Understanding price volatility in electricity markets

Fernando L. Alvarado, The University of Wisconsin
Rajesh Rajaraman, Christensen Associates

Abstract

This paper illustrates notions of volatility associated with power systems spot prices for electricity. The paper demonstrates a frequency-domain method useful to separate out periodic price variations from random variations. It then uses actual observed price data to estimate parameters such as volatility and the coefficient of mean reversion associated with the random variation of prices. It also examines spatial volatility of prices.

Keywords: electricity spot pricing, risk management.

Introduction

Prices in power markets vary both as a function of time of day (and week) and location. Prices also vary as a result of random and less predictable variations in demand, temperature and various market and system conditions. The two key questions of interest in this paper are:

• Can the periodic components of prices be identified in a simple and systematic manner?

• Can the random variations be characterized in some manner involving just a few parameters?

This paper develops procedures to estimate price volatility in an actual market characterized by efficient spot pricing [1]. The assumption is that the prices are available (retrospectively) for every location in the system on an hourly basis for a one-month period. The system prices observed over a one-month period are decomposed into a periodic and a random component using frequency domain analysis. The random component is then further characterized as a specific type of random process borrowing from techniques in widespread use for options pricing calculations.

Separating out periodic components

We begin the analysis by a characterization of the observed prices within the market of interest. The period analyzed involves one 31-day month (744 hours). The system marginal price for the period of study is illustrated in Figure 1. A certain periodicity of prices can be qualitatively observed. In the first phase of our analysis, we establish the periodicity characteristics of this “signal.” To do this, we perform a Fourier Transform of the signal in question. The absolute value of the signal under consideration is given in figure 2.

Figure 1: Actual price variability of the system price for a one month period.

Figure 2: Spectral content of the price variation. At least five periodic harmonics are evident as indicated. Also, an exponential fit to the spectral magnitude amplitudes is illustrated.
The periodic harmonics identified from the graph correspond to periods of 31 (daily variation), 62 (every other day), 124 (every fourth day), 155 (every fifth day) and 217 (weekly variation).

The next step in our analysis is to separate the random component from these periodic variation components. In order to do so, we perform a curve fit in the frequency domain spectrum, and adjust the height of these components to the value of the fit curve. The fit process proceeds as follows:

- Determine all the frequencies for which a periodic component is suspected. In our example, this involves harmonics 31, 62, 124, 155 and 217.

- At each of these frequencies, determine a scaling factor as the ratio of the amplitude of the exponential to the amplitude of the corresponding harmonic. This ratio should always be less than one, otherwise the harmonic cannot possibly be considered a periodic component, rather it must be considered part of the noise.

- Scale the complex frequency domain amplitudes of each of these harmonics (including scaling their aliased counterparts in the upper portion of the spectrum) so that their amplitude is precisely equal to the exponential amplitude. This is illustrated in Figure 3(a).

- The “remainder” after scaling consists of a simple spectrum of a few isolated frequencies (5 in our example) that can be illustrated in the time domain. This is illustrated in Figure 3(b).

- After constructing the periodic signal in the frequency domain, restore it to the time domain by inverse Fourier transformation. The results of restoring both the periodic and the random components of the signal to the time domain are illustrated in Figure 3(c).

**Volatility determination**

Having removed the periodic components, it is now of interest to determine the characteristics of the random component, primarily the price. The determination of volatility is a key element of the theory of options pricing [2].

Volatility plays many other important roles in the assessment of risk in markets. We begin our discussion of volatility with a characterization of a Wiener process. To understand an ordinary Wiener process, consider a small interval of time \( \Delta t \) and let \( S \) be a variable function of time. Assume that the change in \( S \) over one time interval is \( \Delta z \). For a Wiener process, \( \Delta S = \sigma N(0,1)\sqrt{\Delta t} \), where \( N(0,1) \) is a zero mean unity standard deviation random distribution. That is, \( S_{k+1} = S_k + \sigma N(0,1)\sqrt{\Delta t} \). A Wiener process is used to describe random walks and Brownian particle motion.

