} \leq \min(k, \frac{a}{c})$. So we assume that $\frac{b^2}{\mu} \leq k \leq \frac{a}{c}$. In addition, It is easy to prove that the uniformly distributed of ξ satisfies assumption 2. Therefore, according to Property 1 and from (30), we can get the unique optimal retailer price p^* as follows:

$$a - 2kp^* + kc + \frac{b^2(p^*-c)}{\mu} + A + \frac{(B-A)(p^*-c)(p^*+c-2v)}{2(p^*-v)^2} = 0 \tag{33}$$

The total supply chain's expected profit is

$$\Pi_T^c(p^*) = (p^*-c)(a-kp^*+A) + \frac{(p^*-c)^2}{2}(\frac{B-A}{p^*-v} + \frac{b^2}{\mu}) \tag{34}$$

Let $\Pi_R^c(p^*)$ and $\Pi_S^c(p^*)$ be the retailer's expected profit and the supplier's expected profit with coordination, respectively. Using (34), we can rewrite (21) and (22) as followings:

$$\begin{aligned}\Pi_R^c(p^*) &= \Pi_R(Q^*,e^*,p^*,r^*,\tau^*) \\ &= (p^*-c)(a-kp^*+A) \\ &+ \frac{(p^*-c)^2}{2}(\frac{B-A}{p^*-v} + \frac{b^2}{\mu}) - (w-c)T\end{aligned} \tag{35}$$

and

$$\Pi_S^c(p^*) = \Pi_S(Q^*,e^*,p^*,r^*,\tau^*) = (w-c)T \tag{36}$$

We next consider the non-coordination contract, i.e., the wholesale price contract. In the wholesale price contract both supplier and retailer are independent, they will try to maximize their own expected profits without considering maximizing the total supply chain's expected profit. In this setting, the supplier charges the retailer a wholesale price w that is higher than c. In turn, the retailer sells the products to the public at the retailer price p and liquidates any overstock at the end of the selling season. The retailer's objective is to choose an optimal order quantity to maximize his expected profit.

In the wholesale price contract, the retailer's expected profit Π_R^w can be expressed as follows:

$$\begin{aligned}\Pi_R^w &= pS(Q,e,p)+vI(Q,e,p)-wQ-g(e)\\ &=(p-v)S(Q,e,p)-(w-v)Q-g(e)\end{aligned} \tag{37}$$

Since this retailer's profit function (37) is the same as the total supply chain's profit function (3) except that c is replaced by w, the optimal retailer's order quantity Q^Δ, effort level e^Δ and retail price p^Δ are the same as total supply chain's except that c is replaced by w. So we have

$$e^\Delta = \frac{b(p-w)}{\mu} \tag{38}$$

$$\begin{aligned}&a-2kp^\Delta+kw+\frac{b^2(p^\Delta-w)}{\mu}+\\ &A+\frac{(B-A)(p^\Delta-w)(p^\Delta+w-2v)}{2(p^\Delta-v)^2}=0\end{aligned} \tag{39}$$

$$\begin{aligned}Q^\Delta &= \Phi^{-1}\left(\frac{p^\Delta-w}{p^\Delta-v}\right)+y(e^\Delta,p^\Delta)\\ &=A+\frac{(B-A)(p^\Delta-w)}{p^\Delta-v}\\ &\quad +a+\frac{b^2(p^\Delta-w)}{\mu}-kp^\Delta\end{aligned} \tag{40}$$

Compare (38) with (31), because $w>c$, we can know that

Property 2. The independent retailer's optimal effort level without coordination is lower than the supply chain's optimal effort level, i.e., $e^\Delta < e^*$.

In the wholesale price contract, the retailer's expected profit is

$$\begin{aligned}\Pi_R^w(p^\Delta) &= (p^\Delta-w)(a-kp^\Delta+A)\\ &\quad+\frac{(p^\Delta-w)^2}{2}\left(\frac{B-A}{p^\Delta-v}+\frac{b^2}{\mu}\right)\end{aligned} \tag{41}$$

The supplier's expected profit is

$$\Pi_S^w(p^\Delta)=(w-c)Q^\Delta \tag{42}$$

The supply chain's expected profit is

$$\begin{aligned}\Pi_T^w(p^\Delta) &= \Pi_R^w(p^\Delta)+\Pi_S^w(p^\Delta)\\ &=(p^\Delta-c)(a-kp^\Delta+A)\\ &\quad+\frac{(p^\Delta-c)^2-(w-c)^2}{2}\left(\frac{B-A}{p^\Delta-v}+\frac{b^2}{\mu}\right)\end{aligned} \tag{43}$$

Property 3. Considering the stochastic demand is influenced by both sales effort and retail price, if the coordinated supply chain exists, then its optimal total supply chain profit is always greater than the total supply chain profit under the non-coordinated supply chain, i.e., $\Pi_T^c(p^*) > \Pi_T^w(p^\Delta)$.

