Investment and Bidding Strategies in Markets for Firm Transmission Rights

Aleksandr Rudkevich
Tabors Caramanis & Associates
arudkevich@tca-us.com

Abstract

The first part of this paper addresses the applicability of the Markowitz portfolio theory to investing in Contracts for Firm Transmission Right (CFTRs) or Transmission Congestion Contracts (TCCs) typical for electricity markets in Northeastern US (PJM, NY ISO, ISO New England). Special emphasis is placed on the use of the principal component analysis providing for a dramatic reduction in the size of the optimization problem, selecting subsets of statistically independent CFTRs/TCCs and on the mathematical formulation of necessary and sufficient conditions for arbitrage opportunities. The second part of the paper is dedicated to the analysis of profit-maximizing bidding strategies available to large players with significant Auction Revenue Rights (ARRs). The use of the Supply Function Equilibrium technique provides for a closed form solution in the case of two strategic players competing for CFTRs over a major interface.

1. Introduction

What is presently known as a deregulated electricity market is not a single market but, in fact, a composite of several markets varying in type of traded products, time domain and geographical scope. One of those markets is the market for transmission rights implemented in one or another form in every deregulated market in the United States with a goal of giving market participants the means to manage their exposure to congestion cost risk. Markets for transmission rights are essentially forward markets for price differentials between various locations on the electrical grid. For example, a power producer generating energy at location G is paid energy price at that location, $P(G)$. However, the same power producer could be bound by a power purchasing agreement with another party to sell generated power at location S at a price $P(S)$. Because of transmission congestion, the sale price $P(S)$ could be lower than the price at the generation location. That would expose the power producer to a financial risk of $P(S) – P(G)$ for each MW of the power purchasing agreement. In order to hedge this risk, a producer could acquire matching the power purchasing agreement quantity of transmission rights from point S to point G and be entitled to the payment of $P(G) – P(S)$ times that quantity which cancels the congestion cost risk exposure for the producer. This risk protection, however, is not free of charge, because transmission rights should be purchased and paid for, in the market for those rights.

Emerged markets for transmission rights attracted substantial attention not only from generators and LSEs directly exposed to congestion cost risk but also from traders and speculators which perceive these markets as investment opportunities with high risks but also with a high return potential. In order to successfully trade in these markets, one has to understand the underlying engineering elements of the power grid and at the same time to able to measure and manage risk caused by highly volatile nature of transmission congestion. Owners of physical assets use CFTRs as hedging instruments. However, physical assets also mitigate their risk exposure in the markets for CFTRs. Unlike LSEs or power producers, traders often do not have access to physical assets such as generating facilities or served load and therefore have to seek other means of risk management when dealing with CFTRs. Objectives of traders are also different. Rather than using transmission rights as a hedging instrument, they perceive this market as an investment opportunity. Trading in transmission rights is in many ways similar to trading shares on the stock exchange. Understanding engineering elements of the power grid is similar to reading balance sheets or studying market fundamental for a particular firms whose share are of interest. In order to diversify risk, it is important to hold a portfolio of transmission contracts and for that purpose study correlation of congestion costs associated with different CFTRs.

The first part of this paper addresses the applicability of the Markowitz portfolio theory to investing in contracts for transmission rights. Special emphasis is placed on the use of the principal component analysis providing for a dramatic reduction in the size of the optimization problem, selecting subsets of statistically independent contracts and on the mathematical formulation of necessary and sufficient conditions of arbitrage.

In some circumstances, markets for transmission contracts may appear particularly “thin” when a small group of players could affect prices of traded products rather than being price-takes. In the second part of this paper, we consider one such situation in which profit-maximizing bidding strategies are available to large players with significant Allocated Revenue Rights (ARRs). The use of the Supply Function Equilibrium technique provides for a closed form solution in the case of two strategic players competing for transmission rights over a major interface.

2. CFTRs as an Investment Opportunity

2.1. LMP-based Markets for Transmission Rights

The design of markets for transmission rights is tightly linked to the design of the day-ahead market for generated power, or DA energy market. DA energy markets in Northeastern United States such as New England, New York and PJM are based on a combination of Locational Marginal Prices (LMP) and Zonal prices of electricity[1]. Power generated at a particular bus is priced at LPM at that bus. Load Serving Entities (LSEs) delivering energy to end users are exposed to zonal prices. A zonal price is defined as a load weighted average of LMPs for load buses comprising the zone. Transmission rights traded on these markets are based on the spread of LMPs at two different buses, difference between an LMP at a bus and a zonal prices for any zone, or a difference between two zonal prices. Although energy markets in California and Texas (ERCOT) are zonal based, proposals are on the table for these markets to move toward the LMP-based design. The energy market expected to emerge in the Midwest ISO is also LMP-driven. The standard market design concept expressed in white papers and notices issued by the United States Federal Energy Regulatory Commission (FERC) revolves around the concept of LMP-driven markets. In sum, the LMP-based market for transmission rights appears of greater importance to study than any other structure.

