Multi-Agent Systems and Microeconomic Theory: A Negotiation Approach to solve Scheduling Problems in High Dynamic Environments

Hans Czap, Marc Becker
University of Trier
{cz, mb}@wiinfo.uni-trier.de

Abstract

Microeconomics offer a far developed theory on the subject of rational choice. This theory is applied to a multi-agent system, which has been modeled in order to establish schedules for operating theatres in general hospitals. A concept based on conjoint analysis is introduced in order to measure preferences of involved individuals and to establish corresponding individual utility functions. Aggregation of individual preferences to find a final compromise schedule is demonstrated following the Nash bargaining solution of game theory.[1]

1 Introduction

Multi-agent Systems (MAS) are an appealing paradigm to support different variants of Computer Supported Co-operative Work (CSCW). The following paper concentrates on environments that simultaneously show high dynamics and high complexity. Typical examples are virtual or quasi-virtual[2] organizations. The Application scenario considered in this paper relates to the scheduling of operating theatres in general hospitals.

In the MAS-setting each individual as well as each separately scheduled resource, for example each physician or each operating room, is represented by an intelligent software agent. This agent represents the specific constraints and the relevant preferences of its principal. The final schedule will be the result of a negotiation process yielding as compromise solution an equilibrium point in the corresponding game situation.

The approach presented is based on microeconomic theory, since microeconomics mainly addresses individual behaviour and asks how it affects social outcomes. Game theoretic approaches, i.e. Nash equilibrium point theory, particularly show fruitful in order to arrive at compromise solutions.

For the mentioned application domain "scheduling of hospital operating theatres" there is shown:

(i) The potential of so called policy-agents, which are able to express individual goals and preferences of the involved parties.

(ii) The fruitfulness of combining MAS-theory with microeconomic-theory in order to arrive at a pareto-optimal reallocation of resources and in finding a fair compromise solution by game-theoretic approaches.

2 The Application Scenario

2.1 General Aspects of Hospital Scheduling

Scheduling in hospitals is done in two phases, in the large and the small. In the long run, patients get an approximate date of operation, which in some cases – if it is not vitally necessary - might be several months ahead. In Germany, as in other European countries, the refunding system of hospitals by public health insurances is highly regulated and limited on a yearly basis, yielding to a backlog of wished operations. This backlog of several months essentially is caused by the limiting effects of the available yearly budget, which yields to a steady number of operations per week independent of actual need.

A few days before the fixed date of the operation, patients enter the hospital to be prepared. Public health insurances are very eager to shorten the time of patient

1 This paper has been supported by the German National Research Foundation "Deutsche Forschungsgemeinschaft" as part of the main research program "Multi-agent Systems and their Business Applications".

2 The term quasi virtual organization hints at the examined application domain: hospitals with centralized operating theatres, that, while centred in one place effectively consist of highly independent departments and individuals with different preferences that are required to work together to achieve a common goal.

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stay at hospitals before operation and will not refund any overtime. Thus, after admission of a patient, there is a definite need to do the planned surgery operation as soon as possible. In the short run, scheduling of patients will happen from one day to the next.

In the following, we will concentrate on this short run scheduling of operation theatres. Short term scheduling is well known for being a process with an outcome that is highly dependent on situational variables:

- The duration of an operation in some cases cannot be determined beforehand. Thus, there is uncertainty of needed time.
- Also the specific tasks to be performed during an operation may depend on situations not observable during diagnosis (planning phase).
- The daily schedule often will be interrupted by incoming emergency cases. The frequency of emergencies depends on the medical department. For example, schedules of orthopedic departments seem to be very stable, whereas schedules of surgical or neurosurgical departments in general have to take incoming emergencies into account.

If a patient is scheduled for operation today, it may happen that by incoming emergencies, he suddenly will find himself in a position to be rescheduled for next day.

In the analyzed hospital this rescheduling results from a contract between nurses and hospital management that limits regular operating time from 8.00 o'clock in the morning to 4.00 o'clock in the afternoon at the latest. Management had been forced to sign this contract, since dissatisfaction of nurses because of working overtime on a regular basis had been overwhelming.

