Bidder Valuation of Bundles in Combinatorial Auctions

Soumyakanti Chakraborty
Indian Institute of Management Calcutta, Joka, Diamond Harbour Road, Kolkata 700104
fp072004@iimcal.ac.in

Anup Kumar Sen
Indian Institute of Management Calcutta, Joka, Diamond Harbour Road, Kolkata 700104
sen@iimcal.ac.in

Amitava Bagchi
Indian Institute of Science, Education and Research, Kolkata, IIT Kharagpur Extension Centre, HC Block, Sector III Saltlake, Kolkata 700106, bagchi@iimcal.ac.in

Abstract

Current combinatorial auctions require bidders to specify the valuations of bundles at the start of the auction. We propose an alternative mechanism called RevalSlot, based on the common value model, which allows a bidder to participate in the auction even if she can only identify a range within which her valuations lie. This will increase bidder participation, and at the same time maximize revenue. As the auction progresses, bidders get information which helps them to converge to a value for each bundle. RevalSlot is a combination of two processes. One helps bidders to zero in on a value for each bundle, and the other is an ascending proxy auction. We present theoretical and experimental results which confirm the efficacy of our mechanism.

1. Introduction

Basic auction mechanisms can cause inefficiencies in situations where bidders demand combinations (bundles) of items. Such situations arise when items are complementary, i.e., when the value of a combination of items is substantially higher than the sum of the values of the individual items. Items 1 and 2 are *complementary* for a bidder (agent) j if $u_j\{1\} + u_j\{2\} < u_j\{1,2\}$ where $u_j\{S\}$ denotes the utility of bidder j as a result of acquiring the set S of items. This bundling problem is important in various business domains. One example is auctioning of take-off and landing slots at airports [15]. An airline's demand for a takeoff slot at the source airport and a landing slot at the destination airport are interdependent. When a flight links more than two cities, the demand for take off and landing slots are sequentially related. In these cases, it is preferable to bundle the takeoff and landing slots together and auction the bundles instead of auctioning the slots individually. Similar problems exist in the transportation and logistics domains ([3], [11]). The domain of supply chain also presents this problem [18]. Each bidder places bids on combinations of different resources in the supply chain. If the bidder does not get all the components from the requested subset, then the transaction has no value for her. Other applications include travel package planning, where the problem is to allocate airline or railway tickets, and hotel accommodation to travelers who have their own preferences with regard to location, cost and hotels. A railway ticket without a hotel reservation is of less value to a buyer than both items together [6]. The FCC spectrum auctions ([5], [10]), in which the government sells rights to use specific segments of spectrum to regional and national buyers, has generated much interest over the last decade.

One solution to the bundling problem is to sell items sequentially or simultaneously, but this can give rise to the exposure problem [8], in which bidders end up with incomplete bundles, i.e., instead of the full set of items they want to procure, they win only some of the items. To guard against this possibility, bidders bid cautiously, which results in inefficiencies. Combinatorial auctions that allow bids on bundles of items can provide a solution to both the bundling and the exposure problems.

Existing combinatorial auction mechanisms assume that the bidder valuation of bundles remains the same throughout the auction. This is not true in the common value model, in which the actual value is the same for everyone, but bidders have different private information about that value. A bidder changes her estimate of the value during an auction in response to signals from other bidders. In contrast, in the private value model, her value remains unaffected by other valuations ([8], [9]). Because the need for change of valuation is not addressed, bidders with inexact or approximate valuations of bundles are discouraged from participating in the existing combinatorial auctions.

In single-unit combinatorial auctions [12], there is only one unit of each item. Let $M = \{1,2,3,\ldots\ldots.,m\}$ be the set of items and $N = \{1, \ldots, n\}$ the set of bidders. Bidders submit bids on each of the $(2^m - 1)$ possible non-empty bundles of items. Let $v_i(S)$ denote the maximum price bidder i is willing to pay for bundle $S \subseteq M$. An *allocation* of items to bidders is described by variables $x_i(S) \in \{0,1\}$, where $x_i(S) = 1$ if and only if bidder i gets bundle S. An allocation $\{x_i(S) \mid i \in N, S \subseteq M\}$ is *feasible* if no item is allocated to more than one bidder. In the Winner Determination Problem (WDP), the task is to allocate the items so that the seller's revenue is maximized:

$$Max \sum_{i \in N, S \subseteq M} v_i(S)\, x_i(S)$$

$$such\ that \sum_{i \in N} \sum_{S \subseteq M,\, j \in S} x_i(S) \le 1 \ \ for\ all\ j \in M,$$

$$x_i(S) \in \{0,1\}\ and \sum_{S \subseteq M} x_i(S) \le 1\ for\ all\ i \in N$$

The XOR formulation [12] of the WDP is shown above. It adds the constraint that a bidder can win at most one bundle. We use this formulation in the paper.