There are principally two types of transmission rights products traded, obligation-type contracts known as Firm Transmission Rights (FTRs) in New England and PJM and Transmission Congestion Contracts (TCCs) in New York and option-type products which are effectively derivatives of obligation-type products. This paper focuses entirely on obligation-type products. We will refer to a contract as a Contracts for Firm Transmission Rights (CFTR).

A CFTR from location A to location B is a right and an obligation to buy power at point A and sell power at point B. Point A is also called a Point of Injection (POJ), point B is called a Point of Withdrawal (POW). POIs and POWs are authorized by the Independent System Operator (ISO) administering the market for CFTRs. CFTRs are sold in whole quantities multiple of 1 MW. There are markets for monthly CFTRs with a holding period of one calendar month, seasonal (summer and winter) and annual CFTRs. Settlement for CFTRs is hourly and based on Day-Ahead LMPs such that in a given hour h of day d an CFTR holder is paid the difference in LMPs between POW and POI:

$$X(h,d) = LMP(h,d \mid POW) - LMP(h,d \mid POI)$$

if this difference is positive, or obligated to pay that amount if the difference is negative[2]. CFTRs are sold in single (monthly CFTRs) or multi-round auctions administered by the ISO which collects bids for specific CFTRs and processes them with the OPF software. The number of CFTRs to be sold is subject to OPF constraints that are unknown to bidders. In each round of a multi-round auction, only a portion of CFTRs are sold and the results of each round are published prior to the inception of the subsequent round providing for price discovery. At each round of the auction, the OPF software determines locational forward prices for each location. A CFTR price is then determine as a difference between forward locational prices:

$$p(POI,POW) = F(POW) - F(POI)\,.$$

There are two oppositely directed CFTRs for every pair of locations. If we denote by $A \to B$ a CFTR with POI at A and POW at B, settlement payments for the oppositely directed CFTR add up to zero, i.e., $X(A \to B) = -X(B \to A)$. The same identity holds for prices they receive at the auction, $p(A \to B) = -p(B \to A)$. A negative price for a CFTR implies accepting an upfront payment for

[1] In New York this is known as Location-Based Marginal Price or LBMP.

[2] In reality, the payment is not equal to the difference in LMPs but to the difference in congestion components of LMPs.

taking that CFTR. Two oppositely directed CFTRs form a simple loop. In general, a loop is a finite ordered set of CFTRs such that each subsequent CFTR in the set has POI equal to POW of the preceding CFTR; and the POW for the last CFTR equals POI for the first CFTR in the set. It is easy to see that the sum of revenues along any closed loop of CFTRs equals zero; similarly sum of prices along the closed loop also equals to zero. Thus, in a market with n traded locations, there are $n^2 - n$ traded CFTRs. However, only $n-1$ of those are algebraically independent. All other CFTRs are simply linear combinations of algebraically independent contracts. This is not a small matter. For example, there are over 3000 traded locations in the PJM market resulting in nearly ten million traded CFTRs. However, of those only about 3000 are algebraically independent. As explained later, even that smaller number could be substantially reduced further.

2.2. CFTRs Portfolio Mathematics and Applicability of the Markowitz Portfolio Theory

Consider a market for with N algebraically independent CFTRs. Let $X_j, j = 1,2,...,N$ be total earnings accrued to each CFTR per MWh during its holding period. $X_j, j = 1,2,...,N$ should be regarded as stochastic variables with expected values $E[X_j], j = 1,2,...,N$ and covariance matrix $S = \|Cov(X_i, X_j)\| i, j = 1,2,...,N$. Let us further assume that for each CFTR its purchase price is known and equal to $p_j, j = 1,2,...,N$ and that all these prices are positive. Since for each negatively priced CFTR there exists an inverse one with the positive price, it is always possible to choose a set of algebraically independent CFTRs with positive prices (CFTRs with zero prices are assumed to have no value and excluded from consideration). Holding period returns, expected returns and their covariance matrix then could be expressed as:

$$r_j = \frac{X_j - p_j}{p_j}; \quad E[r_j | p_j] = \frac{E[X_j] - p_j}{p_j};$$
$$\Sigma = \|Cov(r_i, r_j | p_i, p_j)\| = \left\| \frac{Cov(X_i, X_j)}{p_i p_j} \right\|$$

A portfolio of CFTRs is defined as a vector of weights $\mathbf{w}^\mathbf{T} = \|w_1, w_2, ..., w_N\|$ such that the sum of all weights is equal to unity. A positive weight implies purchase of the corresponding positively priced CFTR. A negative weight implies accepting an upfront payment for taking the inverse, negatively priced, CFTR.

