Competition Between Internet Search Engines

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Abstract

The Internet search engine market has seen a proliferation of entrants over the last few years. While Yahoo! was the early market leader, there has been entry by both lower quality engines and higher quality ones. Prior work on quality differentiation requires that low quality products have low prices, in order to survive in a market with high quality products. However, the price charged to users of search engines is typically zero. Therefore, consumers do not face a trade-off between quality and price. We develop a vertical differentiation model to show that even lower quality engines can survive in this market. A key property of the model is that, due to low costs users who try out one engine may also sample a lower quality engine in the same session. This “residual demand” allows lower quality products to survive in equilibrium.

We consider a two-period dynamic game between an incumbent and an entrant who enters in the second period. The incumbent has the first mover advantage due to early entry and brand loyalty, while the entrant may have a cost advantage, based on superior technology. The interaction of these two effects determines which product has higher quality in equilibrium.

We also consider the issue of strategic invesments in quality and show that incumbent may underinvest or over-invest in its quality depending on entrant’s cost structure.

1. Introduction

The market for Internet search engines has witnessed rapid growth since its inception. Search engines and search engine based portals consistently rank as some of the heavily visited sites in the market (Media Metrix, October 2002). On the consumer side, surveys indicate that search engines are the most important web promotional method used by e-Commerce sites (ActiveMedia Research, 1999), and they represent the most common way that new sites are discovered by users (Target Marketing, April 2000). Users spend a significant amount of time on search engines looking for relevant information (Gandal, 2001). This market has also seen many changes on the side of the firms. Yahoo! was an early entrant and market leader. Subsequently, the market has witnessed the entry of both lower quality engines and higher quality ones (such as Google, now the pre-eminent engine).

That products of different qualities can exist in the market has long been established in the vertical differentiation literature, (See, for example, Mussa and Rosen (1978), Shaked and Sutton (1982), and Moorthy (1988)). However, a key feature of most such models is that a low quality good must have a lower price than a high quality good. Otherwise, all consumers would buy the high quality product. The price charged to users of search engines is typically zero, with revenues being earned instead from third party; i.e. advertisers. Therefore, consumers do not face a trade-off between quality and price. Given that pricing is not a strategic choice, determination of optimal investments in product quality is an important question for both academics and practising e-Commerce managers. We provide a model of competition in this market.

Our model is based on two key properties of the market for Internet-based information goods. First, explicit price paid by the consumers for the use of these products is zero; the primary source of revenue for these products is advertisements. Model of zero pricing is particularly interesting for it generalizes to many other product categories. Despite depressed online ad market, online advertising is still a substantial part of engines’ revenue stream. For example, Yahoo! still earns close to 65% of its revenues (about $1 billion) through advertisements (The Wall Street Journal, January 15, 2003). Despite two rough years, the online ad industry was a $6.5 billion market in 2002, and is witnessing a strong rebound due to rich me-
Second, users may often sample more than one product or service during a single session. As Lawrence and Giles (1998) and Bradlow and Schmittlein (1999) mention, search engines maintain databases that contain only a fraction of the information in the universe. Since they run different algorithms on their database, different engines provide different results for the same search term. This may induce the users of even a high quality engine to visit a low quality one in the same session and restart the search process. This phenomenon, which we call “residual demand” for the low quality product, may allow the low quality product to survive (It will be immediately apparent from our model that this is not a horizontally differentiated market structure even though engines offer different results). Based on actual usage data, Telang et. al. (2000) report that users switched to a second engine during 22% of search sessions. For example, the rate of switching was 15% for those who first went to Yahoo!, while the rate for Infoseek was 31%. Such phenomenon may extend to other product categories such as news sites like Cnet or the New York Times, and shopbots like Mysimon and E-compare as well.

We consider a dynamic game of sequential entry in the search engine market. In the first period, only incumbent is present in the market. An entrant enters in the second period, and competes with the incumbent. Even though consumers incur the same costs for using the product, products of different qualities can survive in the second period. If there is no brand loyalty, all consumers sample the highest quality product first. This corresponds to the Shaked and Sutton (1982) and Moorthy (1988) models in the following manner: In these models, if all goods were offered at the same price, all consumers would choose the highest quality good. However, in these models, no lower quality good could exist in this circumstance. In our model, some proportion of consumers then go on to sample a lower quality product. We show that this residual demand enables a lower quality product to survive, even when all consumers agree that it is a lower quality good, and prefer to first visit the higher quality good.

Two properties drive the competition in the second period. The incumbent has a first-mover advantage in the form of brand loyalty. Conversely, the entrant either may have a cost advantage because there has been rapid improvement in technology in this sector, or simply a better technology. A late entrant can enter with the newest technology, whereas an incumbent is locked into an earlier technology. The issue of strategic investment in technology and its timing has been studied in the Information Systems literature by, among others, Barua et al (1991) and Clemons (1996) Interestingly, we show that if the entrant’s cost advantage is low, the incumbent overinvests in quality in the first period, and remains the high quality provider in the second period, with the entrant offering a low quality. Conversely, if the entrant has a large advantage in technology, the incumbent may choose to not compete head on and underinvests in quality in the first period. The entrant then takes over as the high quality provider, much as Google did over Yahoo!