Here we propose an iterative combinatorial auction mechanism called *RevalSlot* which tries to address the problem of exact valuation of packages. The auction starts with the auctioneer providing *slots* (i.e., ranges of values) to the bidders. A bidder submits slot numbers for the bundles of interest. An ascending proxy auction is applied with bid increment 1. The provisional winning allocation, the winning slots, and the final ask prices of bundles obtained through the ascending proxy auction are announced, but no allocations are actually made. In the next round the slot size is reduced, bidders submit revised valuations, and the auction is executed again. This procedure helps bidders to converge to a price for each bundle as the auction progresses. *RevalSlot* is thus a blend of two processes. The first process accepts values from bidders for a specified slot size and feeds the values to the second process, which runs an ascending proxy auction. The alternate execution of the two processes helps bidders to zero in on a value for each bundle, at the same time making it possible for the auctioneer to maximize revenue. To summarize, *RevalSlot* has the following three objectives:

- To increase bidder participation by allowing bidders to take part in the auction even when they do not possess exact valuations of packages at start;
- To enable the seller to attain a high level of revenue by the use of the ascending proxy auction;
- To make it possible for bidders to achieve price discovery by providing them information from time to time on the valuations of other bidders, which enables them to adjust and align their valuations to prevailing market rates.

2. Combinatorial Auctions and the Problem of Valuation

Some well known combinatorial auction mechanisms are described below. None of these allow bidders to change the valuations of bundles during the auction.

VCG Auction: This type of auction integrates Vickrey's seminal ideas [16] with the Clarke-Groves design ([4],[7]) in which bidders report their valuations and the auctioneer solves the WDP and allocates the bundles. The winners however, do not pay what they have bid, rather they pay the marginal negative effect that their participation has on the reported values of the other bidders [17]. Though this mechanism ensures truthful bidding, the seller's revenue can be very low and can decrease with increase in participation, and shill bidding and collusion can be profitable strategies [2].

First Price Sealed Bid Auction: Here the payment rule is "pay your bid", which forces the bidders to always make guesses about the bids of others and generally leads to inefficient outcomes [2]. This is a single round auction like the previous one, and bidders must be ready at start with exact valuations of bundles.

Ascending Proxy Auction: In this case, bidders submit their valuations at start to a proxy agent, which bids on their behalf by determining the packages of highest utility [2]. In each round the ask prices of the packages are increased and the proxy agent submits fresh bids on behalf of each bidder until a termination condition is reached. iBundle [13] is an example of such an auction. It can be adapted to a live auction scenario by assuming myopic bidding behavior of human agents.

Clock-Proxy Auction: The initial Clock Phase [1] helps bidders to discover the prevailing market prices of individual items. At its end, bidders know the price of each item and can form lower bound estimates of the prices of bundles. But the scheme is of limited help for highly complementary goods [5]. The Proxy Phase is run only once, so bidders are forced to resolve the problem of accurately determining the valuations of bundles at the close of the Clock Phase.

Simultaneous Multi-Round Auction with Package Bidding (SMRPB): In this auction [5], at the end of every round a minimum price for each item is computed and bidders are informed. Bidders then bid for bundles in the next round. At the end of the auction, payments equal to the winning bids are made by the winners. As in the other cases, this mechanism assumes that valuations of bundles remain unchanged through the auction.

Thus we find that existing methods for combinatorial auctions do not allow bidders to modify the valuations of bundles. All the schemes described above assume that bidders are absolutely sure about the exact valuations of bundles at start, i.e., that bidders are perfectly rational. But in practice, bidders are bounded rational, with limited computing power, and it is difficult for them to value bundles exactly.

It would seem that a bidder would find it easier to specify lower and upper bounds on the valuation of a bundle rather than an exact value. She does not expect the bundle to be sold at a price below the lower bound or above the upper bound. If bidding information is provided to her in the course of the auction, she can narrow down the difference between the two bounds. Our new mechanism, described below, is based on the above observation.

3. Details of the New Mechanism

We now describe *Revalslot* in more detail. At the beginning of each round the auctioneer announces a slot size. The bidders submit their valuations as slot numbers. These are validated and an ascending proxy auction is run on them. The outcome is then communicated to the bidders, together with the highest ask price (in terms of slot number) of each bundle. The auctioneer then reduces the slot size, and bidders submit slot numbers for the next round. This continues until one of the termination conditions is satisfied. The auctioneer then announces the winning allocation and the prices that winners must pay for their winning bundles. The detailed algorithm is given in the Appendix. The main components of the algorithm are *setting the initial parameters, bidding, validation, ascending proxy auction, disclosure of information to bidders, termination,* and *payment rules.*

3.1. Setting the Initial Parameters

Before the auction starts the auctioneer has to decide on the initial slot size, the final slot size, and the reduction factor (the factor, say half, by which the slot size is reduced in each round). All of these would depend on the type of items to be auctioned. If there is much uncertainty about the price, the initial slot size can be large, and if the uncertainty is less the initial slot size can be small. The time taken to reach the final allocation depends on the final slot size. The auctioneer can use this and the reduction factor as parameters to control the time taken by the auction to complete. To guard against unusual bidding behavior at the end, the final slot size need not be made known to the bidders in advance.