A portfolio return $r_P = w^T r$ is equal to

$$r_P = w^T r \tag{1}$$

A portfolio variance V_P is equal to

$$V_P = w^T \Sigma w \tag{2}$$

In general, it seems plausible to apply the Markowitz portfolio theory to in order to choose the portfolio of CFTRs by computing the efficiency frontier and defining a portfolio that, at least in theory, can balance risk and return (see, for example [1]). Historical data and market simulation tools such as GE MAPS and alike could be used to estimate expected earnings $E[X_j], j = 1,2,...,N$. Estimating a covariance matrix is more challenging. However, recall that we deal with a physical power grid. Although it might undergo changes such as construction of new power plants, new transmission lines, or there could be a change in the software used for computing prices, it still seems plausible to rely on historical data in order to estimate the covariance matrix. Historical estimates are much more relevant to the electricity market in general and CFTRs market in particular than the market for financial securities, a more traditional area of application of the Markowitz theory. Composing a portfolio of CFTRs appears to be similar to composing a portfolio of securities. Purchasing a unit of a positively priced CFTR is similar to taking a long position on a security. Purchasing a unit of a CFTR in an opposite direction or an inverse CFTR is similar to taking a short position on a security.

On the other hand, unlike the market for financial securities, CFTRs markets lack liquidity and an efficient price discovery mechanism. Prices tend to change significantly between auctions even to the extent of reversing the sign. However, in a multi-round auction, prices tend to be stable from one round to another. As a result, estimates of prices at which CFTRs may be available are not reliable in the first round, but become relatively reliable starting with the second round. Another problem which makes the use of the Markowitz theory difficult is that an investor willing to participate in the auction for CFTRs should know how many contracts of each particular type could be realistically available for purchase. This information is not directly available; however the number and types of CFTRs available for sale could be estimated by tracking previous auctions and knowing when their results expire.

2.3. Principal Components Analysis

Algebraic independence of CFTRs does not guarantee statistical independence. Indeed, a persistent congestion pattern in the grid could result in a much smaller number of statistically independent CFTRs than the overall number of algebraically independent ones. A principal component analysis could be used to test this hypothesis. Consider the eigenvalue problem for the covariance matrix $S = \|Cov(X_i, X_j)\|$. Since the matrix is symmetrical and positively semi-definite, its eigenvalues are real and non-negative. Each positive eigenvalue has an associated eigen-vector and any two such eigen-vectors are orthogonal.

Let $\lambda_1 > \lambda_2 > ... > \lambda_k$ be all positive eigenvalues of matrix S and $e_1,...,e_k$ be associated normalized eigen-vectors:

$$e_i^T e_j = \begin{cases} 1, \text{ if } & i = j \\ 0, \text{ otherwise} \end{cases}$$

All other *N-k* eigenvalues are assumed to be zeros. Technically, a special statistical test should be implemented in order to determine how many eigenvalues must be considered significantly above zero (see for example [2]). The number of statistically positive eigenvalues would indicate the number of principal components needed as explanatory variables for the entire set of CFTRs. A preliminary statistical analysis of covariance matrices for CFTRs indicates, that in New York the number of principal components could be chosen between 50 and 100. In PJM, the number of statistical components is somewhere between 100 and 200. The number will ultimately depend on the type of CFTRs analyzed. It will vary from one month to another and would be different for seasonal and monthly CFTRs. In sum, using principal component analysis provides for a dramatic reduction in the size of the problem and makes in mathematically tractable.

When k is found, the matrix S could be presented as

$$\tilde{S} = \sum_{i=1}^{k} \lambda_i e_i e_i^T \qquad (3)$$

(3) is known as a spectral representation of the covariance matrix. It is important to note that in theory $\tilde{S} = S$. In reality, these two matrices are not identical, because some eigenvalues were dropped as insignificant. One may argue that for further analysis $\tilde{S}$ must be used instead of S, because the former matrix is cleaned from the influence by insignificant eigenvalues which are more attributable to noise than to valuable information. Although we continue to use the same notation, from now on S stands for the matrix obtained from (3).

