Optimal Bidding and Contracting Strategies in the Deregulated Electric Power Market: Part II

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Abstract

We study the interaction of long-term contracting and spot market transactions between Multi-Gencos and Multi-Discos for electric power. Gencos and Discos may either contract for delivery in advance or they may sell/buy some or all of their output/input in a spot market. Contract pricing involves both a reservation fee per unit of capacity and an execution fee per unit of output if capacity is called. Discos’ optimal portfolios are shown to follow a merit order (or greedy) shopping rule. When Gencos properly anticipate demands to their bids, then bidding a contract execution fee equal to variable cost dominates all other bidding strategies. The optimal capacity reservation fees are determined by Gencos to trade off the risk of underutilized capacity against unit capacity costs. Existence and structure of market equilibria are characterized for the associated competitive game between Gencos.

1. Introduction

In the newly restructured electricity market, Gencos (Generators) and Discos (Load Serving Entities and Distribution Companies) can sign long-term bilateral contracts to cover Disco needs, which are in turn derived from the demands of the Discos’ customers. Alternatively, Gencos and Discos can interact “on the day” in a spot market. How much of their respective capacity and demand Gencos and Discos should or will contract for in the bilateral contracting market, and how much they will leave open for spot transactions, is a fundamental question. Interestingly, the underlying theory for addressing this question has not been fully developed. This paper makes the following unique contributions in the literature. First, and most importantly, we consider Gencos with heterogeneous production costs; second, we consider uncertain access to the spot market by Gencos; third, we consider the two-goods problem in which both capacity and output are bid into the contracting market.

In the problem we model (which might be thought of as the “month ahead” market), Gencos and Discos interact through an electronic bulletin board, posting bids and offers until agreement has been reached. Capacity not committed through this contracting market is assumed to be offered on the spot market, but may go unused because of the risk of not finding customers or transportation capacity at the last minute. Discos face another type of risk for demand not contracted for in the bilateral market, namely price volatility in the spot market. Such price volatility can be quite severe. To illustrate, we note the experience in the midwest electric power market in the United States (the so-called ECAR Region or East Center Area Reliability Coordination Agreement Region) in the summer of 1998 (and again in the summer of 1999!) where prices as high as $8,000/mWh materialized in the spot market (“normal” prices would be in the $20-$50 range/mWh). Such experiences as this have caused Load Serving Entities (our Discos) in the electric power market to pay close attention to the proper balance in their supply portfolio between long-term contracting and spot purchases.

The rest of this paper is organized as follows. Section 2 generalizes our earlier results to the Multi-Genco, Single-Disco case, solving this case completely. Section 3 generalizes these results to the Multi-Genco, Multi-Disco case,
again solving this case completely. Section 4 concludes with a number of suggestions for future research.

Note: Unless noted otherwise, all proofs can be found in the full versions of this paper [3]. These are left out due to space limitation.

2. Multi-Genco, Single-Disco, Radial Network

This section considers the Multi-Genco-Single-Disco radial network case. Assume there are I Gencos and one Disco. We use the subscript i to denote Genco’s costs ($\beta_i$, $b_i$) and Capacity $K_i$. We assume Genco i has knowledge on its own cost structure ($b_i$, $\beta_i$), where $b_i$ is the unit variable cost of production and $\beta_i$ is the unit cost of capacity. Genco i bids the following information on an electronic bulletin board, [$s_i$, $g_i$, $L_i$], $i = 1, \ldots, I$, while the Disco posts its demand information on the bulletin board, [D($P_s$, $s_i$, $g_i$, $Q_i$), $Q_i$, $q_i$]. As noted, we assume that the demand function of the Disco is common knowledge among Gencos.

The problem confronting the Disco is to choose an optimal portfolio of contracts from those available on the bulletin board. Some of the contract offers carry high subscription rates $s_i$ but low execution fees $g_i$. These must be compared to other offers carrying lower subscription fees but higher execution fees. We will show below that the Disco’s optimal solution is basically to rank contracts in increasing order of a certain index. The index reflects the cost of reserving capacity at price $s_i$ plus the cost of maintaining the option to execute the contract at price $g_i$ rather than utilizing the spot market.

Imitating the single-Genco case [1, 2], define the Disco’s utility as:

$$V(D, q, x, \phi) = U(D) - \sum_{i=1}^I s_i Q_i - \sum_{i=1}^I g_i q_i - P_s x, \quad (1)$$

where $\phi = (P_s, Q_i (i = 1, \ldots, I))$ is the spot price and the vector of contract capacities, $q = (q_1, \ldots, q_I)$ is the vector of purchases under contract from Genco $i = 1, \ldots, I$, $x$ is the amount purchased in the spot market, and $D$ is the total consumption of the Disco, so that

$$D = x + \sum_{i=1}^I q_i.$$  

Denoted $D_s(P_s)$, to maximize $\{U(D) - P_s D | D \geq 0\}$, thus, $D_s(P_s) = U^{-1}(P_s)$, which would be the demand of the Disco if only the spot market were available for sourcing.

In keeping with decreasing marginal utility of consumption (i.e., assuming the normal demand curve $D_s$ is downward sloping), we will assume that the Disco’s WTP $U(z)$ is strictly concave and increasing so that

$$U'(z) > 0, \quad U''(z) < 0, \quad \text{for } z \geq 0. \quad (2)$$

We will also assume throughout the remainder of the paper that the following regularity condition on the contract market holds for the Disco.

No Excess Capacity Condition: Let Gencos’ offers be indexed so that $g_1 \leq g_2 \leq \ldots \leq g_I$. Then the No Excess Capacity Condition is said to hold if and only if

$$Q_i[D_s(g_i) - \sum_{i=1}^I Q_i] \geq 0, \quad i = 1, \ldots, I.$$  

This condition says that if $Q_i > 0$ then the sum of all contracted capacity with execution fees less than or equal to $g_i$ must not exceed $D_s(g_i)$.