3.2. Bidding

After the initial slot size has been decided, the auctioneer asks the bidders to submit their valuations for the bundles in terms of slot numbers. For example, if the slot size is 100 and a particular bidder's estimate of a bundle's value is between 200 and 300, then she would submit a valuation of 3 for the bundle. The slots in this case are 0-99, 100-199, 200-299 and so on. In the next round, the slot size is reduced and the bidders are asked to submit their valuations in terms of the new slot size. Continuing with the above example, if the slot size is reduced to 50 in the next round, the bidder would bid either 5 or 6. In this case 5 means 200-249, and 6 means 250-299. This holds if we assume that the bidder does not increase her upper bound. If she decides to bid in the next slot, then she has to decide among 5, 6, 7 and 8.

3.3. Validation

Two validations are performed in each round.

a) First, any bidder who has bid 0 for a bundle cannot bid a positive value for the same bundle in later rounds. A bid of 0 indicates that the bidder is not interested in the bundle.
b) Second, a bidder cannot in general reduce the value of a bundle to a level that is lower than its value in the previous round. There are two exceptions to this rule: i) In the second round, a bidder is allowed to reduce her valuation to the winning slot number of the first round. ii) A bidder can also decide, in any round, to cease bidding for a particular bundle; she indicates her intention to do so by bidding 0 for it.

These rules prevent bidders from manipulating the auction. In their absence, a bidder can alternate between very low and very high values, and thereby prevent other bidders from correctly estimating the price of a bundle, thus defeating the very purpose of the mechanism. Rule b(i) permits a bidder who has overestimated the value of a bundle to correct her estimate in the next round. However, reduction of valuation is allowed only once, and only in the second round. Rule b(ii) gives a bidder the freedom to withdraw her bid on a bundle in any round during the auction.

3.4. Ascending Proxy Auction

After the submitted values are validated, an ascending proxy auction [13] is run on them with a bid increment of 1. The ask price of each bundle is 0 at start and increases by 1 after each iteration. The ask price of a bundle is updated after each round as explained below.

In every round of an ascending proxy auction, we need to determine for each bidder the packages for which her utility is a maximum. The utility, u, of a package is given by $u = v - p$, where v is the bidder's valuation of the package and p is its ask price. The proxy agent bids on the maximum utility packages on behalf of each bidder. Bidders in the winning allocation repeat their bids even if the ask prices of the bundles have increased. The (provisional) allocation that maximizes revenue is then determined. The bidders in the winning allocation are designated *happy* bidders; others are *unhappy* bidders.

The new ask prices are computed from the valuations received from the unhappy bidders by adding the bid increment to the highest bid on that bundle. The ascending proxy auction terminates when any one of the two conditions given below is satisfied:

a) All the bidders are happy. Since every bidder has received a bundle that maximizes her utility, there is no incentive for her to increase her bid.
b) All bidders submit the same bids in two consecutive rounds.

Ties are broken by assigning bundles to more bidders, and if more than one such alternative is available, one of them is chosen at random.

3.5. Disclosure of Information to Bidders

After each round of the auction, the winning allocation, the winning slot number and the final ask prices of all the bundles are communicated to the bidders, but without identifying any bidders individually. The winning slot number specifies the range within which the ask price of a winning bundle lies. The ask price of a bundle at the end of a round provides an indication of the valuations of other bidders. Consider the following example. An auction has 3 items and 10 bidders, It provisionally allocates ABC to bidder 2 and the winning slot number is 4 (with slot size, 100). This information is of interest to bidders who want ABC, but it is of not of much interest to others. A bidder who wants AC needs feedback information on how other bidders value AC.

3.6. Termination Conditions

The auction terminates when either of the following two conditions is satisfied:

a) The slot size has reached its minimum value.
b) The winning allocation of the current round is the same as that of the immediately previous round, and the highest losing bid for any winning bundle in the current round does not lie in the winning slot.

3.7. Payment Rules

Since prices are specified in slot numbers, the final price that the winners pay must be computed from the slot numbers at the end of the auction. The winners can be assumed to be indifferent to the price range within a slot, so we can take the mean of the final slot size when calculating the final payment. If the final slot size is Z_f and the winning slot number is k, the payment is $(k-1) * Z_f + Z_f/2$. For example, if Z_f is 10 and k is 21, the final payment is 20 * 10 + 10/2 = 205. The seller's revenue is the sum of the prices of the winning bundles.