In practice, one may expect product positioning (that is, horizontal differentiation) to matter in the search engine market as well. Such horizontal differentiation would also imply that different engines will survive in the market. In this paper, to focus on the effects of residual demand, we abstract away from explicit horizontal differentiation. As we show, even in its absence, products of different qualities can coexist in the market. Hence, embedding residual demand in a model of horizontal differentiation is likely to lead to even greater product variety in equilibrium. Our results extend to competition within a specific segment of the market, such as, for example, search engines specialized for medical information.

The rest of this paper is organized as follows. Section describes the model in detail, and presents a preliminary result that helps us find the equilibrium of the game. In Section , we determine the equilibrium when the entrant has no cost advantage, and show that the incumbent is the high quality engine. Section considers the case of low brand loyalty, and Section that of high brand loyalty. Implications of our model and some concluding remarks are presented in Section 2. All proofs are relegated to the Appendix, Section 3.

2. Model

Two firms compete in quality in the market. We interpret quality as the ability of the engine (We use the term engine or product interchangeably in this paper) to provide results or information that satisfy a user. For example, for search engines, this definition of quality includes the two common attributes of quality: (1) the quantity of information retrieved, and (2) its relevance to the user. Our definition of quality may be interpreted as a reduced form notion that encompasses both these attributes. A higher quality is more valued by the user, because it implies either a higher quantity of information, or information of

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2Note that these costs are modelled only implicitly, via a demand curve.
greater relevance, or both. We assume that the quality of the firms are common knowledge amongst consumers.

Using the product requires both time and effort on the part of a consumer. Therefore, the lower the quality of a product, the less likely it is that a consumer will be willing to invest her time and effort. Hence, we model demand as increasing in quality. If a firm is a monopolist and offers quality $q$, the demand in a given period is $D(q)$. We assume that the demand curve is linear, with $D(q) = a + bq$, where $a \leq 0$ (so that a firm offering a zero quality has no customers) and $b > 0$.

Our model encompasses two periods, 0 and 1. The demand curve is assumed to be the same in each period. In period 0, firm 1 (the incumbent) is the only firm in the market, and chooses a quality level $q_0$. In period 1, firm 2 (the entrant) enters, and offers quality level $q_2$.

Most digital products require an upfront investments in hardware and software development (Jones and Mendelson 1998). The firms need to set up the infra-structure in terms of web servers, content gathering and provision. For example, in case of search engines, web robots crawl the world-wide web, and index the information in a local database. When a user inputs a search term, queries are run on the local database, and results are returned. Once the up-front investments have been made, the marginal costs are relatively low for such information goods (Bhagava and Chowdhary 2000). We assume it to be zero. Therefore, we assume that firm 1 can at least remain at its original quality level $q_0$ by incurring small marginal costs. Hence, we restrict $q_1$ to be at least as high as $q_0$.

Firm 1 has a cost function $C(q)$ that is assumed to be strictly increasing and strictly convex. Firm 2 has a cost function $\lambda C(q)$, where $\lambda \in [0, 1]$. $\lambda$ captures the idea that the costs associated with technology are constantly falling (or, alternatively, that the technology itself is improving). When $\lambda = 1$, neither firm has a cost advantage. When $\lambda < 1$, the entrant has a cost or technology advantage. The incumbent, since it established its infrastructure in period 0, is locked into the old technology.\footnote{As we will show that even if the incumbent could use new technology, it will not change our results significantly}

On any given visit to a particular engine, a user may not be fully satisfied with the outcome. The higher the quality $q$ of the engine, the greater the chance that the user will be satisfied. We assume that for a given visit, a user is satisfied with the results with probability $p(q)$. Suppose a user in period 1 tries out firm 1 first. Then, with probability $p(q_1)$, she finds the information she needs and her quest for information ends. With probability $(1 - p(q_1))$, she is dissatisfied with the information received.\footnote{Without any loss of generality, we can also assume that users has tried different refinement of same search words and has been dissatisfied with all such results}

Then, if firm 2’s product is of sufficiently high quality, she proceeds to use it. We say that firm 2 has a “residual demand” in this case.

To facilitate analytical solutions, we assume that $p(q) = q$, and $C(q) = kq^2$. Parameter restrictions are imposed to insure that chosen quality levels satisfy $0 < q < 1$. As shown below, interestingly, firms’ demand and profit functions are discontinuous, depending on whether the firm has higher or lower quality than its competitor. Therefore, though equilibria with similar qualitative features exist with more general functions for demand, cost and probability, characterizing these analytically is quite difficult.

Revenues of each firm are directly linked to its demand; we assume that a firm earns an advertising revenue of $r$ in each period per consumer that visit. As mentioned by Hoffman and Novak (1996), advertising rates depend on the number of “hits” a web site obtains. For convenience, we define $c = \frac{b}{r}$, and renormalize the profit function by dividing by $r$. This renormalization affects the level of profit earned by a firm, but not its profit-maximizing quality level (i.e., its strategy). Since our main concern is with the strategies of the firms, and whether profits are greater or less than zero, renormalization does not affect our results.

Finally, we allow for brand loyalty in this market. Loyalty has been shown to be an important factor in product choices and pricing (see, for example, Narasimhan 1988). We model loyalty in our paper such that $\beta \in [0, 1]$ proportion of the users return to their previous choice, irrespective of its quality, as their first choice.