Lemma 1: Let $\phi = (P_s, Q_i (i = 1, \ldots, I))$ be given. W.o.l.g., assume that Gencos’ offers are indexed so that $g_1 \leq g_2 \leq \ldots \leq g_I$. Let the Disco’s optimal total demand be given by the solution $D(\phi)$ to the following problem:

$$\text{Maximize}_{D, q, x} \quad V(D, q, x, \phi), \quad (3)$$

subject to:

$$D = x + \sum_{i=1}^I q_i, \quad x, q, D \geq 0. \quad (4)$$

Then (under the No Excess Capacity Condition) the solution $D(\phi)$ to (3)- (4) is

$$D(\phi) = (D_s(P_s)) - \max_k \{\frac{\chi(P_s - g_k)}{\sum_{i=1}^k Q_i} | 1 \leq k \leq I\} \quad (5)$$

and optimal purchases under contract from Genco $i = 1, \ldots, I$ and from the spot market are given by

$$q_i(\phi) = Q_i\chi(P_s - g_i), \quad x(\phi) = D(\phi) - \sum_{i=1}^I q_i(\phi). \quad (6)$$

Alternatively $D(\phi)$ may be expressed as $D(\phi) = \max_k [D_s(P_s), \sum_{i=1}^k Q_i]$, where Genco $k = k(\phi)$ provides the last unit of contract output to the Disco. Defining $g_0 = 0$ and $g_{I+1} = \infty$, Genco $k$ is determined as the first Genco (in the indicated order of increasing $g_i$) satisfying $g_k < P_s \leq g_{k+1}$. If $k = 0$, no contract capacity whatsoever is used.

Proof: See Appendix.
The optimal quantities (6) purchased under contract from Gencos follow the normal “merit order” indicated by the execution fee rank order in the Lemma. Contracts are executed in order up to the point at which the spot price dominates any further available contracts. This implies also, as seen in (6), that either all units of a contract or none are executed on the day, depending on the spot price. We note from Lemma 1 that when facing dual sources (spot market and contract market) for procurement, the Disco’s demand curve is kinked, as captured in (5).

Define $G(p)$ as the “effective price function” or “expected unit total price”

$$G(p) = \int_0^p (1 - F(y)) dy = E\{\min(P_s, p)\}, \quad (7)$$

and $G^{-1}$ as the inverse function of $G$. We have the following Theorem 1.

**Bid Tie Allocation Mechanism:** If there is a tie in bids among any subset of Gencos, then the Disco’s total demand for that subset of Gencos is allocated to the Gencos’ in proportion to their respective bid capacities.

The most important results of this paper build on the characterization of Disco demand in response to Genco bids. To derive this demand, we introduce some additional notation. Let $\Xi = \{1, \ldots, I\}$ and, for every $k \in \Xi$, define the following sets:

$$M^1_k(s, g) = \{i \in \Xi | s_i + G(g_i) < s_k + G(g_k)\};$$
$$M^2_k(s, g) = \{i \in \Xi | s_i + G(g_i) \leq s_k + G(g_k)\};$$
$$M^3_k(s, g) = \{i \in \Xi | s_i + G(g_i) = s_k + G(g_k)\};$$
$$M^4_k(s, g) = \{i \in \Xi | s_i + G(g_i) > s_k + G(g_k)\};$$

where $s = (s_1, \ldots, s_I)$ and $g = (g_1, \ldots, g_I)$. We will usually suppress the dependence of the sets $M^i_k$ on $(s, g)$ whenever $(s, g)$ is fixed and clear from the context.

**Theorem 1 (Disco’s Optimal Contract Portfolio):** Let $(s, g, L) = \{(s_i, g_i, L_i), i = 1, \ldots, I\}$, be posted bids by the Gencos. Wo.l.g. assume that Genco bids are ranked in order of the index $s_i + G(g_i)$, so that $s_1 + G(g_1) \leq s_2 + G(g_2) \leq \ldots \leq s_i + G(g_i)$. If $U_x(0) \leq s_1 + G(g_1)$, then the Disco’s solution will be to set $Q_{i} = 0$, for all $i$, i.e., no contracting is optimal. Otherwise, **Greedy Contracting** in order of the given index is optimal for the Disco, i.e., the optimal Portfolio of contracts has the form: $\forall i \in M^1_k$, $Q_i = L_i$; $\forall i \in M^2_k$, $Q_i(s, g, L) = 0$; and for $i \in M^3_k$, $Q_i(s, g, L) = \frac{L_i}{\sum_{j \in M^2_k} (D_k(G^{-1}(s_i + G(g_i))) - \sum_{j \in M^2_k} L_j)}$, where $h \in \{1, \ldots, I\}$ is any Genco (there may be more than one in the case of tied bids) with the largest value of the index $s_i + G(g_i)$ satisfying $s_h + G(g_h) < G(U'(\sum_{i \in M^2_h} L_i))$. \quad (9)

**Proof:** See Appendix.

The structure of the optimal portfolio captured in Theorem 1 is relatively simple. It calls for the Disco to rank all offers in terms of a single index $s_i + G(g_i)$ and then to pull off as much capacity as allowed by Genco $i$, proceeding in rank order of the contract index until the marginal WTP is exceeded by the contract index. Since $G$ (and therefore $G^{-1}$) is strictly increasing ($G'(x) = 1 - F(x) > 0$), and since by concavity (see (2)) $D_k$ is (strictly) decreasing, and therefor $Q_i$ is decreasing in the contract index for all $i = 1, \ldots, I$. Before WTP is exceeded, the Disco takes all capacity offered by Gencos from whom it contracts. Of course, WTP may be exceeded with the first Genco and the Disco may, in fact, sign no contracts whatsoever (if $G(U'(0)) \leq s_1 + G(g_1)$).

The contract index $s_i + G(g_i)$ reflects the cost of reserving a unit of capacity plus the cost of maintaining the option to execute the contract at the price $g_i$ rather than using the spot market at price $P_s$. This captures the industrial marketing practice in paying the cost of reserving capacity, which is the payment to the Genco for the opportunity cost of their committing their capacity to a specific contract. Optimal contracting for the Disco is determined by adding to this reservation price the risk hedging benefit of maintaining the option to purchase at the fixed execution fee $g_i$ rather than facing spot market volatility. From the noted properties of $U$ and $G$, Disco’s contract amounts are decreasing in both $s_i$ and $g_i$ as expected.

We now turn to the Gencos’ optimal bidding strategies for $(s_i, g_i, L_i)$. We assume throughout that Gencos face stringent penalties for non-performance under contract so that they will, in fact, post no more than $L_i \leq K_i$ as available capacity and that they will set prices $(s_i, g_i)$ so that contracted amounts will not exceed $L_i$.

