On the nonlinear control of TCSC

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Abstract—**The paper reviews modeling of thyristor–controlled series capacitors (TCSC) which are highly nonlinear systems with continuous and discontinuous states, and proposes three nonlinear controllers for it: (1) An approximate feedback linearization control (FLC) (2) A sliding surface–like design (3) Control in transformed coordinates. Detailed (switched) time–domain simulations are used to verify the control performance achieved by the three controllers.**

Keywords— **Series capacitive compensation, reactive power control, adaptive control, nonlinear systems.**

I. Introduction

Series capacitive compensation in AC transmission systems can yield several benefits, such as increased power transfer capability and enhanced transient stability. Thyristor Controlled Series Capacitors (TCSC) are beginning to find applications as adjustable series capacitive compensators [1], as they provide a continuously variable capacitance by controlling the firing angle delay of a thyristor controlled reactor (TCR) connected in parallel with a fixed capacitor. Besides controlling the power flow, TCSCs have a potential to provide other benefits, such as transient stability improvement, damping power swing oscillations, mitigating subsynchronous resonance (SSR) and fault current reduction. Hence, effective firing control strategies are required to exploit all advantages that a TCSC installation might offer.

In this paper we present a comparative study of three nonlinear controllers following different approaches to guarantee (locally) stable regulation and to speed up the response of a *Thyristor–Controlled Series Capacitor* (TCSC) used to control the power flow in a distribution line.

Instrumental in our developments is the use of a more complete model in the control design. The new model is an extension to the phasor model proposed in [4] for the TCSC circuit. This extension consists in the introduction of a nonlinear term which replaces the "heuristic" modification made to the control signal, before closing the loop, when the original model was used in the control design process. We show that such a "heuristic" modification, whose effect was not clear, corresponds to a nonlinear damping term. Indeed, this term concentrates the damping that may appear in many thyristor based circuits inherent to the switching process [8]. The proposed model thus represents the behavior of the real TCSC system more closely, and with the advantage that it is written in terms of more familiar variables –the dynamic phasors.

II. Modified model of TCSC

Consider the TCSC circuit shown in Fig. 1, which is used to control the power flow in a distribution line.

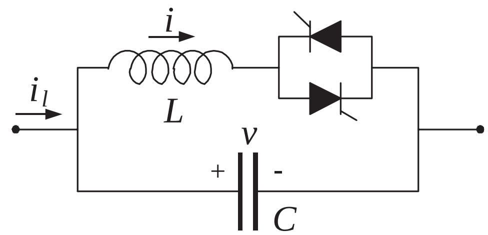

Fig. 1. Thyristor Controlled Series Capacitor

The following model for the TCSC circuit in terms of the phasor quantities was proposed in [4] where the authors assumed that the dynamics of current i is much faster relative to the dynamics of v and thus can be neglected.

$$C\frac{d}{dt}V = I_\ell - \mathcal{J} w_s C_{\text{eff}}(\beta) V \tag{1}$$

where C is the capacitance; w_S is the line fundamental frequency; vectors V and I_ℓ are the fundamental Fourier coefficients or 1–phasors for voltage across the capacitor v and current in the line i_ℓ, respectively. In this model we have used matrix $\mathcal{J} = -\mathcal{J}^\top = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ instead of the complex number j to express all phasor quantities as real vectors $V = [V_1,\, V_2]^\top$ and $I_\ell = [I_{\ell 1},\, I_{\ell 2}]^\top \in \mathbb{R}^2$ (with real and imaginary part of the corresponding complex phasor as the entries); $C_{\text{eff}}(\beta)$ is the equivalent capacitance of the circuit in terms of angle β which is equivalent to the half of the prevailing conduction angle, and thus term $w_s C_{\text{eff}}(\beta)$ is the *effective* (also called apparent) *quasi–steady–state (qss) admittance* of the TCSC (which represents the actual control input).

The effective qss capacitance $C_{\text{eff}}(\beta)$ can be approximated by the expression

$$\begin{aligned} C_{\text{eff}}(\beta) = \Big[\tfrac{1}{C} - \tfrac{4}{\pi} \Big(\tfrac{1}{2C}\mathcal{S} \left(\beta + \tfrac{\sin(2\beta)}{2} \right) + \\ + w_s^2 L \mathcal{S}^2 \cos^2(\beta) \left(\tan(\beta) - \eta \tan(\eta\beta) \right) \Big) \Big]^{-1} \end{aligned} \tag{2}$$

where we have defined $\eta = \frac{w_0}{w_s}$, $w_0 = \sqrt{\frac{1}{LC}}$ and $\mathcal{S} = \frac{\eta^2}{\eta^2 - 1}$.