In period 0, $D(q_0) = a + b q_0$ consumers use firm 1, as it is the only product in the market. As mentioned, of these consumers, a fraction $\beta \in [0, 1]$ remain loyal and return to the product 1 in period 1 as their first choice, regardless of $q_1$ and $q_2$. The rest (fraction $1 - \beta$) of old consumers, plus any new users in period 1, choose the product which has higher quality as their first choice in period 1. Any user may be dissatisfied with the results offered by her first choice product. As mentioned before, all dissatisfied users will switch to the other product if its quality is high enough.

Brand loyalty in this model, therefore, refers only to the first choice of product at period 1. Therefore, in our model, some proportion of loyal users may also sample both products. Users loyal to the incumbent return to it first in the next period regardless of its quality. Other users (potentially disloyal) always use the product which has higher quality (either incumbent or entrant). Note that none of the key results of this paper are affected by how
loyalty enters the equation. To illustrate the demand of both firms in the presence of brand loyalty, consider the following two cases.

**Case 1:**
Let \( q_2 > q_1 > q_0 \). Then, firm 2 has the high quality product and firm 1 the low quality one in period 1. Figure 1 shows the demand curve.

\[ D(q) \]

![Figure 1: Consumer demand when \( q_2 > q_1 > q_0 \).](image)

Consider the demand faced by firm 2 in period 1. The users \( D_2 - D_0 = b(q_2 - q_0) \) are all new users in this period (that is, they did not use any product at time 0). All new users will use the higher quality product first. In this case, since \( q_2 > q_1 \), all these users visit firm 2 as their first choice in period 1. In addition, of the users \( D_0 \) who visited firm 1 in period 0, the fraction \( 1 - \beta \) (which is not loyal), will also switch to the higher quality entrant in period 1. Hence, the total demand for firm 2 as a first choice is \((1 - \beta)(a + bq_0) + b(q_2 - q_0)\). With probability \((1 - q_2)\), each of these users will be dissatisfied with the output of firm 2. Users in the region \([D_1, D_2]\) end their search, since \( q_1 \) is too low for them. The remainder will switch to firm 1.

Of the period 0 users, \( D_0 \), the fraction \( \beta \) (loyal users) continue to visit firm 1 as the first choice in period 1. These users switch to firm 2 if dissatisfied (that is, with probability \((1 - q_1)\)). Table 1 represents the demand for both products in period 1 alone.

**Table 1: Demand for firms in period 1**

<table>
<thead>
<tr>
<th>First Choice</th>
<th>Second Choice</th>
</tr>
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<tbody>
<tr>
<td>( \beta(a + bq_0) )</td>
<td>( (1 - q_2)((1 - \beta)(a + bq_0) + b(q_1 - q_0)) )</td>
</tr>
<tr>
<td>( (1 - \beta)(a + bq_0) + b(q_2 - q_0) )</td>
<td>( (1 - q_1)\beta(a + bq_0) )</td>
</tr>
</tbody>
</table>

The profit of firm 2 in period 1 is the sum of its profits in periods 0 and 1, with no discounting (Adding a discount factor affects the levels of profit, but does not change any of the key insights provided by the model). When period 1 profits are discounted, firm 1 is less eager to be the high quality provider at time 1. This changes the parameter regions in which different equilibria emerge, but not the characteristics of those equilibria. The cost of firm 1 is \( c q_1^2 \) in period 0, and \( c(q_1^2 - q_0^2) \) in period 1, for a total cost of \( c q_1^2 \). Its demand in period 0 is \( a + b q_0 \), and demand in period 1 comes from the above table. Hence,

\[
\pi_1^f(q_0, q_1, q_2) = a(2 - q_2 + \beta q_2 + bq_1 + b q_0 (1 + \beta q_2 - q_2)) - cq_1^2. \tag{2}
\]

**Case 2:**
Now firm 1 is the high quality in period 1, and firm 2 the low quality one. In particular, suppose that \( q_1 > q_2 \), and \( q_1 \geq q_0 \), where \( q_0 \) can be greater or less than \( q_2 \). Then, the first and second choice demands for the products change in an appropriate manner. The profit function of firm 1 is now determined to be

\[
\pi_1^f(q_0, q_1, q_2) = (a + bq_0) + (a + b q_1) - cq_1^2. \tag{3}
\]

Notice that, in this situation, all users who used firm 1’s product in period 0 come back to it as the first choice in period 1, regardless of \( \beta \), since it is the higher quality firm. Hence, there is no residual demand for firm 1: any user dissatisfied with firm 2 has already tried firm 1’s product, and found it unsatisfactory. The profit function of firm 2 is

\[
\pi_2^f(q_0, q_1, q_2) = (1 - q_1)(a + bq_2) - \lambda c q_2^2. \tag{4}
\]

If \( q_2 = q_1 \), then firms are assumed to share new customers equally (that is, those consumers in the region \( b(q_1 - q_0) \)). As before, of the experienced consumers, in the region \( a + b q_0 \), the fraction \( \beta \) use product 1 as the first choice.
Notice the discontinuity in the profit functions associated with being a high quality instead of being a low quality provider. This discontinuity implies, among other things, that in equilibrium it cannot be that \( q_1 = q_2 \). If \( q_1 \neq q_2 \), each firm has an incentive to increase its quality by \( \epsilon > 0 \). This discontinuity has the same flavor as the discontinuity in the Bertrand model of duopoly. The difference is that, in a standard Bertrand model, if one firm has a higher price than the other, its demand falls to zero. In our model, if one firm offers lower quality than the other, its demand does fall, but can remain strictly positive because no firm can perfectly satisfy its consumers.