**Lemma 2 (Optimal Gencos’ Bidding Capacity):** $L^*_i = K_i$, $\forall i$.

**Lemma 3 (Optimal Execution Fee Bids by Gencos):** $g^*_i = b_i$, $\forall i$.

**Proof:** See Appendix.
Thus, we will refer to \( g_i = b_i \) as a \( g \)-dominant bidding strategy for Gencos, since it is optimal no matter what total effective bid is made, i.e., no matter what the value of \( s_i^* + G(g_i) \). [Note that this strategy is not "dominant" in the usual game-theoretic sense since the unqualified term "dominant strategy" means that it is optimal for a player no matter what \( s_i \) is and no matter what the strategies are played by other players in the game. As is rather clear, and will be made explicit below, the optimal value of \( s_i \), and therefore also of \( s_i^* + G(g_i) \), bid by a particular Genco will depend on what bids are made by other Gencos. However, the term "\( g \)-dominant" seems appropriate here given the strong optimality properties of the strategy \( g_i = b_i \).] The rationale for Lemma 3 is that there is a tradeoff for Gencos between charging higher \( s_i \) and higher \( g_i \), depending on their market power relative to competitors. Charging higher \( g_i \) erodes the options value (recall the contract index \( s_i + G(g_i) \)) of the benefit Discos see from contracting more quickly than the marginal benefits associated with increases in \( s_i \). The lowest level for \( g_i \), namely \( b_i \), is therefore the result.

**Lemma 4 (Lower Bound for Reservation Charge):** Let \( g_i = b_i, \forall i \). Then

\[
s_i^* \geq s_i^* \overset{def}{=} E \{ m_i(P_s)(P_s - b_i)^+ \}, \forall i.
\] (10)

Moreover (when \( g_i = b_i \)), Genco \( i \) will find it in his interest to participate in the contract market at any reservation fee \( s_i \geq s_i^* \).

The rationale for Lemma 4 is that the Genco does not want to make any less per unit of capacity than it could by committing to sell its capacity into the spot market (although access to the spot market is imperfect and occurs on the day only with probability \( m_i(P_s) \)). Equating the two possibilities leads to Lemma 4. From Lemma 4, we know that when there is a long-term contract market, Genco \( i \) could still be used even if \( \beta_i > E \{ m_i(P_s)(P_s - b_i)^+ \} \), but Genco \( i \) may or may not break even if \( \beta_i \) is sufficiently large, though participation in the contract market, when it is appropriate, will lead in any case to increased profitability. Recall that we are dealing with the short-run problem here in which capacities are not variable, so that negative profits are indeed possible if investments are sunk and capital costs sufficiently high relative to what the contract and spot market will bear. Note also from Lemma 4 that if \( m_i(P_s) \) is with very high probability near unity, then the Genco will also face diminished incentives to contract, since the spot market then provides a viable alternative to contracting.

The reason the Genco finds participating in the contract market attractive at any \( s_i \geq s_i^* \) is that the contractor has nothing to lose at such a reservation fee. If any units of the contract are purchased at \( (s_i, g_i) \geq (s_i^*, b_i) \), then the Genco earns at least as much from such units as in the spot market. Whatever portion of the bid capacity is not purchased in the contract market can always be sold on the spot market, incurring precisely the same risk as if the capacity had not been offered in the contract market. The reason is that we assume Gencos can always sell unused capacity in the spot market (subject to market access risk \( m_i(P_s) \), which is not affected by the amount they bid in the contract market). We will refer in the sequel to \( s_i^* \) as the minimum contract-feasible (reservation price) strategy for Genco \( i \).

We are now in a position to derive market equilibrium prices in the short-term and long-term contract market. Several definitions of market equilibrium might be used. The approach we take is to assume that Gencos all know the total demand function of the Disco, consistent with sophisticated Gencos and well-developed markets. Gencos only know their own costs and capacities and bid these into the market via an electronic bulletin board. Gencos adjust their bids until they achieve a Nash equilibrium in this market. We refer to this equilibrium as a von Stackelberg Leader-Follower Equilibrium (vSLFE) to account for the assumption that Gencos anticipate Disco responses to their actions, given what they observe other Gencos to be bidding on the electronic bulletin board. The normal form game of interest is straightforward. The players are the Gencos. The strategies are the triples \( \{(s_i, g_i^*, L_i) \mid i = 1, \ldots, J \} \) and the utility functions of the players are the expected profit functions. The von Stackelberg assumption is evident in this game through our substitution into the expected profit functions of the anticipated Disco demand functions \( Q_i(s, g, L) \) as derived from Theorem 1. Given the results of Lemmas 1-4, we will actually restrict attention to a simpler game in which strategies are only concerned with the capacity reservation charges and in which certain knife-edge cases are ruled out for simplicity.

We have the following profit functions (in the short run, i.e., neglecting adjustments in capacity, see [3]):

\[
E \pi_i(s, b, K) = [s_i - s_i^*]Q_i(s, b, K),
\]

where \( s, b, K \) are, respectively, the vectors of \( \{s_1, \ldots, s_J\}, \{b_1, \ldots, b_J\}, \{K_1, \ldots, K_J\} \). We are interested for the moment only in short-term pricing strategies satisfying Lemmas 1-4. For such strategies, the only interesting parameter is the "price" index: \( p_i = s_i + G(b_i) \). From Lemma 4, we know that this price index must satisfy \( p_i \geq c_i \) with \( c_i = s_i + G(b_i) \). Using this notation, we can express the profit functions for Gencos in the following simplified form:

\[
E \pi_i(p) = [p_i - c_i]Q_i(p),
\]

where \( p \) is the vector of \( \{p_1, \ldots, p_J\} \), and \( Q_i(p) = Q_i(s, b, K) \) is given by our Theorem 1. We will characterize contract equilibria of interest in \( p \)-vector space.
For notational simplicity, define \( D(x) = D_s(G^{-1}(x)) \).

Definition: For any potential equilibrium set \( M \subseteq \Xi \) and any \( k \in M \), define \( f_k(p_k) = (p_k - c_k)(D(p_k) - \sum_{i \in M^0_k} K_i) \), where \( M^0_k = M - \{ k \} \).

Lemma 5: If there exists an equilibrium, then it must be symmetric for all Gencos providing positive capacity in the contract market. That is, every equilibrium \( p^* \) must be of the form \( p^*_i = x, \forall i \in M \), where \( Q_i(p^*) > 0, i \in M \) and \( Q_i(p^*) = 0, i \in \Xi \setminus M \).