On the other hand, any postponing of an operation from one day to the next is a major source of dissatisfaction of patiens, physicians, and management. It essentially contributes to the increase of the backlog and causes possible shortcuts of refunding since public insurances will not pay overtime in front of surgery operations.

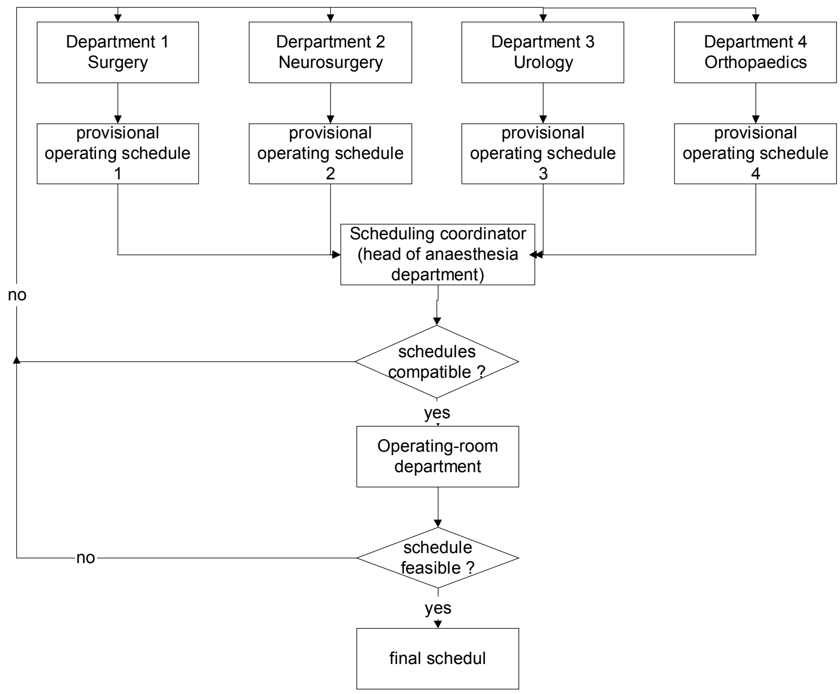

Figure 1: Conventional process of scheduling

2.2 The Conventional Process of Hospital Scheduling, Preferences and Constraints

Classically each medical department had its own operating room. This has been changed in actual hospital organizations where operating rooms are combined to form a centralized operating theatre. Thus, the operating rooms form an independent organizational complex, which is used by the different medical departments. This leads to a better utilization of rooms, devices and personnel as well as to higher flexibility of planning and reaction to emergencies. On the other hand, transaction costs for manual scheduling dramatically rise, which causes the need for automated planning systems.

The conventional manual planning process relies heavily on direct communication between different departments and follows strict formal rules.

In the observed case, the complexity of the planning procedure is reduced by a sequential procedure. In the first step, each medical department gets specific operating rooms at their daily disposal. In the second step, the department heads make a preliminary schedule of planned operations separately for each operating room. This preliminary schedule will obey department specific preferences (department policies). Some of them will vary from department to department. Typical examples of these policies are [CzBeSch 02]:

- Patients to be scheduled will be selected by urgency and availability of surgeries.
- Operation of children in general will be done before operation of adults. This reduces the probability of delays and reschedules for children.
- Performance of septical operations will be done at the end of day. This measure results from the time needed to disinfect the operating room afterwards, i.e. the room cannot be used immediately for a subsequent operation.

These preliminary schedules are transmitted to the operating theatre coordinator. In the conducted case study this function had been fulfilled by the head of anesthesia[3]. The operating theatre coordinator first checks compatibility of the preliminary plans with scarce resources. For example, in some cases the intensive care unit has shown to be a bottleneck. Also, some operations need a microscope that is only available in a limited number.

At the next step, the operating theatre coordinator decides about needed anesthetists and makes an assignment. If incompatibilities occur, the coordinator has to contact the involved departments and negotiate a compromise solution.