The solution concept we use is pure strategy subgame-perfect equilibrium. Subgame-perfection is natural here since it is a dynamic sequential game. We do not consider mixed strategies as pure strategy equilibria exist in both periods.

First, we show that, in equilibrium, firm 1 sets \( q_1 = q_0 \). Suppose, instead, that \( q_1 > q_0 \). Then, firm 1 incurs a total cost of \( c q_1^2 \) over the two periods. Increasing first period quality to \( q_1 \) leads to additional demand (hence additional revenue) in period 0, at no extra cost (since the amount \( c(q_1^2 - q_0^2) \) is incurred anyway in period 1). Hence, a strategy that has \( q_0 < q_1 \) is dominated by one that has \( q_0 = q_1 \).

Note that the proposition below holds for all values of \( \lambda \) and \( \beta \). All proofs are in the appendix, in Section 3.

**Proposition 1** In equilibrium, the incumbent chooses the same quality in both periods, so that \( q_1 = q_0 \).

For the rest of the paper, we set \( q_1 = q_0 \). Next, we define the quality levels the incumbent and entrant would offer if each were a monopolist. Let \( q_1^m \) be firm 1’s monopoly level, and \( q_2^m \) that of firm 2. Then, \( q_1^m \) solves the problem \( \max_{q_1} \pi_1(q) = 2(a + bq) - c q_1^2 \). The first order condition is \( 2bq - 2c q_1 = 0 \), from which we have \( q_1^m = \frac{b}{2c} \). The monopoly level of the entrant, \( q_2^m \), is found as the solution to \( \max_{q_2} \pi_2(q) = a + bq - \lambda cq^2 \). The first order condition is \( bq - 2\lambda c q_2 = 0 \), from which we have \( q_2^m = \frac{b}{2\lambda c} \).

We maintain the following assumptions on the parameters:

**Assumption 1** (i) \( \lambda c > (a + b) \). (ii) \( a + \frac{b^2 (1 - \frac{b}{2c})}{4c} > 0 \).

Part (i) of the assumption ensures that the cost stays within a range such that the entrant will not be profitable at a quality of 1. Therefore, it will choose a quality strictly less than 1. Part (ii) ensures that the entrant makes a strictly positive profit when \( \lambda = 1 \), the incumbent offers its monopoly quality \( q_1^m = \frac{b}{2c} \), and the entrant plays a best response. If \( \lambda \) is strictly less than 1, then this condition is

3. **No Cost Advantage for Entrant**

First, consider the case of \( \lambda = 1 \); that is, the entrant has no cost advantage over the incumbent. Under the assumptions made, we show that the incumbent, firm 1, will offer its monopoly quality level, and the entrant will be the follower at time 1. In this situation, the presence of the entrant does not affect firm 1 at all. It continues to offer a monopoly quality and earn a monopoly profit. However, firm 2 still finds it profitable to enter the market, because of demand from customers of firm 1 that are dissatisfied with the results of their search query.

**Proposition 2** Suppose the entrant does not have a cost advantage. Regardless of the strength of brand loyalty for the incumbent, in equilibrium the incumbent engine offers the monopoly quality \( q_1^m = \frac{b}{\lambda c} \), in both periods and the entrant chooses a lower quality \( q_2 = \frac{b(1-q_1^m)}{2c} < q_1^m \).

This proposition holds for all values of \( \beta \). When \( \beta = 1 \), the incumbent enjoys complete brand loyalty, and is not affected by the entrant’s strategy. However, even at the other extreme of zero brand loyalty, when \( \beta = 0 \), the incumbent has a first-mover advantage, and the entrant offers low quality in equilibrium.

The interesting part of this equilibrium is that despite no price difference, two engines of differing qualities can survive without any explicit horizontal differentiation. Therefore, an entrant can choose to be a lower quality engine and survive in this market. For example, Lycos and Infoseek, which entered the market after Yahoo!, are widely perceived as lower quality engines.

To demonstrate the nature of the strategic interaction between the firms, we consider a numeric example and depict the best response of the entrant to every possible \( q \in [0, 1] \) that the incumbent might offer. This example represents a base case, and all other examples in the paper consider variants of the same parameters. The parameters for this example are \( a = -0.05, b = 1, c = 2, \lambda = 1, \beta = 0 \).

Figure 2 demonstrates the reaction function of firm 2, given a quality level \( q_1 \) chosen by firm 1. When \( q_1 \) is low, close to zero, firm 2 has the high quality product, and chooses its monopoly level, \( q_2^m = 0.25 \). As \( q_1 \) rises to just above 0.25, firm 2 prefers to compete with firm 1, and choose \( q_2 = q_1 + \epsilon \). However, at \( q_1 = \hat{q} \) (0.43 in this example), firm 2 is indifferent between being a high qual-
ity and low quality provider. For \( q_1 > \hat{q} \), firm 2 has the low quality, and its quality falls as \( q_1 \) increases. Finally, when \( q_1 \) rises above 0.6, firm 2 exits the market altogether (i.e., sets \( q_2 = 0 \)).