The authors in [4] also propose to include the following modification to the model above to consider the transient behavior

$$\beta = \beta_0 + 2\phi \;,\;\; \phi = \text{ atan}\left(\frac{V_2}{-V_1} \right) \tag{3}$$

Thus, for simulation purposes, once angle β_0 has been computed following any control design, we add the effect of angle ϕ computed as in before entering to (2).

Unfortunately this "heuristic" modification made to the control signal before closing the loop cannot be included in the control design process, and moreover, its effect is not clear at al

We propose then to use an approximation to such modification which leads to the addition of a nonlinear term into the model and we show that the effect of this term is to introduce damping.

Consider the following approximation

$$C_{\text{eff}}(\beta) = C_{\text{eff}}(\beta_0 + 2\phi) = C_{\text{eff}}(\beta_0) + 2\phi \frac{\partial C_{\text{eff}}}{\partial \beta}\Big|_{\beta_0}$$

Notice that, in the capacitive region

$$f(\beta_0) \triangleq \frac{\partial C_{\text{eff}}}{\partial \beta}\Big|_{\beta_0} < 0 \quad \forall \beta \;\; \langle \text{In capacitive region} \rangle$$

For $\phi \ll 1$ we can approximate $\phi = \arctan\left(\frac{V_2}{-V_1}\right) \cong -\frac{V_2}{V_1}$ The voltage subsystem equation is now

$$\begin{aligned} C\frac{d}{dt}V &= I_\ell - \mathcal{J}wC_{\text{eff}}(\beta) \\ &\cong I_\ell - \mathcal{J}wC_{\text{eff}}(\beta_0)V + 2\mathcal{J}w\frac{V_2}{V_1}f(\beta_0)V \end{aligned}$$

The third term on the RHS can also be written as

$$2\mathcal{J}w\frac{V_2}{V_1}f(\beta_0)V = 2w\frac{V_2}{V_1}f(\beta_0)\begin{bmatrix} -V_2 \\ V_1 \end{bmatrix} =$$

$$= 2wf(\beta_0)\begin{bmatrix} -\frac{V_2^2}{V_1^2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathcal{R}(V,\,\beta_0)V$$

Notice from the structure of matrix $\mathcal{R}(V,\,\beta_0)$ that such a nonlinear term is adding damping on the second row but not in the first row, for this reason efforts should be carried out in the control design to introduce damping in the dynamics of V_1 to dominate such a bad term.

We observe that $\mathcal{R}(V,\,\beta_0)$ depends on both, the control and state, besides, we would like to extract the control angle β_0 by considering only $C_{\text{eff}}(\beta_0)$ and disregarding its effect in $\mathcal{R}(V,\,\beta_0)$, that is, it would be easier to define a $u = wC_{\text{eff}}(\beta_0)$ and compute u following a given control design technique, and then extract the angle β from a table $u \mapsto \beta_0$, which is the final control to be applied to the system. If we try to reconstruct β_0 from the system considering the effect on both terms $C_{\text{eff}}(\beta_0)$ and $\mathcal{R}(V,\,\beta_0)$ the process becomes extremely difficult. With the idea of facilitating the control design process by avoiding the effect of the control signal in term $\mathcal{R}(V,\,\beta_0)V$ we consider the evaluation of such a term in the desired equilibrium point, which yields the following linear term

$$2wf(\beta_0)\begin{bmatrix} -\frac{V_2^2}{V_1^2} & 0 \\ 0 & 1 \end{bmatrix}V \quad \rightarrow \quad -\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 2w|f(\overline{\beta}_0)| \end{bmatrix}}_{\mathcal{R}(\overline{\beta}_0)}V$$

where $\mathcal{R}(\overline{\beta}_0) = \text{diag}\{0,\,R\} \geq 0$ is a positive semidefinite matrix, $R = 2w|f(\overline{\beta}_0)|$ and the constant $\overline{\beta}_0$ is the equilibrium value of β_0.

The system then reduces to

$$C\frac{d}{dt}V = I_\ell - \mathcal{J}wC_{\text{eff}}(\beta_0)V - \mathcal{R}(\overline{\beta}_0)V \tag{4}$$

The control design process can now be carried out in a more conventional way, as the damping term appears explicitly in the model and no further modifications are needed.