Finally the operating theatres plans are handed to the nurse personnel who have to assign the needed nurses. Again, incompatibilities respectively a shortcut of available operating room personnel has to be resolved by negotiation between the operating room department and the involved medical departments. The result of this negotiation process is the final operating room schedule.

This sequential planning procedure leads to measurable dissatisfaction of the lower working ranks, especially the operating room personnel, which can only react to the decisions made in earlier planning stages. Also, these operating room nurses like to have a definite time, when their regular work finishes in the evening. They are strongly dependent on the punctuality of the involved physicians, who in general are not very sensitive to the schedule. Therefore, a poor working climate results showing high fluctuation of operating room personnel[4]. Figure 1 depicts the different planning steps.

3 The head of the anaesthesia department (or another physian of this department) seems to be a rational choice for coordinating the different schedules, because he combines medical knowledge with direct involvement in the centralized operating complex.

2.3 Requirements for a Multi-Agent System

The scheduling problem presented here leads to different requirements for a multi-agent system. First, human interaction should be reduced to a minimum, thus reducing the needed time to resolve incompatibilities by phone. Second, the planning procedure should be done simultaneously, thus allowing for more flexible solutions, which for example utilize the available operating rooms more intensively. Third, a multi-agent system should take care of the individual interests of the involved personnel whenever possible. This will give strong evidence for the acceptance of the planning system and will allow a better degree of satisfaction of staff.

The paper will concentrate on preferences of involved organizational units and individuals and the question how these preferences can be combined to yield an acceptable solution. The case study above has shown a subtle preference structure of the scheduling process, partly expressed by the sequence the process is conducted, partly explicitly expressed by the involved parties, but not taken care in a satisfactory manner.

2.4 Preferences and Desires

To describe motives and behavior of persons, one should distinct between preferences and desires. Desires form a simpler and more basic notion than preferences. A desire for something involves only one object and refers to a pro-attitude toward this object, whereas a preference for one object over one or more other objects involves at least two objects and indicates that the decision making person assigns a higher priority to his pro-attitude toward the first object than towards the others. Preferences therefore not only indicate the priority a person assigns towards his various desires, but indicate also the relative importance, which the decision maker assigns to his objects of desire [Ha 97].

Because the utility function of a person can be defined only in terms of his preferences[5], economic theory con-

4 A first examination of the relationship between the organisational structure of operating theaters and working climate was done by [Se 84].

5 Harsanyi [Ha 97] states that we cannot assign a higher utility to an option A than to an option B unless we know that the individual prefers A to B. We cannot infer his preference from facts about the person's desires alone. Thus, even when the person has a much stronger desire for A then for B, it does not follow automatically that he really will

centrates on people's preferences rather than on desires. The following chapter, therefore, deals with microeconomic theory. This will lead to the foundations needed to model decision making agents.

3 Application of the Microeconomic Theory of Decision Making to Hospital Scheduling

„Economics is the science which studies human behaviour between ends and scarce means which have alternative uses".
(Lionel Robbins 1935)[Ro 35, p. 16]

3.1 Preferences and Utility

The microeconomic theory of decision making, mostly referred to as the theory of choice, allows economists to "look beyond" the well known market demand and supply curves [which are essentially price-consumption and price-sales curves] and to derive these curves based on a theory of individual behavior. But the theory of choice is also interesting in its own right. It provides a general framework for understanding important aspects of human behavior and can be applied to a much wider range than simple price changes.

Basically, utility is understood as a measure of a persons well being. This concept dates back to the eighteenth and nineteenth century and relies on the work of British utilitarian philosophers and economists. In their view the concept of well being relates directly to happiness, i.e. they tried to explain a person's behavior as his attempt to maximize his well being and happiness all at once.

Facing the difficulties of measuring a utility function, modern economists try to explain people's behavior in terms of preferences. In so far, the utility function of a person becomes a convenient mathematical representation of his preferences [Ha 97, p. 130f].