Proof. From (43), we know that

$$\begin{aligned}\Pi_T^w(p^\Delta) &< (p^\Delta-c)(a-kp^\Delta+A)\\ &\quad+\frac{(p^\Delta-c)^2}{2}\left(\frac{B-A}{p^\Delta-v}+\frac{b^2}{\mu}\right)\\ &=\Pi_T^c(p^\Delta)\end{aligned} \tag{44}$$

Since the p^* is the optimal solution to maximize the $\Pi_T^c(p)$, we can get $\Pi_T^c(p^\Delta)\le \Pi_T^c(p^*)$ for any $p^\Delta\in[c,\overline{p}]$. Therefore, from (44), we have $\Pi_T^w(p^\Delta)<\Pi_T^c(p^*)$. □

From (36) and (42), we are able to show:

Property 4. When $T>Q^\Delta$, the supplier's profit with coordination is greater than the supplier's profit without coordination, i.e., $\Pi_S^c(p^*)>\Pi_S^w(p^\Delta)$.

From Property 3 and Property 4, we can see that:

Property 5. By choosing an appropriate sales target T, we can let $\Pi_S^c(p^*)>\Pi_S^w(p^\Delta)$ and $\Pi_S^c(p^*)>\Pi_S^w(p^\Delta)$.

Property 5 implies that the supplier can choose an attractive sales target T to induce the retailer to accept the coordination contract while the retailer's expected profit with coordination is higher than without coordination, in this way, the coordinated supply chain will be Pareto improving, i.e., both parties of the supply chain are no worse off (and at least one of them is strictly better off) with the coordination contract in place than with the non-coordination contract.

7. Simulation studies

Our objective in this section is to gain further insights based on a simulation analysis of the supply chains with and without coordination. We assume the parameters take the following base values: $c = 5.0$, $v = 1.0$, $w = 15$, $a = 400$, $b = 10$, $k = 10$, $\mu = 100$, $A = -80$, and $B = 80$.

7.1. Variation in T

First, we keep other parameters constant and change the sales target level T.

Considering the coordination contract of the returns policy with SRP, let Π_R^c be the retailer's expected profit, Π_S^c the supplier's expected profit, and Π_T^c the total supply chain's expected profit.

Correspondingly, in the wholesale price contract, we let Π_R^w be the retailer's expected profit, Π_S^w the supplier's expected profit, and Π_T^w the total supply chain's expected profit.

We plot these profits at different sales target levels in Figure 2.

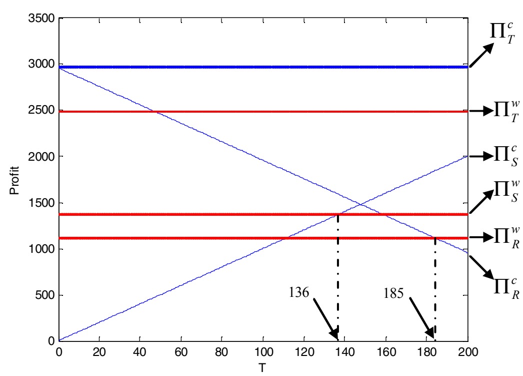

Figure 2. Effect of the sales target level on profits

As we proposed in subsection 6.3, Figure 2 shows that Π_T^c is always higher than Π_T^w. And when $136 \le T \le 185$, we can find that $\Pi_S^c \ge \Pi_S^w$ and $\Pi_R^c \ge \Pi_R^w$. So by adjusting the value of T, the coordinated supply chain can reach Pareto improving. In this example, we assume the $T = 160$ as the base value for following analysis.

7.2. Variation in w, c, and v

In this subsection we compare profits without coordination with profits with coordination when we change w, c, and v. The " % profit increase" in Figures 3-5 shows the percentage increase in profits with coordination as compared to profit without coordination. This amount is calculated as:

$$\Pi_i = \left[(\Pi_i^c - \Pi_i^w)/\Pi_i^w\right] \cdot 100\% \quad , \quad i = S, R \text{ or } T$$

where Π_S, Π_R and Π_T represent the" % profit increase" of the supplier, the retailer and the total supply chain, respectively.

In Figures 3-5 we consider the effects of w, c, and v on the increase in profits with coordination.