Proof: Take any supposed equilibrium \( p \) and let \( M \) be the subset of Gencos at this equilibrium supplying positive capacity in the contract market. Suppose that the equilibrium \( p \) is not symmetric, so that \( p_1 = \min\{ p_i \mid i \in M \} < \max\{ p_i \mid i \in M \} = p_2 \), and such that \( Q_1 > 0 \) and \( Q_2 > 0 \). Then, as an equilibrium strategy for player 1, \( p_1 \) must be a best response, and moreover \( Q_1 = K_1 \). But consider the new bid for player 1: \( p'_1 = (p_1 + p_2)/2 \) which is now strictly greater than \( p_1 \) and strictly less than \( p_2 \). Now \( Q_1 = K_1 \) still obtains, but clearly profits are strictly higher for player 1 at \( p'_1 \) than at \( p_1 \), keeping all other players’ strategies fixed at their equilibrium values. Thus, the original \( p \)-vector cannot have been an equilibrium. This contradiction obtains as long as the lowest bid is lower than the highest bid for Gencos in \( M \). Q.E.D.

Given Lemma 5, we will abuse notation somewhat in what follows and denote by \( p^* \) the equilibrium price, such that for those Gencos \( i \in M \) having positive contract capacity at equilibrium \( p_i = p^* \), \( \forall i \in M \).

Theorem 2 (Short-Term von Stackelberg Leader-Follower Equilibrium): For any equilibrium price \( p^* \), denote by \( M(p^*) \subseteq \Xi \) the equilibrium set of all Gencos having positive capacity contracts, i.e., \( Q_i(p^*) > 0, i \in M \) and \( Q_i(p^*) = 0, i \in \Xi \setminus M \). Assuming \( |M| \geq 2 \) and \( \min\{ c_i \mid i \in \Xi \} < G(U'_i(K_i)) \); then the necessary and sufficient conditions for an equilibrium \( p^* \) to exist are (i) \( D(p^*) = \sum_{i \in M} K_i \); (ii) \( f_k(p_k) < 0 \) if \( p_k > p^* \); and (iii) \( \forall j \in \Xi \setminus M, p^* < c_j \).

Proof: See Appendix.

Corollary 1 (Short-Term sSLFE for Single Element Equilibrium Group): When \( |M| = 1 \), the only Genco providing positive contract output (which we denote as Genco 1) satisfies \( c_1 = \min\{ c_i \mid i \in \Xi \} < G(U'_i(0)) \). The necessary and sufficient conditions for a single-Genco equilibrium \( p^* \) to exist are (i) \( p^* = \max\{ p_i^H, x_i^H \} > c_1 \), where \( p_i^H \) as the solution to \( \max f_i(p_i) = (p_i - c_i)(D(p_i)) \), and \( x_i^H \) as the solution to \( D(x^H) = K_1 \), and (ii) \( p^* < \min\{ c_i \mid i \in \Xi \setminus \{ 1 \} \} \).

Example (Identical Cost Gencos): In the case where all Gencos have the same cost, i.e., \( \forall i, j \in M, c_i = c_j \), then if an equilibrium exists (see Theorem 2 for conditions) it will entail \( M = \Xi \).

Theorem 3 (Uniqueness of set M): If there exists any equilibrium group \( M \subseteq \Xi \), it must be unique.

Proof: First we notice that, for any equilibrium group, it must accept group members along the index line, i.e., if Genco \( k \) is in the equilibrium group \( M \), then, \( \forall i < k \) must be in \( M \). Therefore, if there are two separate equilibrium groups \( M_1 \) and \( M_2 \), then one must be the strict subset of the other, assume \( M_1 = \{ 1, \ldots, l \} \subseteq M_2 = \{ 1, \ldots, l, l+1, \ldots, m \} \), denote \( p'_1(i = 1,2) \) as the equilibrium price of \( M_i(i = 1,2) \), then we must have \( p'_1 > p'_2 \). Consider any Genco \( k \in \{ l+1, \ldots, m \} \) which belongs to \( M_2 \) but not \( M_1 \), since it does not belong to \( M_1 \), we have \( p'_1 \leq c_k \). On the other hand, \( M_2 \) is a large set than \( M_1 \), we have \( p'_2 < p'_1 \), thus we obtain \( p'_2 < c_k \), this suggests Genco \( k \) is price out of \( M_2 \), which is a contradiction to the fact that Genco \( k \) is indeed a member in \( M_2 \).

Theorem 4 (Computation of set M): \( M \) can be computed in the following way.

(i) We index Gencos in order of \( c_i \), i.e., \( c_1 \leq c_2 \leq \ldots \leq c_N \), for convenience, define \( c_{N+1} = \infty \), then \( M = \{ 1, \ldots, h-1 \} \), where \( h \) is the smallest index that satisfies,

\[
G(U'_i(\sum_{i=1}^{h-1} K_i)) < c_h = s_h + G(b_h).
\]

(ii) For any \( k \in M \), if \( f'_k(p_k) < 0 \), when \( p_k > p^* \), then \( M \) is the unique equilibrium group, otherwise, there does not exist any equilibrium subsets of \( \Xi \).

Proof: From (i) of Theorem 2 we have \( D(p^*) = \sum_{i \in M} K_i \), which is equivalent to \( D_s(G^{-1}(p^*)) = U'_i(G^{-1}(p^*)) = \sum_{i \in M} K_i \), therefore \( p^* = G(U'(\sum_{i=1}^{h-1} K_i)) \). From (iii) of Theorem 2, we know that \( \forall j \in \Xi \setminus M, p^* < c_j \), since Genco \( h \) does not belong to \( M \), we have \( p^* < c_h = s_h + G(b_h) \). Hence we obtain \( G(U'(\sum_{i=1}^{h-1} K_i)) < c_h = s_h + G(b_h) \), and identify set \( M \). If every member in this set can past the test of condition (ii) of Theorem 2, then it satisfies all necessary conditions of theorem 2, thus it is the unique equilibrium group. Q.E.D.
3. Multi-Genco, Multi-Disco, General Network

This section considers multiple Gencos and multiple Discos, containing the most general results in this paper. These results will be seen to follow very closely the structure of previous results, so we will spare the reader the full development of the arguments.

From the point of view of a particular Disco, the existence of other Discos does not affect its own demand. Thus, on the day, each Disco’s problem remains the same as in the Multi-Genco-Single-Disco case. Hence the following Lemma 6 is a direct consequence of Lemma 1.