4. Theoretical Results

Result 1: Bidder participation in *RevalSlot* will be higher than in other combinatorial auction mechanisms.

Proof: Consider a bidder i who wants to participate in an auction but has been unable to arrive at a precise valuation for a bundle. She has only been able to identify an upper bound U and a lower bound L for her valuation. Then the valuation uncertainty of the bidder is $\Delta_i = U - L$. Let us denote the valuation uncertainty that an auction can tolerate as $\Delta_{auction}$. Current combinatorial auction mechanisms does not provide for uncertainty in valuation, i.e., $\Delta_{auction} = 0$. Since bidder i is likely to participate in an auction only if $\Delta_{auction} \geq \Delta_i$, only bidders with $\Delta_i = 0$ can participate in such auctions. *RevalSlot* allows $\Delta_{auction}$ to be significantly greater than 0 at start, and as a result, more bidders can take part.

Result 2: *RevalSlot* maximizes the seller's revenue.

Proof: *RevalSlot* employs an ascending proxy auction such as iBundle to determine the provisional winners at the end of each round. iBundle terminates with an allocation that is within $3\min\{|M|,|N|\}\varepsilon$ of the optimal solution [14], where M is the set of items, N is the set of bidders, and ε is the minimum bid increment. As ε gets closer to 0, the seller's revenue approaches its maximum value.

Remark 1: Some uncertainty in the bidders' valuations still persists at the termination of the auction, and this affects the computation of the seller's revenue. However, if the final slot size is small, the revenue generated is close to maximum.

Result 3: (**Dominant Strategy of Bidders):** Assuming bidders are utility maximizing agents, a happy bidder at the end of a round will place her bid in the lower half of the slot in the next round, while an unhappy bidder will place her bid in the upper half.

Proof: Let us suppose that the provisional winning slot number for a particular bundle is 4 (slot size = 100). At the end of the round, bidders get to know that the price of that bundle lies between 300 and 400. We now reduce the slot size to 50. Bidders have two options, either to bid in the range 300 - 350 or to bid in the range 350 – 400. The happy bidder, who is the provisional winner of the bundle, knows that she cannot lose the auction in the next round. Either she will win or the auction will continue (see Section 3.6). Therefore, she will place her valuation in the range 300 – 350, as this will maximize her utility. An unhappy bidder also sees this, but she cannot take it for granted that the happy bidder/s valuation will lie in the range 300 – 350. She knows that if the happy bidder's valuation lies in the range 350 – 400, and she herself bids in the range 300-350, then the auction might terminate, with the happy bidder becoming the final winner. The risk involved in choosing in the higher slot is small, as we are running an ascending proxy auction, and placing the valuation in the range 350 - 400 does not necessarily mean paying that price. Thus the unhappy bidder will opt for the higher slot, and the happy bidder for the lower slot. Valuations at a level higher than the provisional winning slot can be costly, as no reduction in bids is allowed after the second round.

Remark 2: The only way *RevalSlot* can terminate at a suboptimal price is when all the bidders collude to bid low in the first round.

Result 4: *RevalSlot* helps in the discovery of the market price of a bundle by reducing the valuation uncertainty of the bidders.

Proof: The price discovery factor r of a winning bundle can be expressed as $r = \frac{L}{U}$, where L is the lower bound of the current slot and U is the upper bound. The mechanism ensures that as the auction progresses, r does not decrease. Ideally, r should tend to 1. The price discovery of the entire auction can be expressed as

$$R = \sum_{j \in W} w_j r_j$$

where the weight $w_j = \frac{v_j}{\sum_{j \in W} v_j}$, v_j is the valuation (slot number) of the provisionally winning bundle, W is the set of all provisionally winning bundles and r_j is the price discovery factor of the winning bundle j.

5. Worked Out Example

In this section, we first illustrate how the mechanism works and then provide a detailed example to describe how it would function in practice. We make an attempt to explain the probable bidding behavior of a bidder.

Illustration: Assume there are 3 items and 3 bidders. A hypothetical valuation chart is shown in Table I. The values in the table are merely illustrative and have no practical significance, the only purpose of the chart being to explain the working of the mechanism. We assume here that valuations do not change in the course of the auction. The optimal allocation is [2, ABC] and the optimal revenue is 247.

Table I: Valuation Chart

Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	49	22	11	99	88	118	212
2	58	29	78	101	118	159	247
3	25	35	45	67	95	99	185

Let us suppose the starting slot size Z_i is 100 and the final slot size Z_f is 10. We have taken the slot sizes to be 100, 50, 20 and 10. The auctioneer asks the bidders to value the packages in slots of 100. The valuation chart that she gets is shown in Table II.