Principal components present a useful theoretical notion, but they are not traded in the market. From that perspective, it appears helpful to identify k tradable CFTRs that could be used as a statistically independent basis such that all other CFTRs could be represented as a linear combination of CFTRs forming that basis. These CFTRs will be called the *basic CFTRs or the basis* and the remaining *N-k* CFTRs will be dependent upon the basis. In determining a subset of basic CFTRs, we need to identify columns of the covariance matrix S that are independent. Theoretically, this could be done in many ways. In practice though, it is essential to select the basis in such a way that a corresponding submatrix of the covariance matrix S will have a determinant that is significantly above zero. Otherwise the portfolio optimization problem would be ill-posed. The 'best' or 'most independent' system of vectors in a linear space is the orthogonal system. Thus, a simple way of finding a good subset of independent columns of matrix S would be to find such vectors that best resemble the orthogonal basis $e_1,...e_k$. That could be achieved by first finding the column which has the largest projection on the first eigen-vector, then the column having the largest projection on the second eigen-vector and so on. Columns identified in this manner determine a subset of independent CFTRs.

Assuming that basic CFTRs are numbered from 1 through k and dependent CFTRs are numbered from k+1 through N, linear dependency would indicate that there exists a set of coefficients $m_{ij}, j = 1,..., N-k, i = 1,...,k$ such that

$$X_{k+j} = \sum_{i=1}^{k} m_{ji} X_i \quad j = 1,..., N-k \qquad (4)$$

expressing the revenues on dependent CFTRs as a linear combination of revenues for basic CFTRs[3]. If representation (4) takes place, the covariance matrix for revenues could be presented in the following form

[3] In general, one might assume a constant term in that formula also. However, if revenues for basic CFTRs are zero, that should indicate an absence of congestion in the system and therefore no revenues for non-basic CFTRs also, hence the constant term should be zero. Analysis of actual data for New York confirms this hypothesis.

$$S = \begin{Vmatrix} G & GM^T \\ MG & MGM^T \end{Vmatrix} \quad (5)$$

where G is a $k \times k$ covariance matrix of revenues for basic CFTRs and $M = \|m_{ji}\|$ is the matrix of dependency coefficients used in formula (4) such that in matrix form it could be re-written as

$$x_{dep} = Mx_{ind} \quad (6)$$

where x_{dep} is a *N-k-* dimensional vector of revenues for dependent CFTRs and x_{ind} is a *k*-dimensional vector of revenues for basic CFTRs.

Similar relationships between basic and dependent CFTRs could be established in terms of returns. In order to establish these relationships, let us define two notations that will be used throughout this paper. For any positive integer m define an m-dimensional vector $\mathbf{i}_m = \|1,...,1\|^{\mathrm{T}}$ with all coordinates equal to unity. For a given m-dimensional vector $\mathbf{u}^{\mathbf{T}} = \|u_1,...,u_m\|$ define a diagonal matrix $\Delta(\mathbf{u}) = diag(u_1,...,u_m)$ whose elements on the main diagonal are coordinates of the given vector and all elements outside of the main diagonal are zeros. Obviously,

$$\Delta(\mathbf{u})\mathbf{i}_m = \mathbf{u} \quad \text{and} \quad \mathbf{i}_m^T\Delta(\mathbf{u}) = \mathbf{u}^T .$$

For revenues and prices associated with independent and dependent CFTRs the following identities will hold:

$$\begin{aligned} x_{ind} &= \Delta(p_{ind})r_{ind} + p_{ind} \\ r_{ind} &= \Delta^{-1}(p_{ind})x_{ind} - \mathbf{i}_k \\ x_{dep} &= \Delta(p_{dep})r_{dep} + p_{dep} \\ r_{dep} &= \Delta^{-1}(p_{dep})x_{dep} - \mathbf{i}_{N-k} \end{aligned} \quad (7)$$

The covariance matrix for revenues S and covariance matrix for returns Σ are linked in the following way:

$$S = \Delta(p)\Sigma\Delta(p) \quad (8)$$

Moreover, matrix Σ could also be presented in the block form

$$\Sigma = \begin{Vmatrix} \sigma & \sigma L^T \\ L\sigma & L\sigma L^T \end{Vmatrix} \quad (9)$$

where

$$\boldsymbol{\sigma} = \Delta^{-1}(p_{ind})G\Delta^{-1}(p_{ind}) \quad (10)$$

$$L = \Delta^{-1}(p_{dep})M\Delta(p_{ind}) \quad (11)$$

A linear relationship for returns can now be expressed in the following way:

$$r_{dep} = Lr_{ind} + \xi \quad (12)$$

where

$$\xi = L\mathbf{i}_k - \mathbf{i}_{N-k} \quad (13)$$

We will call vector ξ an *arbitrage vector* for reasons that become apparent from the subsequent section of the paper.