For control design purposes, we define $u \triangleq w_S C_{\text{eff}}(\beta_0)$, i.e., the effective admittance, as the control input. The *control objective* consists of finding the controller u (and hence an angle β_0 through the mapping $u(\beta_0)$) such that regulation of the voltages vector $V = [V_1,\,V_2]^\top$ is guaranteed towards some predefined constant reference $V^* = [V_1^*,\,0]^\top$. Notice that the effective admittance u is restricted to take positive values if the TSCS circuit is asked to behave as an capacitor.

We assume throughout the paper that the line current has the form $i_\ell(t) = I_{\ell 2}\sin(w_s t)$, with $I_{\ell 2}$ a positive constant. Thus, $I_\ell = [0,\,-I_{\ell 2}]^T$, out of which $V^* = [V_1^*,\,V_2^*]^T = [-\frac{I_{\ell 2}}{u^*},\,0]^T$.

Using the proposed model we design three nonlinear controllers following different approaches:

(i) An approximate feedback linearization control (FLC)
(ii) A sliding surface–like design
(iii) Control in transformed coordinates

In these three techniques we have stressed the requirement of making the response of the system faster than the response obtained by operating the system in open loop. Moreover, we have include adaptations, which result in the addition of integral terms, to cope with parameter uncertainties.

III. An approximate feedback linearization control (FLC)

We found that the linearizing output for system (4) is the energy of the system [5], given by

$$y = \frac{C}{2}V_1^2 + \frac{C}{2}V_2^2 \tag{5}$$

The output first and second time derivatives are respectively

$$\begin{aligned} C\dot{y} &= -CV_2(I_{\ell 2} + RV_2) &(6) \\ C\ddot{y} &= -2R\dot{y} + V_1(I_{\ell 2} + 2RV_2)u + I_{\ell 2}(I_{\ell 2} + RV_2) &(7) \end{aligned}$$

Expressions for V_1 and V_2 can be obtained from (5)–(6) which will results in highly nonlinear expressions. To complete the linearization process we propose a control u that cancels the nonlinearities in the right hand side by partially inverting the second term and cancelling the third nonlinear term as follows

$$u = \frac{\psi - I_{\ell 2}(I_{\ell 2} + RV_2)}{V_1(I_{\ell 2} + 2RV_2)}$$

where ψ is designed to guarantee stability.

Unfortunately, the linearizing process cannot be fully completed as some of the parameters involved in the linearization are unknown, namely $I_{\ell 2}$ and R. We thus propose a controller which approximately linearizes the system, that is, we remove from the system the vanishing perturbation terms and concentrate on the compensation of only the nonvanishing ones.

Let us rewrite (7) as follows

$$C\ddot{y} = -2R\dot{y} + V_1 I_{\ell 2}u + I_{\ell 2}^2 + RV_2(I_{\ell 2} + 2V_1 u) \tag{8}$$

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Notice that the fourth term in the RHS of (8) vanishes once reached the desired equilibrium point. This motivates the proposition of the following PID–type controller

$$u = -\frac{1}{V_1 I_{\ell 2}^{nom}} \left((I_{\ell 2}^{nom})^2 + K_v y + K_p \tilde{y} + K_i \phi \right) \;\; , \;\; \dot{\phi} = \tilde{y}$$

where K_v, K_p and K_i are positive design constants and $I_{\ell 2}^{nom}$ is a constant nominal value of the line current which is used to distress the integral action. Notice that any discrepancy of $I_{\ell 2}^{nom}$ with respect to the real value $I_{\ell 2}$ can be absorbed by the design constants K_v, K_p and K_i.

The controller above can be modified to facilitate the implementation if the derivative term $K_v \dot{\tilde{y}}$ used to introduce damping is substituted by an approximation $K_v \nu$, also referred in control literature as "dirty derivative", where ν is the result of low pass filtering $\dot{\tilde{y}}$ as follows

$$\nu = \frac{b\, s\, \tilde{y}}{s + a} \qquad (9)$$

where a and b are two positive design constants and s is the Laplace complex variable.

In resume the controller can be rewritten as

$$u = -\frac{1}{V_1 I_{\ell 2}^{nom}} \left[(I_{\ell 2}^{nom})^2 + K_v \nu + K_p \tilde{y} + K_i \phi \right] \;\; , \;\; \dot{\phi} = \tilde{y}$$
$$\nu = b(\tilde{\beta} - \vartheta) \;\; , \;\; \dot{\vartheta} = a(\tilde{\beta} - \vartheta)$$

where we have used the state space realization of the proper filter (9). Following a linear analysis it can be shown that this controller locally stabilizes the system.