**Lemma 6:** Let \( \phi_j = (P_s, Q_{ij} \ (i = 1, \ldots, J)) \), \( j \in \{1, \ldots, J\} \) be given, where \( Q_{ij} \) is contracted capacity by Disco \( j \) with Genco \( i \). Define Disco \( j \)'s demand function \( D_j(\phi_j) \) as the solution to Maximize \( \{ V_j(D_j, q_j, x_j, \phi_j) | D_j \geq 0, q_j \geq 0, x_j \geq 0 \} \), where \( q_j \) and \( x_j \) are purchases by Disco \( j \) under contract and from the spot market, respectively, and where

\[
V_j(D_j, q_j, x_j, \phi_j) = U(D_j) - \sum_{i=1}^{I} s_i Q_{ij} - \sum_{i=1}^{I} g_i q_{ij} - P_s x_j. \tag{11}
\]

Then \( D_j(\phi_j), q_{ij} \) and \( x_j \) are given by Lemma 1 (with obvious adjustments for the subscripts \( j \)).

Although the presence of other Discos will not affect the structure of demand, a key issue where multiple Discos are present is who will have precedence for the more preferable Gencos (those for which the shopping index \( s_i + G(g_i) \) is lowest). As we note below, this will not be an issue in equilibrium, since just as in Theorem 2, so too here the shopping index of all Gencos in the money will be set equal to the “market index”. But clearly the operation of the market can depend on how competing Discos are allocated among Gencos. We follow the standard economic assumption that whenever two or more Discos compete for the same Genco contract, the Disco with the highest WTP will be the Disco awarded the Genco’s capacity, at least up to the point at which some other Disco does not have a higher WTP. Under this allocation procedure, the Genco offering capacity at \( (s_i, g_i) \) will sell just as much as he would to a single Disco with aggregate demand equal to the sum of the demands of all the Discos. Thus, the consequence of this normal economic allocation procedure is the following Corollary.

**Corollary 2 (Disco’s Optimal Consumption Portfolio):** Given the demand functions \( D_{ij}(p), p \geq 0, j = 1, \ldots, J \), define the aggregate Retail demand as \( D_s(p) = \sum_{j=1}^{J} D_{ij}(p) \) and aggregate marginal WTP as \( U'(Q) = D_s^{-1}(Q), Q \geq 0 \). Greedy Contracting is optimal for every Disco \( j \), as specified in Theorem 1, in the sense that Discos will pick off contracts from Gencos offered to them in the order of the Shopping Index \( s_i + G(g_i) \). Moreover, if Discos are offered contracts in order of their WTP, then total demand for each Genco \( i \) will be the same as when the Genco faces a single Disco with demand \( D_s(p) \) and marginal WTP \( U'(Q) \).

In a similar fashion, Lemmas 2, 3 and 4 in the previous sections hold since the Gencos’ problems all remain the same, where now \( Q_i \) is to be understood as aggregate supply to all Discos by Genco \( i \). In particular, from Lemma 2 and Lemma 3, \( L_i = K_i \) and \( g_i = b_i \) continue to be, respectively, dominating and \( g \)-dominating strategies for the Gencos. Most importantly, Corollary 1 implies that the game among Gencos, and its equilibria following the von Stackelberg model assumed here, is given by Theorem 2, with aggregate Retail demand, as specified in Corollary 1, and its inverse aggregate WTP \( U'(Q) \) playing the role of the “Retail sector”. The remaining logic of Theorem 2 and its proof is then identical. The result is the following.

**Corollary 3 (von Stackelberg Leader-Follower Equilibrium (vSLFE) for the Multi-Disco, Multi-Genco Case):** Consider the Multi-Disco, Multi-Genco Case in which competing demands by Discos for a particular Genco are allocated in order of Disco WTP. For this case, the conditions for the existence of a vSLFE to the Multi-Disco, Multi-Genco Case \( \{(s_i', g_i', L_i') | i = 1, \ldots, I\} \) are identical to those specified in Theorem 2 for the given \( D_s \) and \( U' \).

Thus, under the assumption that competing Disco demands for the same Genco contract are rationed in order of Disco WTP, the solution to the general network case is solved. This general solution results from using the demand and equilibrium contracting building blocks of Disco contract demand and of the market equilibrium results for the Multi-Genco, Single-Disco case. The equilibrium characterized in both Theorem 2 and Corollary 3 has important prescriptive properties to it that should be mentioned in concluding our discussion. The \( g \)-strategy is \( g \)-dominant for the contract capacity provided, the capacity strategy is a dominant strategy, and the \( s \)-strategy is the transparent outcome of the pivot Genco \( h \) who is the price setter in the contract market. Whether this equilibrium is stable or has other important properties behaviorally than the dominance properties noted remains to be seen through further theoretical and experimental work. The key matter to note here is that once the game is understood to be completely specified in terms of the shopping index \( s_i + G(g_i) \), considerable degrees of freedom are removed from the problem. Most importantly, the above results establish this index as the critical focal point for rational strategies for a particular Genco, whatever
strategies are played by other Gencos.

4. Planned Empirical Studies Based on the Model

The authors have arranged to undertake a case study of this methodology with a large, national power marketing company, denoted here as POWMARCO, active in the PJM pool as well as in several other pools. The authors will use this case study as a benchmark test of the above model and results. The case study envisaged entails the following foundations. First, there must be an assessment of the stochastic process describing forecasted spot prices in the PJM pool for specific future periods (e.g., month ahead prices). This problem is, of course, of independent interest and under active research by POWMARCO and others. In addition, it will be important to obtain data on the structure of current bid prices and costs of available capacity bid into the PJM pool [Preliminary estimates by POWMARCO indicate, for example, that typical plants bidding into PJM have annualized capital costs ($\beta$) of $500/kW, average running costs of (b) $2-3/MWh, day-ahead typical bid-ask prices of $50 to $55/MWh, with estimated mean of spot prices a month ahead for the Western Hub of PJM varying widely (depending on hour of the day, location etc.) being between $26 and $93/MWh, and with access probabilities (m) for specific paths and plants varying between 0 to .99, depending on conditions.]. Finally, transmission access probabilities, under various load conditions and for various plants and paths, will be studied with POWMARCO transmission specialists.