As shown above in Proposition 2, when there is no cost advantage, the entrant is exactly indifferent between \( q_2 = 0 \) and \( \hat{q} \). This threshold is \( \hat{q} = q_2^m = \frac{b}{2\lambda c} \).

We define the critical quality threshold to be the quality level of the incumbent such that the entrant is indifferent between offering a lower quality and a higher quality. This threshold is \( \hat{q} \) in Figure 2 above. Note that the profits and market shares of the firms are readily determined from the equilibrium qualities.

4. Entrant Cost Advantage, Low Brand Loyalty

As shown above in Proposition 2, when there is no cost advantage to the entrant, the incumbent is high quality firm in equilibrium. However, when brand loyalty is low, its effect can be dominated by a high cost advantage for the entrant. That is, under some conditions, the entrant can also be high quality firm in equilibrium.

In this section, we analyze the countervailing effects of brand loyalty for the incumbent (\( \beta \)), and cost advantage to the entrant (\( \lambda \)). We first analyze their effects in isolation, and then demonstrate that, depending on the strengths of these two effects, either the entrant or the incumbent could be the period 1 high quality firm in equilibrium.

The incumbent in our model has a first-mover advantage. By choosing \( q_1 > \hat{q} \) (critical quality threshold), it can always force the entrant to offer the low quality in equilibrium. Hence, the effects of the parameters \( \lambda \) and \( \beta \) can be analyzed through their effect on \( \hat{q} \). If \( \hat{q} \) increases, this is costly for the incumbent: it has to offer a higher quality than otherwise, to force the entrant to offer a low quality product. If \( \hat{q} \) falls, the incumbent is better off.

First, consider the effect of increasing brand loyalty on firm 2’s best response, for a fixed value of \( \lambda \). As \( \beta \) increases above zero, \( \hat{q} \) will fall. At some \( \beta \), which we denote as \( \tilde{\beta} \), we will have \( \hat{q} = q_2^m = \frac{b}{2\lambda c} \) (notice that \( \beta \) is a function of \( \lambda \)). When \( \beta = \tilde{\beta} \), if firm 1 offers \( q_1 = \hat{q} \), firm 2 will choose \( q_2^m < \hat{q} \). That is, firm 1 will offer higher quality in equilibrium. Now, if \( \beta \) increases beyond this, brand loyalty is so strong that the incumbent will continue to be the higher quality firm in period 1. In this section, we analyze the case of \( \beta \in [0, \tilde{\beta}] \), which we term low brand loyalty. In the next section, we consider high brand loyalty.

Define the critical value of brand loyalty, \( \tilde{\beta} \), to be the level at which \( \hat{q} = q_2^m = \frac{b}{2\lambda c} \). As an aside, we note that we can explicitly determine the value of \( \tilde{\beta} \) in our model.

**Lemma 1** \( \tilde{\beta} = 1 - \frac{b^3}{4\lambda c(b^2 + 2a\lambda c)} \).

We can show formally that, when brand loyalty is low, \( \hat{q} \) declines in \( \beta \) and increases in \( \lambda \).\(^5\) Note that since \( \tilde{\beta} \) is defined as a function of \( \lambda \), the cost advantage of the entrant, we can interpret the condition \( \beta \in [0, \tilde{\beta}] \) as a joint restriction on \( \beta \) and \( \lambda \).

Note that since \( \hat{q} < q_1^m \) at \( \lambda = 1 \), there will be a value of \( \lambda \), \( \lambda_m \), at which \( \hat{q} = q_1^m \). Until this value of \( \lambda \) is reached, it is clear that the equilibrium of Proposition 2 will continue to hold. The incumbent offers higher quality, and the entrant will offer lower quality. The entrant will offer a real competitive threat to the incumbent only for \( \lambda < \lambda_m \). When \( \hat{q} > q_1^m \), it becomes costly for the entrant to offer a high quality product.

\(^5\)The formal proof is not shown here due to space constraint. Note that \( \hat{q} \) is computed to be \( \hat{q} = \frac{b^2 + 2a\lambda c(a + b) - 2a\lambda c}{2b^2 + 2a\lambda c} \pm \delta \), where \( \delta = \frac{2\sqrt{a^2\lambda c(-b^3 - ab(b + 2a\lambda c) + a^2\lambda c(\beta - 1))(\beta - 1)} + 4b\lambda c}{b^2 + 4\lambda c^2 + 4b\lambda c} > 0 \).
incumbent now compares its own profit as high quality (as a leader, it must offer $\hat{q}$ now) and as low quality (in which case it plays some best response strategy, with the entrant offering higher quality).

Note that when incumbent offers $q_1 = \hat{q} > q_m$, it has to offer higher than monopoly quality when the entrant enters with lower costs. This also outlines the strategic advantage the incumbent has. It can offer higher quality and force the entrant to become a low quality firm even though entrant has cost advantage. Also, note that since $q_0 = q_1$, incumbent overinvests in quality in the first period itself to be a quality leader. In the two-period dynamic game, as long as the entrant does not have too high a cost advantage, the incumbent can overinvest in quality and remain the quality leader.