Table II: Valuation chart for slot size 100

Round 1							
Slot #1: [0 – 99]; #2: [100 – 199]; #3: [200 – 299]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	1	1	1	1	1	2	3
2	1	1	1	2	2	2	3
3	1	1	1	1	1	1	2

The next step is validation of the reported values. There is nothing to validate in the first round. Also, since the values are taken from Table I, there cannot be any violations even in the subsequent rounds. The auctioneer runs an ascending proxy auction and comes up with the allocation [1, ABC] and the winning slot = 3. So all the bidders now know that ABC is worth between 200 and 299. The auctioneer then changes the slot size to 50. The new valuation chart is shown in Table III.

Table III: Valuation chart for slot size 50

Round 2							
Slot #1: [0 – 49]; #2: [50 – 99]; …; #5: [200 – 249]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	1	1	1	2	2	3	5
2	2	1	2	3	3	4	5
3	1	1	1	2	2	2	4

This time the allocation is [1, ABC], and the winning slot is 5. The allocation is the same as that of the previous round, but the highest losing bid is within the slot, so the auction does not terminate. Now the bidders have the information that the value of ABC is between 200 and 249. The auctioneer changes the slot size to 20. The valuation chart of the bidders is shown in Table IV. The new allocation is [2, ABC] and the winning slot is 12. The bidders get the information that the price of ABC lies between 220 and 239.

Table IV: Valuation chart for slot size 20

Round 3							
Slot #1: [0 – 19]; #2: [20 – 39]; ... ; #13: [240 – 259]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	3	2	1	5	5	6	11
2	3	2	4	6	6	8	13
3	2	2	3	4	5	5	10

The slot size is changed to 10. The new valuation chart is shown in Table V.

Table V: Valuation chart for slot size 10

Round 4							
Slot #1: [0 – 9]; #2: [10 – 19]; …; #25: [240 – 249]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	5	3	2	10	9	12	22
2	6	3	8	11	12	16	25
3	3	4	5	7	10	10	19

This time the allocation is [2, ABC] and the winning slot is 22. Also, we find that the winning allocation is the same for two consecutive rounds and the highest losing bid is not in the winning slot. So, the auction terminates and ABC is allocated to Bidder 2. She pays a price of 21 * 10 + 10/2 = 215. Even if the highest losing bid had been in the winning slot, the auction would have terminated anyway since the final slot size is 10.

We now provide an example to help explain the manner in which bidders change their valuations of bundles.

Example: To understand how bundle valuations of bidders might change in practice, we consider a bidder i who has submitted a non-zero valuation on a bundle at the start of a round. There are four possibilities:

Case I: The bundle valuation (i.e., slot number) is similar in value to the valuations of other bidders. Then bidder i has no reason to stray outside the range of her initial slot.

Case II: Bidder i's slot number is lower in value than the valuations of other bidders. In this case, she is likely to increase her valuation.

Case III: Bidder i's slot number is lower in value than the valuations of other bidders, but she cannot increase her valuation because of budget constraints. In this case she withdraws her bid from the auction.

Case IV: Bidder i's slot number is higher in value than the valuations of other bidders. She then lowers her slot number.

We explain how the mechanism takes all the above four possibilities into account. In a live auction, there is no valuation chart (as in Table I) to guide the bidders. We take the bidders to be utility maximizing, i.e., we assume they will submit valuations as low as possible, at the same time ensuring they do not lose the auction.

Suppose the auction starts with a slot size of 100. Let the initial valuation be as shown in Table VI.

Table VI: Valuation chart for slot size 100

Round 1							
Slot #1: [0 – 99]; #2: [100 – 199]; …; #5: [400 – 499]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	1	1	1	1	1	2	5
2	1	1	1	2	2	2	3
3	1	1	1	1	1	1	2

The ascending proxy auction now gives the allocation [1,ABC] and the winning slot number is 3. The auctioneer also announces the final ask prices of all the bundles: [A – 2], [B – 2], [C – 2], [AB – 3], [BC – 3], [AC – 3], [ABC – 4].

The ask price of ABC is 4 which is greater than the winning slot number because of the way the ascending proxy auction operates. At the start of each round, the proxy agent increases the ask prices of the bundles by the bid increment, which is 1 in this case. From Table VI, it is clear that the proxy agent bids bundle ABC for both bidders 1 and 2 until the ask price of ABC becomes 3. When the ask price reaches 3, the proxy agent bids ABC on behalf of both Bidder 1 and Bidder 2, the tie is broken in favor of Bidder 1 and the ask price of ABC goes up to 4 in the next round. But the proxy agent will not bid for ABC on behalf of Bidder 2, as she has submitted 3 as her valuation. Thus, the final ask prices in the ascending proxy auction do not imply that bidders have agreed to go up to that price. The information that a bidder can deduce from the final ask prices is that there are bidders participating in the auction who have gone up to slot number 1 for A, 1 for B, 1 for C, 2 for AB, 2 for BC, 2 for AC and 3 for ABC. So, from this information that the auctioneer reveals to the bidders, we can figure out how the bidders will change their valuations in the next round, assuming each bidder tries to maximize her utility.