2.4. Arbitrage

Day Ahead and CFTRs markets are related but not fully integrated. Even if the bidding behavior in the market for CFTRs is purely rational and based on the expectation of future LMPs, the software used to compute forward prices underlying prices of CFTRs is different from software used to form LPMs. In sum, it is likely that CFTRs prices and expected revenues are not aligned and might provide opportunities for the arbitrage. In this section, we provide a formal definition of arbitrage in the market for CFTRs and formulate criteria of its existence. It is intuitively obvious that if all CFTRs are priced according to expected revenues, no arbitrage should be possible. In other words, it is unlikely to find any arbitrage opportunities if prices for dependent and independent CFTRs are linked by a formula similar to (6):

$$p_{dep} = Mp_{ind} \quad (14)$$

As we demonstrate in this section, equation (14) sets necessary and sufficient conditions for the absence of arbitrage opportunities. If and only if this equation holds, no arbitrage is possible. Using (7) - (12), it is easy to demonstrate that (14) is equivalent to arbitrage vector being zero:

$$\xi = 0.$$

Thus, we will show that arbitrage is impossible if and only if the arbitrage vector is equal to zero.

By definition, arbitrage allows an investor to receive an advance payment of 1 by taking a net short position and at the end of the holding period receive a non-negative return (revenues from positive CFTRs will balance payments made for negative CFTRs) with no risk associated with it (zero variance). In other words, it is possible to form a portfolio $\mathbf{w}^T = (w_{ind}^T, w_{dep}^T)$ where w_{ind}, w_{dep} correspond to weight vectors for independent and dependent CFTRs, respectively, such that

$$w_{ind}^T \mathbf{i}_k + w_{dep}^T \mathbf{i}_{N-k} = -1 \tag{15}$$

resulting in zero portfolio variance

$$V_P = w_{ind}^T \sigma w_{ind} + 2 w_{ind}^T \sigma L w_{dep} + w_{dep}^T L^T \sigma L w_{dep} = 0$$

and a return of no less than unity

$$E(r_P) = w_{ind}^T r_{ind} + w_{dep}^T r_{dep} \geq 1.$$

Consider the following substitution

$$w_{ind} = z - L w_{dep}. \tag{16}$$

Using this substitution, we obtain that

$$V_P = z^T \sigma z \tag{17}$$

and therefore portfolio variance is dependent only on the covariance submatrix for basic CFTRs. At the same time, portfolio return is equal to

$$E(r_P) = z^T r_{ind} + w_{dep}^T \xi \tag{18}$$

and portfolio normalization condition (15) transforms into

$$-w_{dep}^T \xi = -1 \tag{19}$$

By definition, matrices σ and G are positively definite and therefore zero variance could be achieved if and only if $z = 0$. On the other hand, equation (19) for w_{dep}^T would have a solution if and only if $\xi \neq 0$. Finally, holding $z = 0$ and solving (19)guarantees that $E(r_P) = 1$, i.e. zero revenues in exchange for advanced payment of unity. Thus, if $\xi \neq 0$, it is possible to form an arbitrage portfolio in two simple steps. First, we find a set of weights for dependent CFTRs satisfying (19). Second, we set $z = 0$ and substitute this condition addition to (16) which yields the formula for weights for independent CFTRs:

$$w_{ind} = -L w_{dep}.$$

If an arbitrage vector has only one non-zero coordinate, an arbitrage portfolio is unique, because equation (19) will have a unique solution. If arbitrage vector has u+1 non-zero coordinates, there will be an infinite set of arbitrage portfolios all within an u-dimensional plane in the linear space of all possible weights.

3. Strategic Bidding in Presence of Auction Revenue Rights

3.1. Auction Revenue Rights

In general, the proceeds from CFTRs auctions are distributed between LSEs in order to offset their congestion costs of serving the load. However, the actual distribution mechanism varies between markets. In New York, for example, there is no targeted allocation of particular CFTRs to affected loads. In contrast, in the New England market, auction revenues associated with particular CFTRs are allocated between LSEs that are most affected by congestion costs on corresponding paths. This targeted revenue allocation mechanism is known as Auction Revenue Rights (ARRs). ARRs create an interesting incentive for an LSE to participate in an auction for a corresponding CFTR. Indeed, if the price for the CFTR is high (i.e., well above the expected cost of congestion), the LSE would prefer not to buy the CFTRs at the auction and receive a large payment through the allocation mechanism. If, on the other hand, the auction price is low (i.e., well below the expected cost of congestion), the LSE would prefer to buy as many such CFTRs as possible at the auction and then collect congestion revenues in the Day-Ahead market. Finally, in some cases, an LSE could be a dominant player for a particular type of CFTRs and therefore might be able to influence the price for these CFTRs at the auction. This section presents a stylized model of two dominant LSEs with ARRs for a particular type of CFTRs.