Eventually, when the entrant’s cost advantage becomes very high, incumbent has no option but to offer lower quality. For example, with the emergence of Google, Yahoo! is a lower quality player and generates less traffic than Google. We show first that, when the entrant offers high quality, it offers its own monopoly quality level, $q_2 = \frac{b}{2\lambda c}$.

**Proposition 3** Suppose the brand loyalty is less than complete (i.e., less than 1). For a high cost advantage, the entrant becomes a high quality firm in period 1 and offers its own monopoly quality level. The incumbent is now a low quality firm and offers $q_1 = \frac{b}{c} - \frac{b^2(1-\beta)}{4\lambda c^2}$ in both periods.

Therefore, when the entrant is a high quality firm in equilibrium, the quality of the incumbent is increasing in $\lambda$. As $\lambda$ falls, $q_2$ rises. The demand for the incumbent is now the residual demand created by dissatisfied users of firm 2. As this demand shrinks, the quality offered by firm 1 falls as well. Therefore, the prospect of falling technology costs in period 1 affects the quality offered by the incumbent in period 0.

If $\lambda$ is sufficiently low, then $\hat{q}$ can be substantially higher than $q_1^m$. At this point, exercising the first-mover advantage is costly to the incumbent, and it may prefer to be a low quality firm in equilibrium. We can formally show that there exist a high values of $\lambda$ (above some threshold $\lambda_h$), when incumbent will offer high quality. Similarly, for some low value of $\lambda$ (below some threshold $\lambda_l$), the cost advantage of the entrant can dominate the effects of brand loyalty to an extent that the entrant offers a higher quality product. We use our base example, with $\alpha = -0.05, b = 1, c = 2$, and $\beta = 0, \lambda_l = \lambda_h = 0.57$ to highlight this fact. Figure 3 below shows the difference in profits for firm 1, between offering a higher quality product and a lower quality one.

When the difference in profits is positive (for $\lambda > 0.57$), firm 1 chooses to offer higher quality in equilibrium; that is, it sets $q_1 = \max(\hat{q}, q_1^m)$. When this difference is negative, it is too costly for firm 1 to be a high quality firm (that is, $\hat{q}$ is too high), and it chooses to offer low quality. At the critical value of $\lambda$, firm 1 is indifferent between being high quality and low quality firm, and both equilibria exist.

![Figure 3: Difference in Profit of Firm 1, high and low quality](image)

**1 Entrant Cost Advantage, High Brand Loyalty**

First, we consider the case of complete brand loyalty. When $\beta = 1$, the incumbent can just ignore the entrant. By Proposition 1, in equilibrium, it must be that $q_0 = q_1$. In this case, the entrant’s quality level has absolutely no effect on the incumbent’s demand. Therefore, firm 1 chooses its monopoly quality level, $q_1 = \frac{b}{c}$.

Now, firm 2 can choose to be either high quality or low quality, depending on $\lambda$. As shown in the next Proposition, $\lambda = \frac{1}{2} - \frac{b}{4c}$ is a critical value. For higher values of $\lambda$, the incumbent offers high quality, and for lower values low quality.

**Proposition 4** Suppose there is complete brand loyalty, so that $\beta = 1$. Then, in equilibrium, regardless of the cost advantage of the entrant, the incumbent chooses its monopoly quality level. The entrant chooses a lower quality level ($q_2 = \frac{b(1-q^m)}{2\lambda c}$) if its cost advantage is low ($\lambda \geq \frac{1}{2} - \frac{b}{4c}$).
and a higher quality level \( q_2 = q_2^m = \frac{b}{4c_2} > q_1 \) if its cost advantage is high.

When \( \lambda < \frac{1}{2} \), the monopoly level of the entrant exceeds that of the incumbent: \( q_2^m = \frac{b}{2c_2} > q_1^m = \frac{b}{2c_1} \). However, this is not sufficient for the entrant to offer high quality in equilibrium. Consider a value \( \lambda \in (\frac{1}{2} - \frac{b}{4c_2}, \frac{1}{2}) \). At \( \lambda \), the entrant is better off being a low quality firm, and not choosing \( q_2 = \frac{b}{2c_2} \). Because brand loyalty is complete, the entrant only attracts new demand, \( b(q_2 - q_1) \). All users from period 0 return to firm 1 as their first choice.\(^7\)

2 Conclusion

We show that quality differences can exist in a market even when the consumer is paying the same price (zero, for Internet information goods) for all goods. New entrants may enter with low quality, and be viable in the market. This is because, in any given session, some proportion of users will sample more than one product. This leads to a residual demand for lower quality products that creates room for more firms to enter the market. Vertical differentiation as a result of this residual demand is different from that considered in the previous literature, where lower prices are used to sustain low quality products.

The phenomenon of multiple product sampling extends to many information goods on the Internet, including search engines, shopbots and news sites. Some users regularly visit more than one news sites (the content of a single news site may be modeled as probabilistic), even for the same news item. Our results extend more generally to all such goods, where the quality of content or service provided is probabilistic in nature and consumer can engage in product sampling at little cost.