IV. A sliding surface-like design

In this approach we define a surface on the state space which is a linear combination of the states. We prove that this surface contains the desired equilibrium and that all trajectories starting on this curve converge towards the desired equilibrium point. With the idea of maintaining similar convergence rate for both states V_1 and V_2 towards their equilibrium, we propose a straight line with unity slope and design a controller to make such a surface attractive. This design process is very close to that followed in the sliding mode approach, except that we are not forcing the system to create a sliding regime by applying high gain. Instead, we allowed a smooth convergence to the surface and to the desired equilibrium.

Let us define the following output

$$\sigma \stackrel{\triangle}{=} \tilde{V}_1 + V_2 \;\; , \;\; \tilde{V}_1 = V_1 - V_1^*$$

whose desired equilibrium point is $\sigma^* = 0$.

We now consider the quadratic function $H = \frac{C}{2}\sigma^2$ whose time derivative is

$$\dot{H} = \sigma(uV_2 - uV_1 - RV_2 - I_{\ell 2})$$

In case the parameter $I_{\ell 2}$ is known we propose the following controller

$$u = \frac{f(\sigma) + I_{\ell 2}}{V_2 - V_1} = \frac{f(\sigma) + I_{\ell 2}}{2V_2 - V_1^* - \sigma}$$

where $f(\sigma)$ is such that $\sigma f(\sigma) < 0,\ f(0) = 0$, this yields

$$\dot{H} = \sigma(f(\sigma) - RV_2) = -\sigma f(\sigma) - RV_2^2 - R\tilde{V}_1 V_2$$

We have observed from simulations that a considerable improvement in performance can be obtained by selecting $f(\sigma) = -k_1\ \mathrm{atan}(k_2\sigma)$, where k_1 and k_2 are positive design constants.

A. Zero dynamics and stability analysis

Let us analyze the dynamics that remains once the output has reached its desired value, that is, for $\sigma = \tilde{V}_1 + V_2 = 0$. Notice that this constraint defines a straight line in the state space $\tilde{V}_1$ vs V_2 which contains the desired equilibrium $(\tilde{V}_1^*,\, V_2^*) = (0, 0)$. The control signal in this case is reduced to

$$u = \frac{-I_{\ell 2}}{2\tilde{V}_1 + V_1^*}$$

thus the dynamics of $\tilde{V}_1$ restricted to $\sigma = 0$ is given by

$$C\dot{\tilde{V}}_1 = uV_2 = \frac{I_{\ell 2}\tilde{V}_1}{2\tilde{V}_1 + V_1^*} = \frac{I_{\ell 2}}{2}\left(1 - \frac{V_1^*}{2\tilde{V}_1 + V_1^*}\right) \qquad (10)$$

whose equilibrium, obtained by using $1 = \frac{V_1^*}{2\tilde{V}_1 + V_1^*}$, is $\overline{\tilde{V}}_1 = 0$ which in its turns implies that $\overline{V}_2 = 0$.

For easy of presentation consider only the case of capacitive region, and thus consider $V_1^* < 0$ using (10) the following can be proved

i) In quadrant IV $(\tilde{V}_1 > 0\,, V_2 < 0) \;\; \Rightarrow \;\; \dot{\tilde{V}}_1 < 0$

ii) In quadrant II $(\tilde{V}_1 < 0\,, V_2 > 0) \;\; \Rightarrow \;\; \dot{\tilde{V}}_1 > 0$

To study the dynamics of V_2 restricted to $\sigma = 0$ we use $u = \frac{I_{\ell 2}}{2V_2 - V_1^*}$ and $V_1 = V_1^* - V_2$, this yields

$$\begin{aligned} C\dot{V}_2 &= -\frac{I_{\ell 2}(V_1^* - V_2)}{2V_2 - V_1^*} - RV_2 - I_{\ell 2} \\ &= -\frac{I_{\ell 2}}{2}\left(1 + \frac{V_1^*}{2V_2 - V_1^*}\right) - RV_2 \qquad (11) \end{aligned}$$

Using (11) we can prove the following

iii) In quadrant IV $(\tilde{V}_1 > 0,\ V_2 < 0) \;\; \Rightarrow \;\; \dot{V}_2 > 0$

iv) In quadrant II $(\tilde{V}_1 < 0,\ V_2 > 0) \;\; \Rightarrow \;\; \dot{V}_2 < 0$

Hence, from *i), ii), iii)* and *iv)* we conclude that the system is stable. A picture of the qualitative behavior of the system dynamics around the surface $\sigma = 0$ is shown in Fig. 2

Notice that (11) has two equilibrium points $\overline{V}_2 = 0$ and $\overline{V}_2 = \frac{RV_1^* - I_{\ell 2}}{2R}$, nevertheless the latter cannot be equilibrium of the whole system since according to (10) any trajectory starting in $V_2 \neq 0$ will move towards the origin, hence $\overline{V}_2 = 0$, $\overline{\tilde{V}}_1 = 0$ is the only possible equilibrium.