3.2. Simplifying Assumptions

We assume a two-node system with one type of CFTRs over a major interface (e.g., from New England Hub to NEMA Zone). The total quantity of such CFTRs is given and equal $S_{\max}$. We assume that there are two different categories of players in that market. In the first category there are two large LSEs that are awarded ARRs over the interface and referred to as Strategic Players. In this paper, we assume that these two Strategic Players are awarded ARR quantities of a_1, a_2, respectively. The second category is called Other Players. Other Player include small LSEs eligible for ARRs all accounting for the total of a_3, as well as generating companies and speculators not eligible for ARRs. Behavior of Other Players is the most significant uncertainty factor in the analysis of this market. We assume that

$$S_{\max} = a_1 + a_2 + a_3$$

3.3. The Game-theoretical Model of Bidding Behavior

We assume that each of the two Strategic Players would bid into the market a continuous monotonically non-ascending demand curve $d_i(p)$ $i = 1,2$ and where p is the price bid into the auction. While Strategic Players attempt to predict the behavior of each other, they would consider the bidding behavior of Other Players as an uncertainty factor in their prediction. From their perspective, the size of the market (i.e., quantity of CFTRs) available to them is uncertain at any price. In total, two Strategic Players could obtain S CFTRs. All we know is that S could be as high as $S_{\max}$ but the low limit is not known and could be as low as zero.

The payoff function for a Strategic Player can be represented in the following form:

$$\pi_i[d_i] = pa_i + (x - p)d_i(p) \quad i = 1,2 \tag{20}$$

where x is the expected level of day-ahead congestion revenues per unit of CFTRs held. The first term in (20) reflects the ARR allocation and represents the payment received by the Player at the end of the auction per each allocated unit of CFTRs assuming that the auction cleared at price p. The second term indicates that for each unit of CFTRs purchased at the auction, the Player will receive net revenues equal to the difference between congestion revenues in the Day-Ahead market x and price p paid at the auction. The objective of each player is to maximize its payoff function. The market clearing condition is

$$d_1(p) + d_2(p) = S \quad 0 \le S \le S_{\max} \tag{21}$$

The advantage of using bidding curves is that it allows Strategic Players to achieve the optimal (equilibrium) outcome in this game without knowing the behavior of Other Players. An equilibrium pair of demand curves would yield the optimal outcome for any level of S. This framework is similar to the Klemperer-Meyer formulation [3] of the supply function equilibrium (SFE) with uncertain demand, although in this case we are dealing with demand curves facing uncertain supply and a payoff function with a structure that differs from that considered by Klemperer and Meyer. However, the analytical technique they used to derive equilibrium conditions is fully applicable to this case. Similarly, to the SFE problem, equilibrium conditions on demand bid curves could be expressed as a pair of differential equations:

$$d_1'(p) = \frac{a_2 - d_2}{x - p} \qquad d_2'(p) = \frac{a_1 - d_1}{x - p} \tag{22}$$

In order to derive these equations, consider the payoff optimization problem of Player 2 subject to its residual demand $d_2(p) = S - d_1(p)$:

$$\max\left[pa_2 + (x - p)(S - d_1(p)\right]$$

Differentiating this payoff function by price and equating the derivative to zero yields the first equation in (22). Repeating the same process for Player 1 would yield the second equation.

If the system of differential equations (22) yields a pair of monotonically non-increasing functions, such a solution would represent a Nash equilibrium in the game (20)-(21).

It is important to note that equation (22) has a singularity point when the denominator equates to zero ($p = x$). As a result, most of trajectories in system (22) cannot pass the singularity point (and are non-monotonic) and thus would not represent a feasible set of bidding strategies. There is, however, a special family of monotonically descending solutions that pass through the singularity point.

$$d_i(p) = a_i + \frac{d_1(0) - a_1}{x}(x - p)\ . \tag{23}$$

These linear demand curves are the only feasible solutions of system (22). They form a one-parameter family given that at any price the following identity must holds to ensure feasibility

$$d_1(p) - d_2(p) = a_1 - a_2 \quad (24)$$

and therefore $d_1(0) - d_2(0) = a_1 - a_2$. The boundary condition for one of the bid curves uniquely defines the boundary condition for another bid curve and therefore defines both curves.