3.2 Stages of Individual Decision Making

Figure 2 shows the three steps an individual will perform in reaching a decision according to the theory of rational choice:

- First, preferences of each individual i with respect to given alternatives ϕ_1 and ϕ_2 must be identified, i.e. any individual i has to provide some order:

choose option A rather than B in every imaginable situation. The person may have good reasons to resist his stronger desire for A and take option B depending on circumstances, i.e., instead of following their strongest desire, people sometimes prefer to overrule this desire and may have very good reasons for doing so.

$$\phi_1 \succ_i \phi_2 \text{ or } \phi_1 \prec_i \phi_2 \text{ or } \phi_1 \sim_i \phi_2.$$

  In the case of hospital scheduling we show how to arrive at a cardinal utility function $u_i(.)$. In this case, preferences are implied by the respective value of the utility function, i.e.:

$$\phi_1 \succ_i \phi_2 \text{ if and only if } u_i(\phi_1) > u_i(\phi_2) \text{ and}$$
$$\phi_1 \sim_i \phi_2 \text{ if and only if } u_i(\phi_1) = u_i(\phi_2).$$

- Second, one has to identify the set of available alternatives. In general, this corresponds to observing the imposed constraints. In the typical situation of consumption theory these constraints are given by the prices of the goods and the budget constraint of the individual. In the case of hospital scheduling, one has to deal with scarce resources, like the number of available surgeons and anesthetists, as well as legally imposed constraints.

  For example, any surgery operation must be done by a certified surgeon and assisted by a certified anesthetist. Also, in the regular case at least two operating room nurses are needed, one as an assistant for the surgeon, the other to get the needed instruments.
- In the third step, the combination of the decision space and the preference structure of the software agent form the base for maximizing utility of the individual agent.

3.3 Collaborative Decision Making

- **Stability of individual preferences**

In the first step of a collaborative setting, each individual is considered separately. It is assumed, that the individuals find themselves in a position to make up their individual preferences, which are assumed to be stable for the decision at hand. Stability of individual preference structure seems to be given in often repeated situations, where environmental variables do not change heavily, and, consequently, specific involvement is not required. For the scheduling task to be considered in this paper, this assumption fits very well.

In general, individual preferences will cause conflicts. For example, if S_1 and S_2 are valid schedules, individual i might prefer S_1 to S_2, $S_1 \succ_i S_2$, but individual j S_2 to S_1, $S_2 \succ_j S_1$. This situation is covered by the impossibility theorem of Arrow [Ar 51/63], where, under very reasonable assumptions, he has shown, that individual preferences cannot be aggregated to a collective preference without violating the individual ones.

Conflicting individual preferences offer the possibility of negotiations.

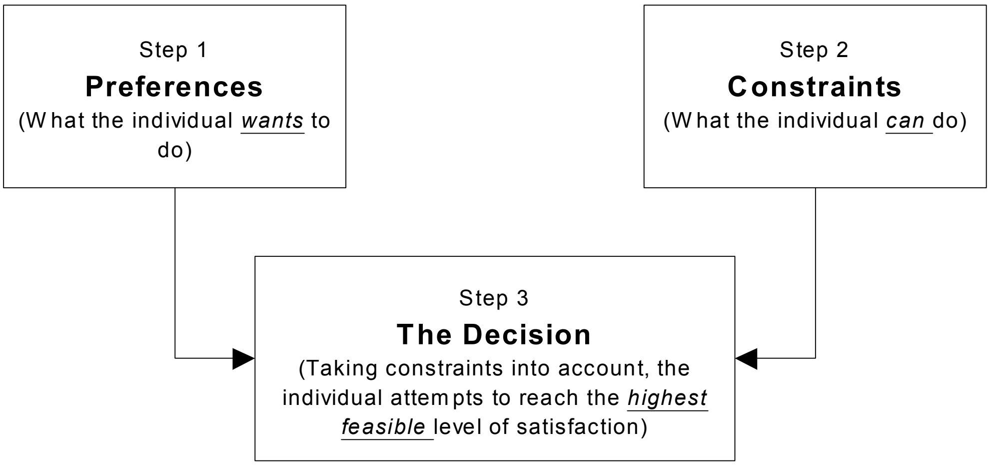

Figure 2: A model of individual decision making

- **Distribution of property rights**

  Any negotiation may be ended without a compromise solution, if one of the participating individuals does not show further interest. In decision theoretic terms, this means that the set of alternatives contains the alternative "status quo", i.e. no action taken.