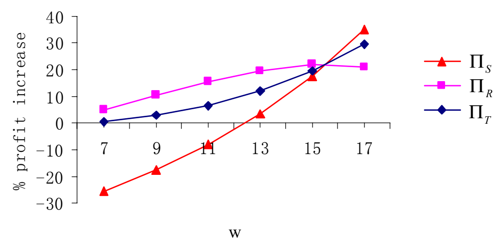

Figure 3. Effect of the wholesale price on the increase in profits with coordination

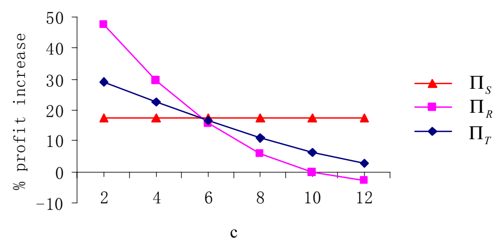

Figure 4. Effect of the supplier's manufacturing cost on the increase in profits with coordination

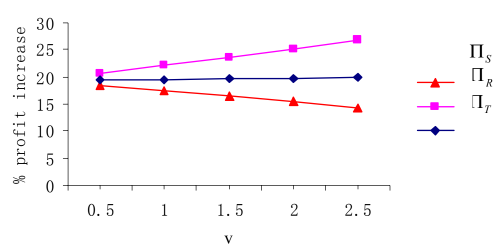

Figure 5. Effect of the salvage value on the increase in profits with coordination

In Figure 3 we change the wholesale price w from 7 to 17 with an increment of 2 while keeping other parameters constant. We can find that the percentage increase in each profit with coordination increases as w. In other words, the supplier, the retailer and the total supply chain all can gain more profit from coordination when the wholesale price are higher. But there exists a critical value of w. When the wholesale price is below this critical value, the supplier's profit with coordination is lower than the supplier's profit without coordination. In this case of this example, this is occurs at $w = 12.43$. For $w < 12.43$, the supplier has no motivation to exert supply chain coordination.

In Figure 4 we increase the supplier's manufacturing cost c from 2 to 12 with an increment of 2 while keeping other parameters constant. We can see that the percentage increase in each profit with coordination decreases as c increases. And there also exists a critical value of $c = 9.91$. If $c > 9.91$, the retailer profit will reduce because of coordination. Under such circumstances, the retailer has no interest to coordinate the supply chain.

In Figure 5 we increase the salvage value v from 0.5 to 2.5 with an increment of 0.5 while keeping other parameters constant. We can find that Π_R increases as v increases, Π_S decreases as v increases, and Π_T increases slightly as v increases. That is to say, when the salvage value is higher, the retailer has a stronger desire to coordinate than the supplier, but it has little impact on the total supply chain's decision about coordination.

From Figures 3-5, we also can find that when we change the value of w, c, or v within a rational range, the percentage increase in total supply chain's profit with coordination is always positive. So supply chain coordination is always benefit to the total supply chain's profit. This further proves the conclusion of Property 3.

8. Conclusion

In this paper, we have studied the returns policy with SRP to coordinate the newsvendor under effort and price dependent demand. Coordination with the effort and price dependent demand model is complex when the firms are not allowed to contract on the retailer's effort level and retail price directly, i.e., any contract that specifies an effort level and retail price for the retailer is either unverifiable or unenforceable. When demand is not influenced by sales effort and retail price, a properly designed returns policy or SRP contract achieves supply chain coordination and a win-win outcome. When demand is influenced by retailer sales effort and retail price, both returns policy and SRP contract fail to coordinate the retailer's action because they all distort the retailer's marginal incentive to exert effort and change price.

Although each of the returns policy and the SRP contract does not coordinate on its own, we demonstrate it can coordinate the supply chain if they are combined each other: the returns policy reduces the retailer's incentive to exert effort and lower the retail price, which counteracts the retailer's excessive incentive to exert effort and lower the retail price with a SRP contract alone. Therefore, coordination can be restored by combining the returns policy with SRP.

Then we employ the Newsvendor Problem framework to build the model of returns policy with SRP. Our objective is to use the model to understand how the effort and price influence the coordination behavior of such supply chains. In this setting, we show that the supplier can offer a set of appropriate contract parameters to the retailer such that the retailer's objective becomes aligned with the supply chain's objective.

By comparing the coordinated supply chain with the non-coordinated supply chain, we find that the total supply chain's profit with coordination is always higher than the total supply chain's profit without coordination. We also can see that if the supplier uses the returns policy with SRP to coordinate the supply chain his profit is fixed, while the retailer faces all the market demand risk. Therefore, if the supplier wants to let the retailer accept this coordination contract, he should give a favorable sales target to the retailer so that the retailer can obtain more profit than without coordination. Through the simulation studies, we find that the Pareto improving can be achieved if the supplier sets a proper sales target for the retailer.

Also through the simulation analysis, we find that the percentage increase in total supply chain's profit with coordination increases with wholesale price, increases slightly with the salvage value, and decreases with the supplier's manufacturing cost. These results are very useful for the supplier to decide how to coordinate the supply chain.

In this paper, we assume the supply chain has only one retailer. The next step in future research is to extend this model by allowing one supplier sells to multiple competing retailers. We expect the retail prices will be lower and the retailer effort level will be higher due to the competition, but the model also becomes more complex. A rigorous study is necessary to confirm this and we leave that for future work.

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