Of course, a firm in such a market is aware that some visitors to its site will soon be consulting a competitor. Hence, by either placing a direct link to a competitor (perhaps in the form of an advertisement), the firm can accrue additional revenue. This phenomenon is evidenced among search engines, for example, Yahoo! and Netscape, which offer links to many other search engines.

Firms may even benefit from partnering with other firms so that, when a user switches, the switch is to a partner in the same family. Such partnerships (such as Yahoo! and Google or Lycos and Hotbot) are becoming more common.\(^8\) By offering viable alternatives to the user, more than one firm can capture revenues from the same user.

Finally, our model also considers the role of loyalty and new technology. Many of the challengers today are firms with some technical superiority. For example, Google is now considered a top engine because of the new technology it uses in indexing pages (http://www.google.com/why_use.html). Eventually, market equilibrium depends on a combination of all the factors mentioned above. Hence, today we see a number of firms in the content provider market which capture some traffic without being either a first mover or having a higher quality than competitors. By allowing lower quality firms to survive in the market, our model points to the fact that more firms can survive in a market than otherwise.

One of the issues we do not consider is what happens to competition in the absence of residual demand. In such a case, pure strategy equilibrium fails to exist. We also have not modelled horizontal differentiation explicitly. We leave them as future research avenues.

3 Appendix: Proofs

Proposition 1

By assumption, it cannot be that \( q_0 > q_1 \). Suppose, then, that firm 1 chooses \( q_0 = \bar{q} \) and \( q_1 = \tilde{q} \), with \( \bar{q} > \tilde{q} \). Consider the effect of choosing \( q_0 = \tilde{q} \) and retaining \( q_1 = \bar{q} \). This has no effect on total cost, which remains \( c(\bar{q})^2 \).

We show that equilibrium at time 1 is unaffected; that is, firm 2’s optimal choice of \( q_2 \) remains the same. Suppose that \( q_1 > q_2 \). Then, from equation (4), firm 2’s profit function is unchanged as \( q_0 \) changes. Hence, its optimal response is unchanged.

Next, suppose that \( q_2 > q_1 \). In this case, from equation (2), we see that a change in \( q_0 \) does affect the level of profit earned by firm 2. However, again it does not affect the optimal choice of \( q_2 \). This is observed from the first order condition \( \frac{dq_2}{dq_2} = 0 \), which yields the best response to be \( q_2 = \frac{b}{4c_2} \), independent of \( q_0 \). If this \( q_2 < q_1 \), then, to offer high quality, firm 2 will have to set \( q_2 = q_1 + \epsilon \), for some \( \epsilon > 0 \), which is again unaffected by \( q_0 \).

Hence, \( q_2 \) remains unchanged, and so equilibrium at time 1 is unchanged when \( q_0 \) is set to \( \tilde{q} \) instead of \( \bar{q} \). Now, consider the demand for firm 1 at time 0. This increases by \( b(\bar{q} - \tilde{q}) > 0 \). Since the cost of firm 1 does not change, this increase in demand represents an unambiguous increase in profit. Hence, it cannot be an equilibrium for

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\(^7\)The page http://www.searchengineworld.com/engine/partners.htm contains a list of current partnerships.

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firm 1 to set \( q_0 < q_1 \).

**Proposition 2**

Clearly, firm 1 (the incumbent) can do no better than set its monopoly quality level \( q_1 = q_0 = q_1^m \). This represents the highest profit it can make, even if firm 2 (the entrant) were absent. Hence, we show that this is an equilibrium, we only need to show that firm 2 is playing a best response, and cannot do better by choosing some other \( q_2 \).

Suppose firm 2 is lower quality in period 1. Then, its profit is given by equation (4). Its best response is found from the first order condition \( \frac{d\pi_2}{dq_2} = 0 \), which yields \( \pi_2^f = a(1 - q_1) + \frac{b^2(1 - q_1)^2}{4Ac} \).

If firm 2 is higher quality, it must set \( q_2 \geq q_1 \). Since \( q_1^m > q_2^m \), its best response is to set \( q_2 = q_1 + \epsilon \), for some \( \epsilon > 0 \). Hence, from equation (2), \( \frac{d\pi_2}{dq_2} = (\text{substituting for } q_0 = q_1) \frac{d\pi_2}{dq_2} = a(1 - \beta q_1) + b(q_1 + \epsilon - \beta q_1^2) - \lambda c q_1(1 + \epsilon)^2 \). This expression is maximized at \( \epsilon = 0 \). Consider a decreasing sequence \( \epsilon_n \to 0 \), where \( \epsilon_n \to 0 \) for each \( n \). Since \( q_1^m > q_2^m \), firm 2’s profit increases along this sequence. The maximal profit it obtains as high quality, in the limit, is \( \pi_2^f = (a + b q_1)(1 - \beta q_1) \). Hence,

\[
\pi_2^f - \pi_1^f = -a q_1 (1 - \beta) + \frac{b^2(1 - q_1)^2}{4Ac} - b q_1 + \beta b q_1^2 + \lambda c q_1^2. 
\]  

(5)

At \( q_1 = \frac{b}{c} \) and \( \lambda = 1 \), this reduces to \( \pi_2^f - \pi_1^f = -a q_1 (1 - \beta) + \frac{b^2(1 - \frac{b}{2c})^2}{4c} + \beta b \frac{b^2}{4c^2} > 0 \), where the last inequality follows since each of the three terms is strictly positive (recall that \( a < 0 \)). Therefore, firm 2 prefers to offer low quality. Given \( q_1 = \frac{b}{c} \), from equation (3), its best response is \( q_2 = \frac{b(1 - \frac{b}{2c})}{2c} = \frac{b}{2c} - \frac{b^2}{2c^2} \), which is less than \( \frac{b}{c} \).