  For the hospital scheduling problem the existence of a valid initial schedule S_0 is supposed and necessary, which can be determined by any conventional scheduling algorithm. S_0 realizes an assignment of individuals, resources and time slices, and thus defines the property rights each individual has got. S_0 establishes the "status quo"-solution.

- **Pareto-optimality by negotiations**

  Improvements of S_0 automatically will result by negotiations, if S_0 is not pareto-optimal. That is, if there is an alternate schedule S_1 and with respect to any individual i S_1 is at least as preferred as S_0, i.e. $S_1 \rangle_i S_0$ or $S_1 \sim_i S_0$, and there exists at least one individual j having strict preference, i.e. $S_1 \rangle_j S_0$.

  The assumption for this negotiation is as follows: As far as any individual i is indifferent with respect to two alternatives S_m and S_n, it will not opposite if S_m is exchanged by S_n, and, if any improvement is possible, it will do so.

3.4 Scheduling Decisions and Rationality of Preferences

As stated above, in a typical hospital one has different physicians, for example surgeons, neuro-surgeons, orthopedists etc. Essentially, in the hospital setting, one has to deal with the following sets:

D = set of surgeons (to simplify things slightly, we assume only one kind of surgeons),
A = set of anesthetists,
N = set of nurses,
P = set of patients,
T = set of time intervals,
R = set of operating rooms.

Basic elements of the schedule are operations ϕ, which are an assignment of a surgeon, an anesthetist, two nurses, a patient, a time slice, and an operating room. On the micro-level, to a given schedule S_1 containing the operation ϕ_1 a decision about an exchange of ϕ_1 with an other assignment ϕ_2 will take place. In so far, the assignment ϕ will be called an alternative. Any alternative ϕ is given as an element of the set Φ,

$$\Phi = D \times A \times N^2 \times P \times T \times R.$$

The sets D, A, N and P consist of natural persons. Any element of the union of these sets D, A, N and P will be called an individual or a party of the decision process.

As usual in decision theory the following assumptions about preference orderings for any involved individual are made:

- **Completeness of the order**

  This assumption states, that for any two alternatives ϕ_1 and ϕ_2 of Φ every individual i has a preference order (~ means indifference):

  $$\phi_1 \rangle_i \phi_2 \text{ or } \phi_1 \langle_i \phi_2 \text{ or } \phi_1 \sim_i \phi,$$
  $$i \in I = D \cup A \cup N \cup P,$$

  where $\phi_1 \rangle_i \phi_2$ means, that individual i prefers ϕ_1 to ϕ_2.

  Since the cardinality of Φ has a very big size, the completeness assumption causes problems how to determine preference for any alternative. Instead of interrogating every pair of alternatives, which seems to be impossible, use of conjoint analysis from marketing theory is suggested in the subsequent chapter. This method will reduce the necessary effort considerably.

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- **Transitivity (three-term consistency)**
  The transitivity assumption states, that for any individual and any three alternatives ϕ_1, ϕ_2 and ϕ_3 of Φ the following implication holds:
  $$\text{if } \neg(\phi_1 \prec_i \phi_2) \text{ and } \neg(\phi_2 \prec_i \phi_3) \text{ then } \neg(\phi_1 \prec_i \phi_3),\quad i \in I.$$
  This transitivity requirement simple states for individual i, that, if ϕ_1 is not minor to ϕ_2, i.e. if ϕ_1 will be preferred to ϕ_2 or individual i is indifferent to ϕ_1 and ϕ_2, and if ϕ_2 is not minor to ϕ_3 , then ϕ_1 will not be minor to ϕ_3. Normally, this is seen as an important aspect of rational behavior.

As Debreu [De 54] has shown, slight additional assumptions are sufficient to show the existence of a numerical function $u_i(.)$, called utility function of individual i, which is compatible with the preference ordering of i.