Formula (23) shows that a Strategic Player would buy the allocated ARR quantity if an only if the price is equal to expected congestion cost. If the price exceeds expected revenues, a Strategic Player would buy less than allocated quantity. At very high prices (23) indicates that the demand becomes negative, i.e. the player would be selling CFTRs rather than buying them. Similarly, according to (23), at lower prices, the player would buy more than its allocated ARR quantity.

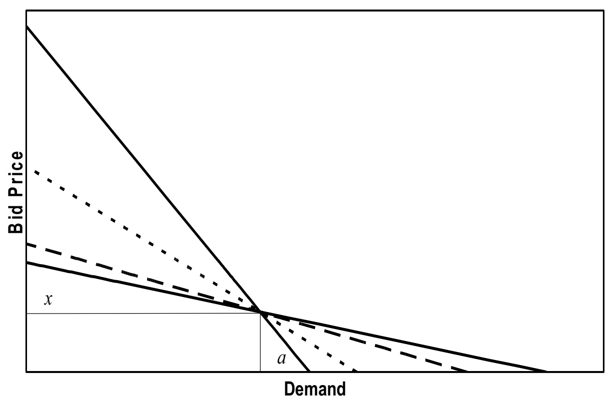

Figure 1. Equilibrium Demand Curves

In general, the game has an infinite set of equilibria, as shown on Figure 1 which depicts a set of demand curves for one player. Let us now explore how the choice of the boundary condition (and a corresponding equilibrium) affects the market outcome and the payoff function of each player. First, let us consider the aggregate demand curve of Strategic Players:

$$D(p) = d_1(p) + d_2(p) = a_1 + a_2 + 2[d_1(0) - a_1]\frac{(x-p)}{x}$$

Solving the market clearing equation $D(p) = S$ for price yields the following formula for the clearing price as a function of S

$$P(S) = x + x\frac{a_1 + a_2 - S}{2[d_1(0) - a_1]} = x + \alpha(a_1 + a_2 - S) \quad (25)$$

where α is the price slope of the aggregate demand curve and

$$\alpha = \frac{x}{2[d_1(0) - a_1]}. \quad (26)$$

As (26) indicates, the boundary value $d_1(0)$ uniquely identifies the price slope of the aggregate demand curve and therefore there is a one-to-one correspondence between the choice of the equilibrium and the price slope α. Expressing the payoff as a function of α allows us to analyze how the payoff depends on the equilibrium chosen. It is easy to demonstrate that if P is the price cleared in the auction, the payoff expressed as a function of α will be equal to

$$\pi_i(P(\alpha), \alpha) = xa_i + \frac{(x - P(\alpha))^2}{2\alpha} \text{ where } i = 1, 2 \quad (27)$$

Equation (27) recognizes the fact that the market clearing price depends upon α. Let us assume that the residual supply faced by Strategic Players could be expressed as a monotonically ascending function $p(S)$. Let us denote the equilibrium slope of the supply function as β such that

$$\beta = \left(\frac{dp}{dS}\right)_{p=P(\alpha)}.$$

Using this definition and equation (25), we get that

$$P'(\alpha) = \frac{\beta}{\alpha(\alpha+\beta)}(P(\alpha) - x). \quad (28)$$

By differentiating (27) by α and using (28) we obtain that

$$\frac{d\pi_i}{d\alpha} = \frac{\beta - \alpha}{2(\alpha+\beta)\alpha^2}(P(\alpha) - x)^2 \quad (29)$$

and therefore

$$\frac{d\pi_i}{d\alpha} > 0 \text{ if } \alpha < \beta; \quad \frac{d\pi_i}{d\alpha} < 0 \text{ if } \alpha > \beta.$$

Formula (29) indicates that payoffs of both players would be highest if they coordinate on the equilibrium with the price slope matching the price slope of the residual supply function ($\alpha = \beta$). However, as stated earlier, behavior of Other Players and therefore the residual supply function is the major source of uncertainty for Strategic Players. On the other hand, the following conjecture appears helpful in addressing this uncertainty. Consider two major possibilities with respect to the overall market for CFTRs in question, the Strong Market hypothesis vis-à-vis the Weak Market hypothesis.