From this information, Bidder 1 finds that she had overestimated the price of package ABC. Her valuation was 5, while the winning bid was only 3 (case IV). So she reduces her valuation in the next round to the range, 200 - 299. She also realizes that the prices of bundles AB and BC are more than what she estimated. Her estimation was 0 – 99 for both the bundles, whereas the other bidders are ready to pay in the range 100 – 199. Bidder 1 decides to go up one slot for AB (case II) but decides to discontinue bidding for BC, i.e. bids 0 for BC (case III). In the case of AB, in the next round with slot size 50, she has two choices, either 3 (100 – 149) or 4 (150 – 199). Since, she has increased her range in this round she will opt for 3 and then wait to see how the auction progresses. As far as the other bundles are concerned, Bidder 1's valuation is within the estimation of the other bidders. So, she will continue bidding for these bundles, and bid within her initial range (case I). Now, she has to decide what valuations she will submit for bundles A, B, C, AC and ABC in the next round, with the slot size of 50. Since, she will be bidding in the range of 200 – 299 for ABC, there are two options, either 5 (200 – 249) or 6 (250 – 299). She is the provisional winner in round 1, and the auction cannot end in the next round, she will maximize her utility and submit a value of 5. Applying the same logic, she will report 1 for A, B and C, and 3 for AC.

As far as Bidder 2 is concerned, she can deduce that Bidder 1 will bid at least 5 in the next round. So, if she has to increase her chances of staying in the auction, she must submit a valuation of 6. Since, the winner and the winning slot number is being decided on the basis of an ascending proxy auction, the risk involved is small, Following the same logic, she will submit 2 for A, B and C, 4 for AB, BC and AC, and 6 for ABC.

If we consider Bidder 3, we can see that her estimation is below the ask price for bundles AB, BC, AC and ABC. Thus, she will bid only for A, B, and C, and following the same logic as for Bidder 2, we can deduce that she will bid 2 for all three. Let us assume that Bidder 2 commits an error while reporting her value for AB and reports 2 instead of 4. Then the valuation chart with slot size 50 will look as shown in Table VII. A value of 0 indicates that the bidder is not interested in that bundle.

Table VII: Incorrect valuation chart for slot size 50

Slot #1: [0 – 49]; #2: [50 – 99]; …; #6: [250 – 299]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	1	1	1	3	0	3	5
2	2	2	2	2	4	4	6
3	2	2	2	0	0	0	0

The mechanism will then validate the reported values. Since Bidder 2 had submitted 2 with slot size 100 (100 – 199), she can submit either 3 or 4 now. The reported value is 2, which violates the second rule (see section 3.3). So Bidder 2 will be asked to resubmit her valuations. Table VIII gives the corrected values.

Table VIII: Correct valuation chart for slot size 50

Round 2							
Slot #1: [0 – 49]; #2: [50 – 99]; …; #6: [250 – 299]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	1	1	1	3	0	3	5
2	2	2	2	4	4	4	6
3	2	2	2	0	0	0	0

Running the ascending proxy auction on the above values will result in the following allocation and winning slot number: [2, ABC] and 6. The ask prices of the bundles are: [A – 3], [B – 3], [C – 3], [AB – 4], [BC – 4], [AC – 4], [ABC – 6]. Based on this information, again the bidders will decide their strategies in the next round with slot size 20, following the logic given above. For example, since Bidder 2 is the new provisional winner, while submitting the value for ABC she will now select 14 (260 – 280) from the two options available to her, which are 14 and 15. However, Bidder 1 will opt for 15 if she wants to win the bundle. Bidder 1 will realize that if she wants to win AB, she has to bid at least 9 for AB, i.e. in the range of 160 – 180. Let us assume that she now decides that she will discontinue bidding for AB (case III). So, in the next round, the valuation chart with slot size is as in Table IX. Again, the mechanism will run the validation process followed by the ascending proxy auction. The result is allocation [1,ABC], winning slot number = 14. The ask prices are: [A – 6], [B – 6], [C – 6], [AB – 10], [BC – 10], [AC – 10], [ABC – 15].

Table IX: Valuation chart for slot size 20

Round 3							
Slot #1: [0 – 19]; #2: [20 – 39]; ...; #15: [280 – 299]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	5	5	5	0	0	10	15
2	4	4	4	9	9	9	14
3	5	5	5	0	0	0	0

As, in the previous rounds, the bidders will decide on their bids in the next and final round (slot size = 10) according to the bidding strategies discussed above. So, the valuation chart for the final round will be as given in Table X.

When we run the ascending proxy auction on this set of values the Allocation is [2, ABC] and the winning slot number is 30.