4 Determination of Utility-Functions

4.1 Utilities with respect to alternative operation assignments

In order to determine for any individual i its utility-function u_i one needs a complete ordering of the set of alternatives Φ, as it has been shown in the last chapter. Remembering that Φ is given by

$$\Phi = D \times A \times N^2 \times P \times T \times R$$

the number of elements (cardinality) of Φ is calculated by the product of cardinality of its components. Also for small hospitals, this number turns out to be very large. Therefore, there is no chance to ascertain a complete ordering of alternatives by the direct comparison of any two alternatives ϕ_1 and ϕ_2. Instead of that, the alternate approach of conjoint analysis is suggested.

Conjoint analysis as an instrument for measuring consumer preferences is frequently used in the marketing sector for over twenty years. It can be defined as "any de-compositional method that estimates the structure of a consumer's preferences given his or her overall evaluations of a set of alternatives that are pre-specified in terms of levels of different attributes"[GrSr 90, p. 4]. Conjoint analysis involves the measurement of psychological judgments, such as consumer preferences as well as perceived similarities or differences between alternatives. The term "conjoint analysis" implies the study of joint effects, which result in marketing applications from the influence of multiple product attributes on product selection.

In the hospital setting, conjoint analysis implies the assumption, that the value (utility) $u_i(\phi_k)$ of any alternative

$$\phi_k = (d_k, a_k, n_{1,k}, n_{2,k}, p_k, t_k, r_k) \in \Phi = D \times A \times N^2 \times P \times T \times R,$$

is determined by the sum of the values (utilities) of the alternatives, i.e.

$$u_i(\phi_k) = u_{D,i}(d_k) + u_{A,i}(a_k) + u_{N1,i}(n_{1,k}) + u_{N2,i}(n_{2,k}) + u_{P,i}(p_k) + u_{T,i}(t_k) + u_{R,i}(r_k).$$

In other words, for any individual i the existence of separate utility functions $u_{D,i}$, $u_{A,i}$, ..., $u_{R,i}$ working on the sets D, A, ... R is assumed. Conjoint analysis provides an algorithm to estimate these separate utility functions.

- Since decision makers are used rather to think in terms of alternatives than to assign numerical values to the relevant attributes, conjoint analysis starts with determination of a reduced, but hypothetical set of alternatives Φ_H, which must be ordered by individual i and which will allow to estimate the separate utility functions $u_{D,i}$,..., $u_{R,i}$.
- The separation of u_i as a sum of attribute-utility functions $u_{D,i}$,..., $u_{R,i}$. has the prerequisite that attribute-utility functions are independent each other. The utility, for example, for surgeon d_i to work with anesthetist a_j should in no way depend on the utility that operating-nurse $n_{1,k}$ will join the team. This independence assumption drastically allows reducing the set of alternatives to be ordered by individual i.
- Corresponding to the last assumption, the sets D, A, N, P, T and R of attributes can be reduced to consider the relevant ones only. For example, patient p_i will be completely indifferent with respect to the two nurses n_1 and n_2 or the operating room r assigned to his operation. Thus, for any patient p_i relevant attributes will vary in the set $D \times A \times T$ at most.

As result of conjoint analysis one gets individual utility functions $u_i(.)$ defined on the set of operation assignments Φ.

4.2 Utilities with respect to the set of admissible schedules

A schedule S consists of a sequence of n operation assignments, $S = (\phi_1, \phi_2,..,\phi_n)$, which satisfy the usual constraints, that the same person cannot be at the same time in different rooms or the same room cannot be in use at intersecting time slices etc.

With respect to the final decision problem these possible schedules S_0, S_1, … constitute the set of alternatives to be considered. That is, having derived individual utility functions $u_i(.)$ defined on the set of combinations Φ of alternative operation assignments, one has to develop for each individual i the utility function U_i defined on the set of all admissible schedules $\{S_0, S_1,.. \}$. U_i will depend on the above determined individual utilities u_i.