Following the above preliminary data matters, several key studies are planned. First, we intend to compare actual bid prices by POWMARCO with optimal bid prices and bid constraints implied by Lemmas 1-4. Second, we will study the relationship between the bid prices of POWMARCO and the best response bid prices embodied in Theorem 2 for a few final energy markets served by the PJM pool. This will require an assessment of the structure of energy demand in the markets of interest, partly building on POWMARCO’s marketing studies. Since there is currently considerable price dispersion in the contract market in PJM (in contrast to the single index equilibrium derived in Theorem 2), contrasting observed results with Theorem 2 should lead both to additional profit opportunities for POWMARCO as well as to additional insights into bidding behavior of market participants.

5. Summary

This paper has provided a general solution to the Multi-Genco, Multi-Disco supply chain contracting problem for electric power. The key question addressed is the structure of the optimal portfolios of contracting and spot market transactions for these Gencos and Discos, and the pricing thereof in market equilibrium. We show that when Gencos properly anticipate demands to their bids, then a g-dominant strategy for the contract execution price is to truthfully reveal the Gencos’ variable cost, with reservation fees determined by Gencos to trade off the risk of underutilized capacity against unit capacity costs. Discos’ optimal portfolios are shown to follow a merit order (or greedy) shopping rule, under which contracts are signed following an index that combines the Gencos’ reservation cost and execution cost. These results are extensions of our earlier theoretical results for the Single-Genco case to the Multi-Genco/Multi-Source case.

It is important to note that the results establish why single-parameter contract structures are not efficient for the electric power market studied here. For example, if the market were for some reason constrained to bids (s, g, L) of the form (s, 0, L) or (0, g, L), then Lemma 3 shows that Gencos would find this bidding structure dominated by the more general two-parameter family of contracts studied here. In addition, the efficiency of the market itself would be adversely affected if contracts were to be constrained to either of these forms. For example, if for some Genco i, g_i ≠ b_i, the Genco would not be called to produce in merit order relative to the spot market leading to ex post inefficiency relative to the optimal contracts characterized in this paper. Thus, heterogeneous production costs and non-scalability of output require for efficiency both a reservation price and a cost-based execution fee.

It would be important to extend this framework to the case where contracting or re-contracting could take place along a temporal continuum. In the latter case, options pricing results can be expected to reflect the changes in information about the value of the spot price as time progresses to the day of physical delivery. We would expect, based on the results of this paper, that not only will the stochastic evolution of the spot market price be important to valuing such options, but also the evolution of predicted access to the market by various Gencos (the m_i(P_s) of our framework). Since one might expect that spot market price and m_i(P_s) are likely to be correlated (depending on congestion and on search intensity of Discos), the resulting framework could be quite interesting. In particular, it could link to the effects of Internet access and shopping since such access is likely to increase the probability of finding customers at the last minute. These belong to ongoing research projects.

6. Appendix

Proof of Lemma 1
Take any $i = 1, \ldots, J$ and any fixed total demand $D$. Con-
tract purchases will be in order of increasing execution fees $g_i$ as long as $g_i < P_s$. We can therefore assume that, for any $l < i$, all capacity under contract $l$ will be used prior to taking any output under contract $i$.

Now let $k \geq 0$ be the Genco providing the last unit of contract output, so that $k$ satisfies $g_k < P_s \leq g_{k+1}$. Under the No Excess Capacity Condition, the Disco will never contract for more than needed, so that for every $k = 1, \ldots, I$, $Q_k = 0$ unless $\sum_{i=1}^{k} Q_i \leq D_s(g_k)$ or equivalently, since $D_s = U'(g_k), g_k \leq U'(\sum_{i=1}^{k} Q_i)$. Thus, on the day, depending on the spot market price, the Disco faces only the following two cases under which $q_k > 0$ and $q_i = 0$ for $i > k$:

1. $q_k \leq P_s \leq U'(\sum_{i=1}^{k} Q_i)$,
2. $g_k \leq U'(\sum_{i=1}^{k} Q_i) < P_s$.

Using the fact that Genco $k$ provides the last unit of contract capacity, we have $q_i = Q_i$ for $i < k$, $q_i = 0$ for $i > k$ and $q_k \leq Q_k$. Thus, noting that $P_s \geq \max\{g_0, \ldots, g_k\}$, we obtain the following expression for Disco’s utility $V$ after substituting the derived expressions for optimal $q_i$ and $x = D - \sum q_i$ as functions of $D$:

$$V(D, q(D), x(D), \phi) = U(D) - \sum_{i=1}^{I} s_i Q_i - \sum_{i=1}^{I} g_i q_i - P_s (D - \sum_{i=1}^{I} q_i)$$

$$= U(D) - P_s D - \sum_{i=1}^{I} s_i Q_i + \sum_{i=1}^{k-1} (P_s - g_i) Q_i + \min_{i=1}^{k-1} D - \sum_{i=1}^{k-1} Q_i, Q_k.]$$ (12)

To determine $D(\phi)$ we need to maximize this concave function $V(D, q, x, \phi)$ over $D \geq 0$. We do this in two steps. In step 1 (which corresponds to case 1 above), we solve this problem where the min is the second quantity in the $[.]$ in (12). This leads directly to $D(\phi) = D_s(P_s)$. In step 2 (which corresponds to case 2 above), we solve this problem where the min is the first quantity in the $[.]$ in (12), i.e., subject to $D \leq \sum_{i=1}^{k} Q_i$. Since in any case $g_k \leq U'(\sum_{i=1}^{k} Q_i)$, this leads to $D(\phi) = \sum_{i=1}^{k} Q_i$. Combining these two cases, shows the validity of the alternative characterization of $D(\phi)$ in the statement of Theorem 1. The characterization (5) is easily seen to be equivalent to this, so that Lemma 1 follows. QED.

Proof of Theorem 1
The reader should keep in mind in this proof that the order assumed in this Theorem, implied by non-decreasing values of the index $s_i + G(g_i)$, may be a different order than the order assumed in Lemma 1 in terms of non-decreasing values of $g_i$.