**Lemma 1**

Consider equation (5) above, which denotes the difference in profit for firm 2 when it offer higher quality compared to lower quality. Set \( \pi_2^f - \pi_1^f \) to zero, and \( \hat{q} = \frac{b}{2Ac} \), to obtain the desired value of \( \hat{\beta} \).

**Proposition 3**

Suppose the entrant offers high quality, and sets \( q_2 > \frac{b}{2Ac} \), that is, higher than its monopoly level. Since the best response of the entrant is above its monopoly level, it must set \( q_2 = q_1 + \epsilon \). Above \( q_2^m \), \( \pi_2 \) is declining in \( q_2 \), so any higher quality will lead to lower profit. However, for any \( q_2 \), the incumbent makes a higher profit by setting \( q_1 = q_2 + \epsilon \) than \( q_1 = q_2 - \epsilon \). The incumbent’s profit at \( q_1 = q_2 + \epsilon \) is \( \pi_1(q_2 + \epsilon) = 2(a + b(q_2 + \epsilon)) - c \epsilon^2(q_2 + \epsilon)^2 \), and its profit at \( q_1 = q_2 - \epsilon \) is \( \pi_1(q_2 - \epsilon) = (2 - q_2(1 - \beta))(a + b(q_2 - \epsilon)) - c \epsilon^2(q_2 - \epsilon)^2 \).

Since \( q_2 > 0 \) and \( \beta < 1 \), in any equilibrium in which the entrant offers higher quality, for \( \epsilon \) close enough to zero, \( \pi(q_2 + \epsilon) > \pi(q_2 - \epsilon) \). Since there is no equilibrium in which \( q_2 = q_1 + \epsilon \), the only other possibility is that the entrant is at its monopoly level, \( q_2 = \frac{b}{2Ac} \).

Next, consider the best response of the incumbent, given that the entrant offer high quality and is choosing \( q_2 = \frac{b}{2Ac} \). Table 1 shows the demand for firm 1 in period 1 when it is a follower. Substituting \( q_0 = q_1 \) (using Proposition 1), adding the first period demand of \( a + b q_1 \), and noting that \( q_2 = \frac{b}{2Ac} \), the profit of firm 1 as low quality can be represented as \( \pi_1^f = (a + b q_1)(2 - \frac{b(1 - \beta)}{2Ac}) - c q_2^2 \). The first order condition, \( \frac{d\pi_1^f}{dq_1} = 0 \), directly yields the optimal value of \( q_2 \) to be \( q_1 = \frac{b}{c} - \frac{b^2(1 - \beta)}{4Ac} \). Again the profits for both firms at equilibrium can be specified as \( \pi_1 = 2a + \frac{b^2}{c} \left( 1 - \frac{b(1 - \beta)}{4Ac} \right) \) and \( \pi_2 = a + \frac{b^2}{4Ac} \).

**Proposition 4**

When \( q_0 = q_1 \), the incumbent’s demand in the second period, regardless of \( q_2 \), is just \( a + b q_1 \). Hence, its profit function is \( \pi_1(1) = 2(a + b q_1) - c q_2^2 \), and its optimal strategy is to set \( q_0 = q_1 = \frac{b}{c} \).

Next, consider the entrant, given that the incumbent offers \( q_0 = q_1 = \frac{b}{c} \). The entrant’s profit as lower quality is \( \pi_2^l(q_2) = (1 - q_1)(a + b q_2) - \lambda c q_2^2 \). As discussed, its best response is to set \( q_2 = \frac{b(1 - \beta)}{2Ac} \). Setting \( q_1 = \frac{b}{c} \), therefore, its profit as a low quality firm reduces to \( \pi_2^l = a(1 - \frac{b}{c}) + \frac{b^2(1 - \frac{b}{2c})}{4Ac} \).

Now, suppose the entrant offers high quality. Its profit as high quality firm is \( \pi_2^h(q_2) = (1 - \beta)(a + b q_1) + \beta(1 - q_1)(a + b q_1) + b(q_2 - q_1) - \lambda c q_2^2 \). From the first order condition, it is immediate that the best response is to set \( q_2 = \frac{b}{2Ac} = q_2^m \). Then, the entrant’s profit reduces to \( \pi_2^h = a(1 - \frac{b}{c}) - \frac{b^2}{4c^2} + \frac{b^2}{4Ac} \).

Hence, the difference in profits, \( \pi_2^h - \pi_2^l \), is greater than zero if \( \frac{4\lambda c^2}{b^2} \left( \frac{b}{c} - 2 + 4\lambda \right) < 0 \), that is, if \( \lambda < \frac{1}{2} - \frac{b}{4c} \). Finally, to complete the proof, note that \( \frac{b}{2Ac} > \frac{b}{c} \) for all \( \lambda \leq \frac{1}{2} \), and hence including all \( \lambda < \frac{1}{2} - \frac{b}{4c} \).
References