The Strong Market Hypothesis. Under this hypothesis, Other Players place a very significant demand for CFTRs such that at relatively low prices their demand for CFTRs will substantially exceed the total number of CFTRs available. In this case the market will likely clear at a price that exceeds *x*. Moreover, the slope of the residual supply curve will likely be very high. Facing such a market, Strategic Players will be better off by coordinating on the equilibrium with a high value of α. In general, this slope is unbounded and in absence of any information on the residual supply curve, Strategic Players would not know which slope to choose. A plausible strategy though might be to coordinate on an aggregate demand curve which crosses the horizontal axis at the demand quantity $S_{\max}$ as shown on Figure 2. This strategy indicates that if the price is zero, players are willing to split the entire market but are not interested in getting more CFTRs than just that quantity:

$$d_1^{str}(0) + d_2^{str}(0) = S_{\max}.$$

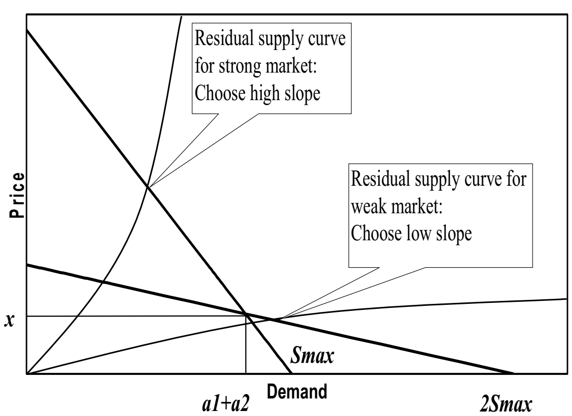

Figure 2. Composite aggregate demand curve for strong and weak markets

Combining the above condition with identity (24), we get

$$d_1^{str}(0) = \frac{1}{2}(S_{\max} + a_1 - a_2),$$

$$d_2^{str}(0) = \frac{1}{2}(S_{\max} + a_2 - a_1),$$

$$\alpha^{str} = \frac{x}{S_{\max} - (a_1 + a_2)}$$

The Weak Market Hypothesis. Under this hypothesis, Other Players show very little interest in the CFTRs in question (for example, neither generators, nor speculators offer strong bids) and the two Strategic Players will share the bulk of the market for that CFTR. In this case, the strategic players will be setting the price and they can drive this price down, well below x. Facing a weak market, Strategic Players will be better off by coordinating on the equilibrium with a low level of α. Again, the price slope is unbounded from below. However, another plausible strategy would be to assume that each Strategic Player is willing to purchase all available CFTRs if price is zero. In this case their aggregate demand will intersect the demand axis at the $2S_{\max}$ level.

$$d_1^{weak}(0) + d_2^{weak}(0) = 2S_{\max}.$$

Combining that with identity (24), we get

$$d_1^{weak}(0) = S_{\max} + \frac{1}{2}(a_1 - a_2),$$

$$d_2^{weak}(0) = S_{\max} + \frac{1}{2}(a_2 - a_1),$$

$$\alpha^{weak} = \frac{x}{2S_{\max} - (a_1 + a_2)}.$$

The Composite Strategy. It is possible for Strategic Players to construct a composite bidding strategy which would allow them to face both types of the market. Indeed, recall the observation made earlier that the strong market is likely to clear at a price that is above *x*, whereas the weak market will likely clear at a price that is below *x*. It seems logical to use one bidding curve to face the strong market and another bidding curve to face the weak market.

As shown on Figure 1, all equilibrium bidding curves intersect at the point (a_i, x) on the quantity-price plane. This means that taking one curve at lower prices and another curve and higher prices would still yield an equilibrium curve, now non-linear with a kink at (a_i, x). Indeed, the resulting bidding curve would still be continuous, monotonically descending and satisfying differential equations (22) at each point except the singularity point. This piece-wise linear bid curve would be targeting the weak market at lower prices and targeting the strong market at higher prices.

The formula for this composite strategy takes the following form:

$$d_i(p) = \begin{cases} a_i + \dfrac{2S_{\max} - (a_1 + a_2)}{2x}(x - p) & \text{if } p < x \\ a_i + \dfrac{S_{\max} - (a_1 + a_2)}{2x}(x - p) & \text{if } p \geq x \end{cases} \quad (30)$$

The use of those composite bid curves could be the best bidding strategy for each Strategic Player. Indeed, should the market appear weak, both strategic players will be well prepared to face each other, reasonably split the market and drive the price down to their benefit. Should the market appear strong, again, both Strategic Players will be well prepared for that outcome and engage in the best bidding strategy effectively as price-takers.

4. References

1. Bodie, Zvi, Alex Kane, Alan J. Marcus. Investments. Fifth Edition, McGraw-Hill/Irwin, 2002.
2. Jobson J.D. Applied Multivariate Data Analysis, Volume II: Categorical and Multivariate Methods. Springer-Verlag, New York, 1992.
3. Klemperer, Paul D. and Margaret A. Meyer, (1989) Supply Function Equilibria, *Econometrica*, 57, p. 1243-1277.