Assume $s_i + G(g_i) < G(U'(0))$ since otherwise the Disco is not willing to pay the effective price of even the lowest cost Genco (i.e., otherwise the Disco will have no incentive to accept any contracts). We first note that since $s_i + G(g_i)$ is nondecreasing in $i$ (by assumption) and since $G(U'(\sum_{i=1}^{I} L_i))$ is nonincreasing in $i$ by the properties of $G$ and $U'$, then if Genco has the largest value of the index $s_i + G(g_i)$ satisfying (9), then every $i < h$ will also satisfy the following condition:

$$s_i + G(g_i) \leq s_{i+1} + G(g_{i+1}) < G(U'(\sum_{i=1}^{i} L_i)),$$ (13)

with the left inequality following from the assumed ordering in terms of the shopping index. For suppose the right inequality in (13) did not hold at some $i < h$. Then again by the ordering of the index and the monotonicity of $U'$ and $G$, we would have:

$$s_h + G(g_h) \geq s_{i+1} + G(g_{i+1}) \geq G(U'(\sum_{i=1}^{h-1} L_i)) \geq G(U'(\sum_{i=1}^{h} L_i)),$$

contradicting the definition of $h$ in (9). We note, in particular, from (13) and the monotonicity of $G$ that the optimal $Q_i, i = 1, \ldots, I$, given in the Theorem statement satisfy the No Excess Capacity Condition.

Now for any $\phi = (P_s, s_i, g_i, Q_i (i = 1, \ldots, I))$, we can substitute the optimal contract and spot purchases $q_i$ and $x$ from Lemma 1 to obtain the following simplified form of the Disco’s utility (1) as a function of $\phi$

$$V(\phi) = U(D(\phi)) - P_s D(\phi) - \sum_{i=1}^{I} s_i Q_i + \sum_{i=1}^{I} (P_s - g_i) Q_i.$$ (14)

where $D(\phi)$ is given by (5). Define $EV(Q)$ as the expected value w.r.t. $P_s$ of $V(\phi)$. Clearly there exists an optimal solution to the problem of maximizing the $EV(Q)$ since $V(\phi)$ is continuous in its arguments and we can, w.o.l.g., restrict attention to the compact set of non-negative contract levels $Q_i$ for which the No Excess Capacity Condition is satisfied. To characterize the optimal contract amounts, we derive FOCs by considering the derivatives of $V(\phi)$ for a fixed realization of $P_s$ and then take the expected values of these derivatives w.r.t. $P_s$.

From (14) we obtain for any $i \in \{1, \ldots, I\}$ that

$$\frac{\partial V(\phi)}{\partial Q_i} = \left(U'(D(\phi)) - P_s\right)\frac{\partial D(\phi)}{\partial Q_i} - s_i + (P_s - g_i)^+,$$ (15)
where $D(\phi)$ is given by (5). We wish to evaluate $\partial D(\phi)/\partial Q_i$.

Let $\hat{k}$ be the index of the Genco with the highest execution fee for which contract capacity is non-zero, i.e. $g_k = \max\{g_i \mid 1 \leq i \leq I; Q_i > 0\}$. Then the following relation holds for any $k$ such that $Q_k > 0$:

$$g_k \leq g_k^* \leq U'(\sum_{i=1}^I Q_i) \leq U'(\sum_{i=1}^k Q_i). \quad (16)$$

The first inequality follows since $g_k^*$ is the maximum execution fee for non-zero callable capacity. The middle inequality is essentially equivalent to the No Excess Capacity Condition since, by definition of $\hat{k}$, $Q_i = 0 \forall i > \hat{k}$, so that $\sum_{i=1}^k Q_i = \sum_{i=1}^\hat{k} Q_i$. The final inequality in (16) follows since $U'' = D^{-1}$ is monotonically decreasing.

From Lemma 1 we know that $D(\phi) = \max(D_i(P_s), \sum_{i=1}^k Q_i)$, where $k$ is the last contract executed given $\phi$. Thus, $D(\phi)$ depends on $Q_i$ if and only if, for some $k \geq \hat{k}$, $\sum_{i=1}^k Q_i = D_i(P_s)$ and $g_k < P_s \leq g_{k+1}$. Using the No Excess Capacity Condition, we further note that if $Q_k > 0$, then $D_i(g_k) \leq \sum_{i=1}^k Q_i$. We conclude from this and (16) that

$$\frac{\partial D(\phi)}{\partial Q_i} = \begin{cases} 1 & \text{iff } \exists k \text{ with } Q_k > 0 \\
\text{and } g_i \leq g_k & \text{iff } U'(\sum_{i=1}^k Q_i) < P_s, \\
0 & \text{else}. \end{cases}$$

But, given the definition of $\hat{k}$ and (16), this is equivalent to the following:

$$\frac{\partial D(\phi)}{\partial Q_i} = \begin{cases} 1 & \text{iff } i \leq \hat{k} \text{ and } U'(\sum_{i=1}^I Q_i) < P_s, \\
0 & \text{else}. \end{cases}$$

We also note from (16) that precisely when $\partial D(\phi)/\partial Q_i = 1$, we have $D(\phi) = \sum_{i=1}^I Q_i$, since if $P_s > U'(\sum_{i=1}^I Q_i)$ for any $k \geq i$ with $Q_k > 0$, then $P_s > U'(\sum_{i=1}^I Q_i)$. Thus, substituting $D(\phi) = \sum_{i=1}^I Q_i$ and interchanging expectation and differentiation, we finally obtain from (15)

$$\frac{\partial EV(Q)}{\partial Q_i} = \int_{U'(\sum_{i=1}^I Q_i)}^{\infty} (U'(\sum_{i=1}^I Q_i) - P_s) dF(P_s)$$

$$+ \int_{U'(\sum_{i=1}^I Q_i)}^{\infty} (P_s - g_i) dF(P_s) - s_i.$$ 

Since $G(p) = E\{\min[p_i, P_s]\} = E\{P_s - (P_s - p)^+\}$, the first two terms in this expression reduce to $G(U'(\sum_{i=1}^I Q_i)) - E\{P_s\}$ and $E\{P_s\} - G(\phi)$, respectively, yielding

$$\frac{\partial EV(Q)}{\partial Q_i} = G(U'(\sum_{i=1}^I Q_i)) - (s_i + G(\phi)). \quad (17)$$

Now note that since the first term is independent of $i$ in (17), $\partial EV/\partial Q_i$ is monotonic decreasing in $s_i + G(\phi)$. Thus, if for some $k \in \{1, \ldots, I\}$, $\partial EV/\partial Q_k \geq 0$, then $\partial EV/\partial Q_i \geq 0$ for any $i$ such that $s_i + G(\phi) \leq s_k + G(\phi)$. Finally, note that for the contract with the maximum such index, say $\hat{h}$, with $Q_{\hat{h}} > 0$, we must have $\partial EV/\partial Q_{\hat{h}} \geq 0$ with inequality unless $Q_\hat{h} = L_\hat{h}$. It follows that every contract with a value of the index $s_i + G(\phi)$ no greater than $\hat{h}$ will have $\partial EV/\partial Q_i \geq 0$ so that for such contracts $Q_i = L_i$. From this, we see that, in fact, $\hat{h} = h$ as defined in (9). The Theorem follows. Q.E.D.