B. Adding adaptation

In the case that current $I_{\ell 2}$ is unknown we may propose instead the following adaptive controller

$$u = \frac{f(\sigma) + \hat{I}_{\ell 2}}{V_2 - V_1}$$

where $\hat{I}_{\ell 2}$ represents an estimate for $I_{\ell 2}$.

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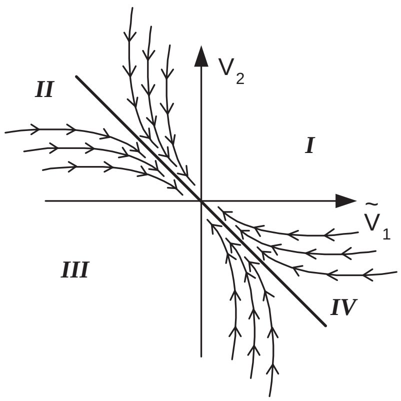

Fig. 2. Dynamics behavior on $\sigma = 0$

The time derivative of function H is now

$$\dot{H} = \sigma(f(\sigma) - RV_2 + \tilde{I}_{\ell 2})$$

where we defined $\tilde{I}_{\ell 2} \stackrel{\triangle}{=} \hat{I}_{\ell 2} - I_{\ell 2}$.

To design the adaptive law we now extend H as $H_T = \frac{C}{2}\sigma^2 + \frac{1}{2\gamma}\tilde{I}_{\ell 2}^2$ whose time derivative gives

$$\dot{H}_T = \sigma(f(\sigma) - RV_2) + \sigma\tilde{I}_{\ell 2} + \frac{\tilde{I}_{\ell 2}\dot{\tilde{I}}_{\ell 2}}{\gamma}$$

we propose then the adaptive law

$$\dot{\tilde{I}}_{\ell 2} = -\gamma\sigma \ \ \rightarrow \ \ \dot{\hat{I}}_{\ell 2} = -\gamma(\tilde{V}_1 + V_2)$$

where γ is a positive design parameter

V. Control in transformed coordinates

We observe that using a polar coordinates transformation we obtain a system where the controller enters linearly. However, the control signal appears only on the angle equation, meaning that the amplitude (the variable of interest) can only be controlled indirectly by first controlling the angle. In other words, the output of interest is of relative degree 2. As in the FLC we propose here an approximate linearization which disregards the vanishing terms.

Consider the following transformation to polar coordinates

$$\beta = \sqrt{V_1^2 + V_2^2} \ , \ \rho = \text{ atan}\left(\frac{-V_2}{V_1}\right)$$

whose inverse transformation is given by

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = e^{-\mathcal{J}\rho}\begin{bmatrix} \beta \\ 0 \end{bmatrix} = \begin{bmatrix} \beta\cos(\rho) \\ -\beta\sin(\rho) \end{bmatrix}$$

The time derivatives are, respectively

$$\dot{\beta} = \frac{1}{\beta}V^\top\dot{V} \ , \ \dot{\rho} = \frac{1}{\beta^2}V^\top\mathcal{J}\dot{V}$$

Applying the above transformations to model (4), yields the following model in terms of the polar coordinates

$$C\dot{\beta} = I_{\ell 2}\sin(\rho) - \frac{R\beta}{2}(1 - \cos)(2\rho)) \qquad (12)$$

$$C\dot{\rho} = \frac{I_{\ell 2}}{\beta}\cos(\rho) - \frac{R}{2}\sin(2\rho) + u \qquad (13)$$

Notice that in this model the control u enters in a linear way.

To fulfill the control objective, the references are now changed to

$$\beta^* = |V_1^*| \ , \ \rho^* = \pi$$

since $V_1^* < 0$ and $V_2^* = 0$.