Table – X: Valuation chart for slot size 10

Round 4							
Slot #1: [0 – 9]; #2: [10 – 19]; …; #30: [290 – 299]							
Bundles/ Bidders	A	B	C	AB	BC	AC	ABC
1	9	9	9	0	0	19	29
2	10	10	10	20	20	20	30
3	10	10	10	0	0	0	0

Thus, the auction terminates with Bidder 2 receiving the bundle ABC and paying a price of 29 * 10 + 10/2 = 295. Although we have executed the same mechanism on practically similar values in both the illustration and the example, we arrive at different revenues in the two cases. In the illustration, the revenue is 215, whereas in the example, we get 295. This is because in the illustration, we assumed that the bidders are guided by the original valuation chart and do not deviate from it. In the example, the bidders modify their valuations after looking at the bidding behavior of the other bidders and deducing their best strategies from the information provided. This results in the bidders drifting towards the upper limit of the range, thus increasing the revenue. As discussed above, the example is a much closer approximation of how bidders would actually behave in practice.

6. Experimental Results

We have implemented the mechanism to test the working of *RevalSlot*. The code for the ascending proxy auction was provided by Parkes (iBundle [13]). Modules to generate sample problem sets were written in C++. While generating the different problem sets, we assumed free disposal, with $v_i(T) \geq v_i(S)$ for all $S \subseteq T$. For a problem set, we randomly generated lower bounds and upper bounds for the bundles for each bidder using the uniform distribution. These bounds gave us the initial uncertainty ranges of the bidder valuations. From these we determined the initial slot size, which was taken to be the maximum range among all the bundles and all bidders. We reduced the slot size by half in each successive round and followed the dominant strategy (Section 4, Result 3) to decide on the bids of each agent. We present a set of representative graphs from one of the problems. It had 5 items and 4 agents. The initial slot size for this problem was 400 and we terminated the auction at slot size = 5.

RevalSlot tries to accommodate the valuation uncertainties that bidders face. It promotes price discovery by providing relevant information to the bidders during the auction. We have measured the price discovery factor *r* (see Section 4, Result 4) of the bundles in the final winning allocation after each round. The graph shows a gradual increase, and by the end of the auction the value is close to 1 (Figure 1). A price discovery factor of 1 for a bundle would mean that the price of that bundle has been discovered accurately. As in the case of *r*, it is also expected that the price discovery factor *R* of the auction would increase with each round of the auction and will approach 1. The value of *R* shows a marked increase during the initial rounds and gradually reaches 1 near the end of the auction (Figure 2). It is clear that if we allow *RevalSlot* to continue indefinitely, we will reach a price discovery factor of 1, which would imply the elimination of all uncertainties among the bidders.

One other interesting characteristic of *RevalSlot* is the change in the revenue with the progress of the auction. In the beginning of the auction, it is expected that there will be a marked uncertainty in the bidder valuations, and we can expect sharp increase or decrease of revenue in the first few rounds. However, as the auction progresses, and the prices of bundles get 'discovered' more accurately, we can expect the revenue to stabilize. We have plotted

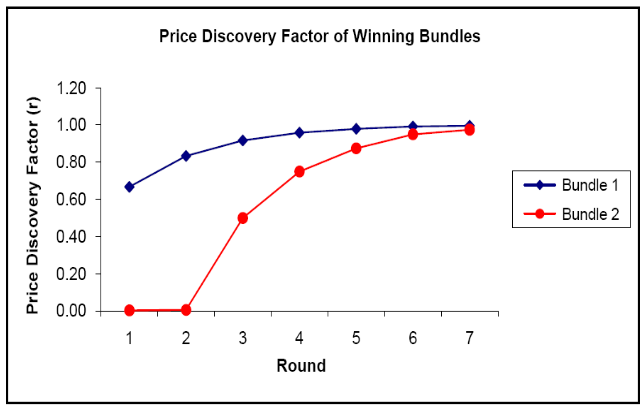

Figure 1. Price Discovery of the Winning Bundles

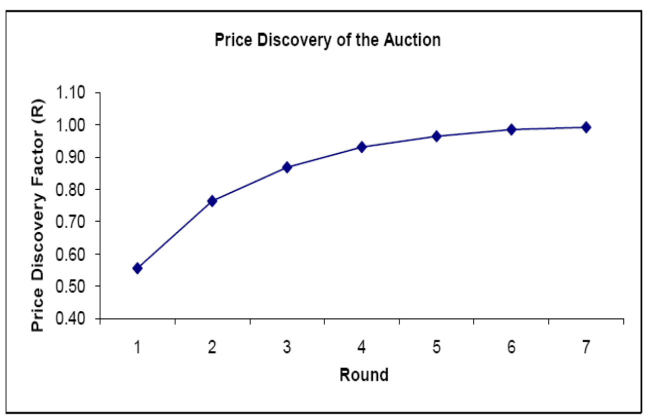

Figure 2. Price Discovery of the Auction

the revenue of each round of the auction, and the graph tallies with our expectations (Figure 3).