It seems to be reasonable to model for any individual i the utility $U_i(\ )$ of schedule $S_k = (\phi_{k,1}, \phi_{k,2},..,\phi_{k,n})$, $U_i(S_k)$, as the sum of utilities of operation assignments, $u_i(\phi_{k,l})$, minus a penalty-term:

$M_i(\phi_{k,1}, \phi_{k,2},.., \phi_{k,n})$: $U_i(S_k) = \sum_{\phi_{k,j} \in S_k} u_i(\phi_{k,j}) - M_i(S_k)$.

The penalty term $M_i(\phi_{k,1}, \phi_{k,2},..,\phi_{k,n})$ measures particularities of the whole schedule S_k. For example, nurses like to avoid overtime. Any schedule having high probability to afford presence in addition to regular working time, will produce for these kind of people a high penalty term $M_i(.)$ resulting in a poor overall utility $U_i(.)$.

Other measures of appropriateness of a schedule S_k by individual i will be to check if there are any gaps between assignments. For example, having a schedule, where a person i is on duty in the morning from 8.00 to 10.00 o'clock, having than a break for 4 hours, and being again on duty for the time from 2.00 o'clock to 4.00 in the afternoon is very likely to be rated lower, expressed by appropriate values of $M_i(.)$, than having continuous duties in the operating theatre from 8.00 o'clock to 12.00 noon. Since there are only a few criteria that influence $M_i(.)$, a separate conjoint analysis might be conducted in order to measure $M_i(.)$.

In the special case, where the individual i considered is a patient, individual utility of operation assignments will equal to zero, $u_i(\phi_{k,l}) = 0$, in all those cases, where $\phi_{k,l}$ is not an assignment of the patient i. Consequently, $U_i(S_k)$ reduces to $U_i(S_k) = u_i(\phi'_{k,j})$, where $\phi'_{k,l}$ designates the only assignment of patient i. Having no further information, the patient will be indifferent to the sequence and duration of individual assignments $S_k = (\phi_{k,1}, \phi_{k,2},.., \phi_{k,n})$ resulting in a penalty term $M_i(\phi_{k,1}, \phi_{k,2},.., \phi_{k,n}) = 0$.

5 The Nash Bargaining Equilibrium Point

As in the chapter on collaborative decision making explicated, the utility functions $U_i(.)$ in general will yield to conflicting preference orders between different individuals. For the following some assumptions are made:

- For each individual i the utility function $U_i(.)$ on the set of admissible schedules $\{S_0,S_1,.. \}$ is known.
- The schedule S_0 defines the initial solution, determined by conventional planning procedure or some scheduling algorithm.

The situation at hand is modeled as an N-person cooperative game, where a suited schedule has to be found.

Definitions:

$T := \{S_0, S_1,..\}$ the ***set of all admissible schedules****.*

Let $S \in T$. The utility (payoff) U of S is given by $U(S) = (U_1(S), U_2(S),.., U_N(S))$, where $U_i(S)$ designates the ***utility (= payoff) of player i****, $i = 1,..,N$.*

The ***payoff space P*** *is defined as the convex hull of the set U(T):*

$P = convex\_hull \{ U(S) = (U_1(S), U_2(S),.., U_N(S)) \mid S \in T \}$.

$U^0 = U(S_0)$ designates the ***payoff of the initial solution*** *S_0.*

The definition of payoff space P as the convex hull of all U(S) has the consequence, that by side agreements payoffs may be realized, which do not coincide with a possible schedule S. That is, in reality the scheduling problem is defined over a discrete solution space, whereas in the case of the Nash equilibrium point a continuous solution space is assumed.