Let us propose the controller

$$u = \frac{-I_{\ell 2}^{nom}\cos(\rho)}{\beta} + \frac{C^2\phi}{I_{\ell 2}\cos(\rho)}$$

where $I_{\ell 2}^{nom}$ is a nominal value for $I_{\ell 2}$ and ϕ a signal yet to be defined.

Then (13) becomes

$$C\dot{\rho} = -\frac{-\tilde{I}_{\ell 2}\cos(\rho)}{\beta} - \frac{R\sin(2\rho)}{2} + \frac{C^2\phi}{I_{\ell 2}\cos(\rho)} \qquad (14)$$

Taking the time derivative of (12) and using (14), we get

$$\begin{aligned} \ddot{\beta} &= \phi - \frac{I_{\ell 2}\tilde{I}_{\ell 2}\cos^2(\rho)}{C^2\beta} - \frac{R\dot{\beta}}{2C}(1 - \cos(2\rho)) - \delta(t) \\ \delta(t) &= \frac{R\beta\sin(2\rho)}{C^2}\left(\frac{C^2\phi}{I_{\ell 2}\cos(\rho)} - \frac{\tilde{I}_{\ell 2}\cos(\rho)}{\beta}\right. \\ &\quad \left. - \frac{R\sin(2\rho)}{2} + \frac{I_{\ell 2}\cos(\rho)}{2\beta}\right) \end{aligned}$$

where $\delta(t)$ is a time varying signal that vanishes in the steady state and thus its effect can be neglected.

We now propose the controller

$$\phi = -K_v\dot{\tilde{\beta}} - K_p\tilde{\beta} - \frac{K_i\cos^2(\rho)\psi}{\beta} \ , \ \dot{\psi} = \tilde{\beta} \qquad (15)$$

where $\tilde{\beta} = \beta - \beta^*$, and K_v, K_p and K_i are positive design constants.

The error model is now

$$\begin{aligned} \ddot{\tilde{\beta}} &= -\left(K_v + \frac{R(1 - \cos(2\rho))}{2C}\right)\dot{\tilde{\beta}} - K_p\tilde{\beta} - \\ &\quad - \frac{K_i\cos^2(\rho)}{\beta}\left(\psi + \frac{I_{\ell 2}\tilde{I}_{\ell 2}}{K_iC^2}\right) \end{aligned}$$

Let us define now $\alpha = \psi + \frac{I_{\ell 2}\tilde{I}_{\ell 2}}{K_iC^2}$ out of which we get the system

$$\begin{aligned} \ddot{\tilde{\beta}} &= -\left(K_v + \frac{R(1 - \cos(2\rho))}{2C}\right)\dot{\tilde{\beta}} - K_p\tilde{\beta} - \frac{K_i\cos^2(\rho)\alpha}{\beta} \\ \dot{\alpha} &= \tilde{\beta} \end{aligned}$$

From a local stability analysis using linearization we can state that the system is locally stable.

In resume the controller will be represented by the following expressions

$$\begin{aligned} u &= -\frac{1}{\cos(\rho)}\left(K_v'\dot{\tilde{\beta}} + K_p'\tilde{\beta}\right) - \frac{K_i'\cos(\rho)}{\beta}\left(\psi + \frac{I_{\ell 2}^{nom}}{K_i'}\right) \\ \dot{\psi} &= \tilde{\beta} \end{aligned}$$

where the constants K'_p, K'_v and K'_i have absorbed the constant $\frac{C^2}{I_{\ell 2}}$, that is, $(\cdot)' = \frac{C^2}{I_{\ell 2}}(\cdot)$. Notice that constant term $\frac{I_{\ell 2}^{nom}}{K'_i}$ can be absorbed by the integral term ψ but we prefer to keep this feedforward term to relax the effort in the integral action.

The controller above can be further modified if the derivative term $K'_v\dot{\tilde{\beta}}$ used to introduce damping is substituted by the "dirty derivative" approximation $K'_v\dot{\nu}$, where ν is the result of low pass filtering $\dot{\tilde{\beta}}$ as follows

$$\nu = \frac{b\, s\, \tilde{\beta}}{s+a} \tag{16}$$

where a and b are two positive design constants.

Rewriting the controller in terms of V_1, V_2 and β, and using the state space realization of proper filter (16), we get finally

$$\begin{aligned} u &= -\frac{\beta}{V_1}\left(K'_v\nu + K'_p\tilde{\beta}\right) - \frac{K'_i V_1}{\beta^2}\left(\psi + \frac{I_{\ell 2}^{nom}}{K'_i}\right) \\ \dot{\xi} &= \tilde{\beta} \\ \nu &= b(\tilde{\beta} - \vartheta)\ ,\ \dot{\vartheta} = a(\tilde{\beta} - \vartheta) \end{aligned}$$

with $\beta = \sqrt{V_1^2 + V_2^2}$ and $\tilde{\beta} = \beta - |V_1^*|$.