**Proof of Theorem 2**

We first note that if the assumption $\min\{c_i \mid i \in \Xi\} < G(U'(0))$ is not satisfied, then there would be no positive contract equilibrium since, as noted in Theorem 1, in this case no Genco would be able offer a contract that would attract non-zero Disco demand.

**Sufficiency.** First we show that if there is a set $M$ that satisfies (i) and (ii), then $\forall k \in M$, the optimal contract price for $k$ is to charge $p_k = p^*$ as computed via (i). Furthermore, denote $p_{-k}$ as the price vector of other players in set $M$, given $p_{-k} = p^*$, $k$ has no incentive to either decrease or increase its price $p_k$ from $p^*$.

Case (a): Choose any $k \in M$, and suppose, $p_k < p^*$. We see immediately that all $k$'s capacity will be used, i.e., $Q_k(p_k, p_{-k}) = K_k$, that gives him a profit of $\Pi_k(p_k, p_{-k}) = (p_k - c_k)K_k$. By definition, we have $\Pi_k(p^*) = (p^* - c_k)D(p^*) - \sum_{i \in M} K_i$, by (i), we get $D(p^*) = \sum_{i \in M} K_i$, therefore, $\Pi_k(p^*) = (p^* - c_k)K_k$. Thus we obtain

$$\Pi_k(p_k, p_{-k}) = (p_k - c_k)K_k < (p^* - c_k)K_k = \Pi_k(p^*), \quad (18)$$

which shows that Genco $k$ has no incentive to decrease its price given that other players are charging $p^*$.

Case (b): Choose any $k \in M$, suppose, $p_k > p^*$. Since $p_k > p^*$ and from (i) we have $D(p^*) = \sum_{i \in M} K_i$, since the demand function $D(\cdot)$ is strictly decreasing, we get $D(p_k, p_{-k}) < D(p^*) = \sum_{i \in M} K_i$, therefore, Genco $k$'s capacity cannot be fully used, i.e., $Q_k(p_k) < K_k$. We notice,

$$\lim_{p_k \rightarrow p^*} \Pi_k(p_k, p_{-k}) = (p_k - c_k)Q_k(p_{-k}) \quad (p^* - c_k)K_k = \Pi_k(p^*).$$

If we take the limit to both side of equation (18), we get,

$$\lim_{p_k \rightarrow p^*} \Pi_k(p_k, p_{-k}) = (p^* - c_k)K_k = \Pi_k(p^*).$$

Hence, $\Pi_k(p_k, p_{-k})$ is continuous at $p^*$. Since $p_k > p^*$, we have

$$\Pi_k(p_k, p_{-k}) = f_k(p_k, p_{-k}) = (p_k - c_k)Q_k(p_k, p_{-k})$$

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Assume there exists an equilibrium set, \( M^0 \), of positive contract Gencos. By Lemma 5, we can further assume all Gencos in \( M^0 \) are able to sell all its capacity \( Q_k(p_k, p_{-k}) = K_k \). Clearly, \( p_k > p^* \) is to say, if we fix other Gencos’ strategy at \( p_{-k} \), then \( k \) has no incentive to increase its price given other players’ charge \( p_{-k} = p^* \).

Condition (iiii) insures that no one outside group \( M \) would have any incentive to charge a price \( p \leq p^* \), since doing so will lead to negative profits by Lemma 4. Q.E.D.

**Necessity.** Assume there exists an equilibrium set, \( M \), of positive contract Gencos. By Lemma 5, we can further assume all Gencos in \( M \) charge an identical price \( p^* \). Consider any Genco \( k \in M \), since \( M \) is an equilibrium, \( k \) has no incentive to deviate from the equilibrium price \( p^* \). That is to say, if we fix other Gencos’ strategy at \( p^* \), \( k \) does not want to decrease or increase his contract price \( p_k \). We discuss these two cases accordingly.

Case (a): Genco \( k \) does not want to decrease its price by charging \( p_k < p^* \). If he decreases his price, then he would either extract all the demand from the Disco by being the first unit and last unit provider \( Q_k(p_k, p_{-k}) = D(p_k, p_{-k}) \), leaving nothing for other Gencos in \( M \), or still be able to sell all its capacity \( Q_k(p_k, p_{-k}) = K_k \). Clearly, \( k \) has incentive to do so in the former case, since

\[
\lim_{p_k \to p^*} \pi_k(p_k, p_{-k}) = (p^* - c_k)D(p^*, p_{-k}) > (p^* - c_k)K_k \sum_{i \in M^0} K_i
\]

The fact that he does not decrease his price indicates that the latter is true, i.e., \( Q_k(p_k, p_{-k}) = K_k \), and \( \pi_k(p_k, p_{-k}) \leq \pi_k(p^*, p_{-k}) \). Therefore

\[
\pi_k(p_k, p_{-k}) = (p_k - c_k)K_k \leq \pi_k(p^*, p_{-k}) = (p^* - c_k)D(p^*, p_{-k}) \sum_{i \in M^0} K_i / \sum_{i \in M} K_i
\]

If we let \( p_k \to p^* \) in both side of the above equation, we obtain, \( K_k \leq D(p^*, p_{-k}) K_k \sum_{i \in M} K_i / \sum_{i \in M^0} K_i \) or equivalently, \( D(p^*, p_{-k}) K_k \sum_{i \in M} K_i / \sum_{i \in M^0} K_i \leq \sum_{i \in M^0} K_i \) or \( D(p^*, p_{-k}) \leq \sum_{i \in M} K_i \).

If we combine the results of case (a) and case (b), we obtain (i) in the Theorem 2, i.e., \( D(p^*, p_{-k}) = \sum_{i \in M} K_i \). Furthermore, the above reasoning shows that \( \pi_k(p_k, p_{-k}) \) is continuous at \( p^* \), and decreases if \( p_k > p^* \), it is straightforward to obtain (ii). Condition (iii) holds since if \( \exists j \in M \setminus i, p^* > c_j \), the Genco \( j \) has an incentive to join in \( M \), which contradicts our assumption that \( M \) is an equilibrium group. Q.E.D.

**References**