Some assumptions are made to this game-theoretic approach

- Linear invariance: Generally, regarding utility functions neither scale nor zero-point is fixed. Therefore, the individual utility functions $U_i(.)$ must be independent of linear transformations, i.e. interpersonal comparisons of utility between the players are not permitted.
- Individual rationality. The individual preference structures must be rational in so far, that the payoffs received as result of negotiation must be at least as high as the payoffs received in the case negotiation fails. Consequently, if a payoff $U = (U_1,..,U_N) \in P$ is negotiated, $U \geq U^0$ must hold, i.e. for $i = 1,..,N$, $U_i \geq U_i(S_0)$.
- Pareto optimality. The final (optimal) solution $\hat{U}$ must be pareto-optimal.
- Symmetry. If the players are identical in their utility functions and starting points, the final (optimal) solution $\hat{U}$ will have identical components: $\hat{U}_1 = \hat{U}_i$ for all $i = 1,..,N$.
- Independence of irrelevant alternatives. If $Q \subseteq P$ is some subset of P and $U^0 \in Q$ and $\hat{U} \in Q$ ($\hat{U}$ = final (optimal) solution regarding the payoff space P), then the reduced problem, where instead of P the subset Q is considered, yields the same solution $\hat{U}$.

Nash has shown that there is exactly one point $\hat{U}$ fulfilling all those requirements and this point is the solution of the following optimization problem [Ha 77, p. 141ff.]:

$$\max \prod_{i=1}^{N} (U_i - U_i^0)$$
$$U_i \geq U_i^0, \; U \in P.$$

$\hat{U}$ is known to be a fair compromise solution. Observe that this solution $\hat{U}$ in general does not correspond to a valid schedule S, since it may contain side payments.

To get a valid schedule corresponding or at least very close to the Nash bargaining solution $\hat{U}$, a compensation mechanism is provided which will be forwarded to the repetition of scheduling the next day.

Define $\hat{S} \in T$ to be the solution to the following minimization problem:

$$\min_{S \in T} \| U(S) - \hat{U} \| .$$

The difference $U(\hat{S}) - \hat{U} = (U_1(\hat{S}) - \hat{U}_1, .., U_N(\hat{S}) - \hat{U}_N)$ defines for i = 1,..,N the utility gain, if $U_i(\hat{S}) - \hat{U}_i > 0$, respectively utility loss against Nash bargaining solution, if $U_1(\hat{S}) - \hat{U}_1 < 0$, the individual i has to tolerate, if $\hat{S}$ is the final accepted schedule.

Determination of Nash equilibrium point $\hat{U}$ strongly depends on the payoff of initial solution $U^0 = U(S_0)$. This initial payoff will be modified in order to compensate for the gain or loss any individual belonging to staff had experienced the day before. With this modification the **algorithm** is as follows:

For day t = 0,1,…

(1) *Define gain (or loss)* $G_t = (g_{1,t}, .., g_{N,t})$, *where* $g_{i,t}$ *is gain (or loss) of individual i of last schedule. Set initially* $g_{i,0} = 0$ *(i=1,..,N).*

(2) *Let* $T_t := \{S_{0,t}, S_{1,t}, ..\}$ *be the set of all admissible schedules of day t,*

(3) $U^0_t := U(S_{0,t})$ *the payoff of initial solution* $S_{0,t}$,

(4) *Define the starting payoff* $\hat{U}^0_t := U^0_t - G_t$. *(compensation by gain or loss of last period) and the payoff space* $P_t :=$ *convex hull of* $U(T_t)$.

(5) *Determine the Nash solution* $\hat{U}_t$ *of:*

$$\max \prod_{i=1}^{N} (U_i - \hat{U}^0_{i,t}) \quad ,$$

$$U_i \geq \hat{U}^0_{i,t}, U \in P_t .$$

(6) *and the corresponding admissible schedule* $\hat{S}_t$ *as solution of*

$$\min_{S \in T_t} \| U(S) - \hat{U}_t \| .$$

(7) *update* $G_{t+1} := U(\hat{S}_t) - \hat{U}_t$.

Summary

In this paper the problem of scheduling of operation theatres in general hospital had been addressed. A special focus was on the application specification. By a case study of a major German hospital the conventional way of scheduling had been demonstrated and typical planning problems identified. A multi-agent system approach to solve the problem is suggested. It consists of different planning steps, first identifying and measuring individual preferences of different operation assignments ending up with individual utility-functions. Second game theory had been applied in order to aggregate individual preferences and to find a compromise solution. Nash equilibrium point theory had been applied as solution procedure.

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