VI. Simulation results

For the purpose of simulation, we use a system model with the following parameters: $L = 6.8$mH, $C = 176.80\mu$F, at a constant frequency $f = 60$ Hz. This parameters correspond to the Kayenta substation [1]. The intervals where the apparent admittance u is restricted are given by

$$\begin{aligned} 0.0151 \geq u \geq 0.0662 \qquad &(\text{capacitive region}) \\ -0.2469 \geq u \geq -0.0182 \qquad &(\text{inductive region}) \end{aligned}$$

The line current has been set to $i_\ell = 600\sin(w_s t)$ Amp. Due to space limitations we will present simulations only for the capacitive boost behavior. The test consists in fixing a reference for the apparent admittance $u^* = 0.06\Omega^{-1}$ at the beginning, and then after time $t = 0.2$ sec, we switch to a new reference $u^* = 0.03\Omega^{-1}$. This corresponds to a step change in the desired voltage reference V_1^* going from -10 KV to -20 KV.

Since dynamic phasor quantities are not available from measurement, we have designed a filter to extract such quantities from the time varying periodical signal. Fig. 3 shows a block diagram of the implemented filter. This filter follows very closely the ideas presented in [7], where similar filters, referred as resonant filters, were used with the same objective (but for three phase systems). We have observed from simulations that the response of the filter used here improves significantly with respect to the conventional "rotation plus low pass filter (LPF)", as the signals generated have a smaller ripple.

The response of all three controllers proposed are more and less the same, we show the simulation responses of state V_1 in Fig. 4 where it can be observed that all them guarantee zero steady state error and a faster response than with simple open loop.

In Fig. 5 we present the open loop behavior of the circuit. In this case the control is simply taken as $u = u^*$. We observe that, due to inaccuracy in the generation of table u–β, a considerable steady state error exists between voltage V_1 and its reference V_1^* (in dashed line).

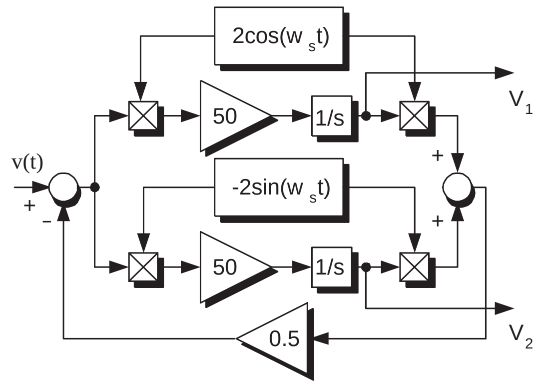

Fig. 3. Filter used to extract the phasor quantities.

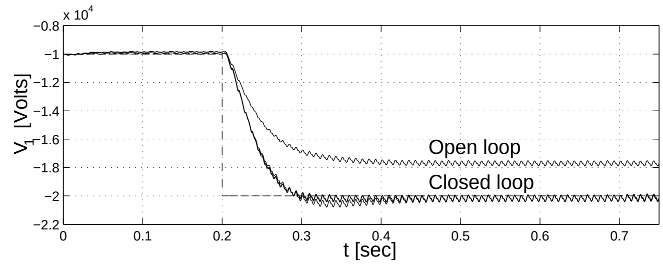

Fig. 4. **(–)** Response of $V_1(t)$ for the system in open loop and under the three different controllers, and **(--)** reference $V_1^*(t)$.

Responses of voltages V_1 and V_2 using the approximate Feedback Linearization based controller are shown in Fig. 6. The corresponding control signal $u = w_s C_{\text{eff}}(beta)$ and the angle β are shown in Fig. 7. The transient responses for current $i(t)$, voltage $v(t)$ and line current $i_\ell(t)$ are shown in Fig. 8. The plots for the rest of the controllers are very similar and are omitted here for reasons of space limitation.