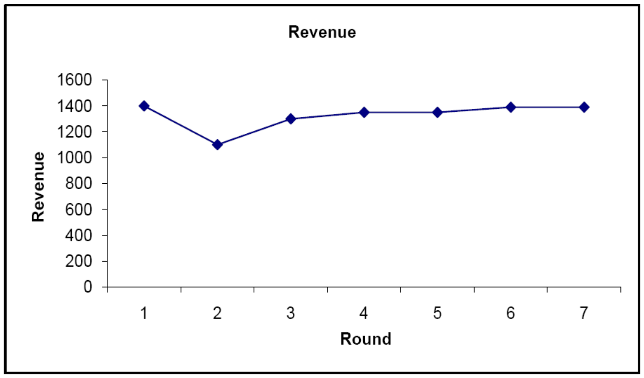

Figure 3. Auction Revenue

7. Concluding Discussion

In this paper, a preliminary attempt has been made to address the uncertainty in valuations of bidders in combinatorial auctions. The proposed mechanism allows a bidder to participate in the auction even if she is not sure about the exact valuations of the bundles. As long as the bidder has some idea of the range of valuation of the bundles in the auction, she can participate. In addition, this mechanism does not require a bidder to reveal her exact valuations to the auctioneer. Bidders might be reluctant to reveal their exact valuations at start in situations of long term strategic interaction. We have assumed that the bidders will bid truthfully and will not attempt to strategically manipulate the auction. This is perhaps not optimal in the game theoretic sense, and bidders would want to develop strategies that can increase their utilities.

The primary issues and challenges that have to be addressed are as follows:

a. More extensive experimental validation of the mechanism is needed [10]. This should be carried out if possible with human participants.
b. An interesting extension to this work would be to modify the mechanism to allow some bidders to provide exact valuations. We can also examine the possibility of automating the entire process by asking the bidders to provide an upper bound and a lower bound for all the bundles and the software would then bid on behalf of the bidders.
c. One of the disadvantages of this mechanism is the time it takes to complete. Although it provides bidders with a price discovery method, the multiple executions of the ascending proxy auction is time consuming. Solving the WDP directly on the submitted values will reduce the time considerably, but that would mean that the bidders would be reluctant to report their true valuations, as in the case of the first price sealed bid auction. One other way in which the time can be reduced is by starting the ascending proxy auction with the ask prices of the previous round (in terms of the new slot sizes). This can substantially reduce the time taken to complete the process.

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Appendix

The *RevalSlot* Algorithm

***Step 1**:* Depending on the nature of the items to be auctioned, decide on the starting slot size Z_i and the final slot size Z_f. Also, decide the factor, f, by which Z_i would reduce after each round.

***Step 2**:* Ask the bidders to submit their valuations in terms of slot numbers based on Z_i.

Step 3: Validate the reported valuations (slot numbers) with the following rules:

a) Any bidder who has bid 0 for a bundle cannot bid a positive value for the same bundle in a later round.
b) The valuation of a bundle by a bidder should not decrease from one round to the next.

***Step 4*:** Run an ascending proxy auction on the submitted values. The algorithm for the ascending proxy auction is given below.

4a. Start the auction with ask price of 0 for each bundle.

4b. For each bidder, determine the package(s) for which the utility is maximum.

4c. Bid for the maximum utility packages (from Step 4b) on behalf of each bidder.

4d. Check if the bids of the current round are exactly equal to that of the earlier round. If yes, go to Step 9.

4e. Solve the winner determination problem (WDP) to determine the allocation that maximizes revenue. This allocation is a provisional allocation. The bidders who are a part of the winning allocation are termed as 'happy' bidders, the others are termed as 'unhappy' bidders.

4f. If all the bidders are happy, terminate the ascending proxy auction and announce the winning allocation and winning prices.

4g. The ask prices for the bundles on which the 'unhappy' bidders had bid are increased by 1 (The bid increment for this ascending auction is always 1).

4h. For the happy bidders, repeat the bids from the previous round.

4i. For the unhappy bidders, continue from Step 4b.

***Step 5*:** If the winning allocation of the present round say, k^{th} round is the same as that of the earlier round, $(k - 1)^{th}$ round, and if the highest losing bid (slot) of the k^{th} round is not within the winning slot, go to Step 9.

Step 6: Announce the provisional winning bundles and the provision winning slots.

***Step 7*:** Reduce Z_i by the factor f, i.e., $Z_i = Z_i$ / f.

***Step 8*:** If Z_i equals Z_f or $Z_i < Z_i$, go to Step 9; else continue from Step 2.

***Step 9*:** Compute the prices of the winning bundles and announce it to the winners. Terminate the auction.