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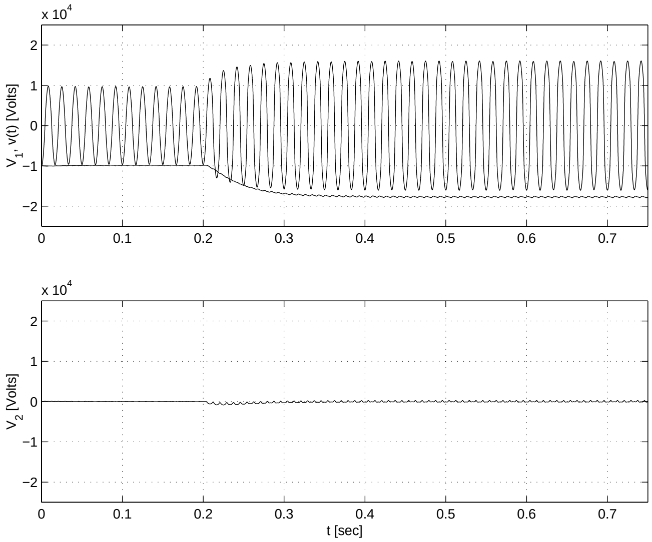

Fig. 5. Open loop: **(top)** $V_1(t)$ and $v(t)$; **(bottom)** $V_2(t)$.

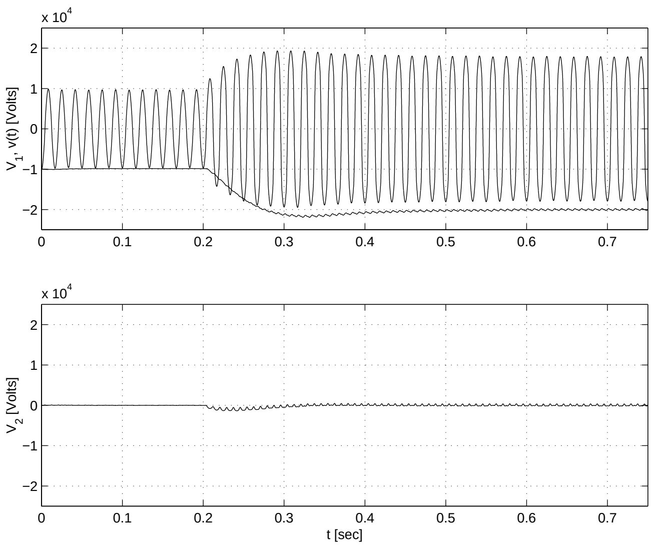

Fig. 6. Approximate FLC : **(top)** $V_1(t)$ and $v(t)$; **(bottom)** $V_2(t)$.

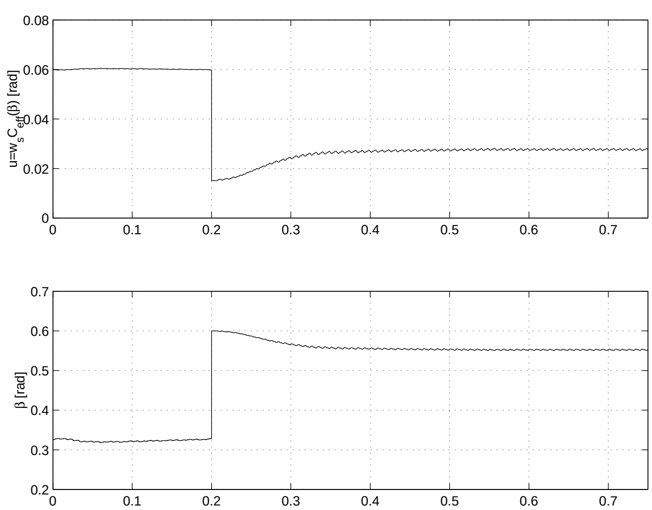

Fig. 7. Approximate FLC: **(top)** $u(t) = w_s C_{\text{eff}}(\beta(t))$ and **(bottom)** $\beta(t)$.

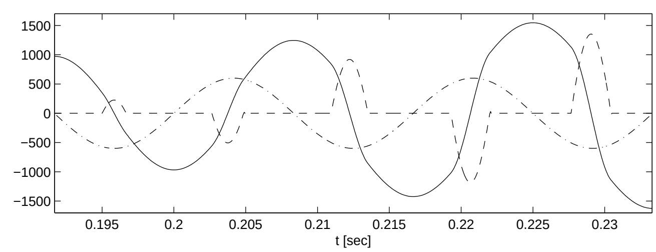

Fig. 8. Approximate FLC: **(–)** scaled voltage $0.1 * v(t)$, **(--)** current across the inductor $i(t)$ and **(-·)** line current $i_\ell(t)$.

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