![Figure 3: Frequency spectrum after separation between periodic and random signal components.](image)

![Figure 4: Time domain signal reconstruction for the time domain and the random components of price.](image)
“unnatural” behavior near zero for items that are never supposed to reach zero or negative value (such as stock prices)\(^1\). A Wiener process will often lead to negative values.

An alternative assumption is to assume that it is the logarithm of the variable that drifts according to a Wiener process. This is sometimes called an Ito process. In this process:

\[
\ln \left( \frac{S_{k+1}}{S_k} \right) = \sigma N(0,1) \sqrt{\Delta t}
\]

That is, the logarithms of the prices follow a Wiener process. This is called a lognormal process. Let there be \(n+1\) prices observed (744 in our example), where the intervals are numbers from 0 to \(n=743\). Let the observed price of interest for interval \(i\) be \(S_i\). For the usual case of estimating prices of a stock, the usual definition of \(u\) is:

\[
u_i = \ln \left( \frac{S_i}{S_{i-1}} \right)
\]

This definition fails in situations where zero or negative prices are expected. Negative prices during any time period can and do occur, sometimes as the result of low demand and nonzero startup and shutdown costs, and other times as a result of transmission congestion. In fact, our example contains zero prices during some hours. Thus, rather than using the lognormal assumption for observed prices, it is perhaps better to assume an ordinary Wiener process with a normal distribution of price differences. In this case, we define \(u\) as follows:

\[
u_i = S_i - S_{i-1}
\]

The estimate of the standard deviation \(s\) of this process is:

\[
s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2}
\]

where \(\bar{u}\) represents the mean value of the \(u_i\)’s. The volatility \(\sigma\) for the random component of the process (expressed in yearly terms) is:

\[
\sigma = \frac{s}{\sqrt{\tau}}
\]

where \(\tau\) is the length of the interval in years (or 1/12, in our example).

Using this methodology for estimating the volatility, for the example at hand we obtain an annualized volatility of 21.8.

In order to better understand the meaning of this volatility, we now generate a completely artificial price process, using this implied volatility. We assume a zero-mean value for the price. The time step is one day, or 1/360 years. Assuming that there is no drift (or rather, that any drift or periodic components is accounted as a deterministic additive term), we then obtain that random prices can be generated as follows:

\[
\Delta S = \sigma N(0,1) \sqrt{\Delta t}
\]

Using a starting price of zero, a sample sequence generated by this random process is illustrated in figure 5(a). Observe the significant qualitative differences between this sequence and the actual observed prices. In particular, observe the tendency of this price to “drift”.

It is clear that the assumption of a simple random walk process does not fit the empirical evidence well. The reason for the discrepancy is that (a) there appears to be a strong “mean reversion” component to the random variation, and (b) there seems to be a “jump” behavior to the prices [3,4,5]. We now repeat the same experiments but we use the following expression instead:

\[
S_i = a S_{i-1} + \sigma N(0,1) \sqrt{\Delta t}
\]

Here \(a\) represents a coefficient of mean reversion (\(a<1\)). The mean is zero in this example. Figure 5(b) illustrates the same simulation as before, but this time with a mean reversion coefficient.

Mean reversion is not sufficient to replicate realistic price data. There seem to be “jump processes” associated with this data. Jump processes have also been thoroughly studied in the literature [Merton].
Figure 5(c) illustrates the same simulation but including jump processes. The jump processes have a separate mean-reversion coefficient and the probability of upward jumps is different from the probability of downward jumps. This model seems to come the closest to replicating actual price behavior.

Volatility by location

The data we have used also includes data for the price at individual buses in the system. A total of more than 2000 locations are considered in the study. For each price at each location the same analysis as has been done for the system prices can be repeated. The result of this analysis for the period of interest (one month) is a distribution of average prices and a distribution of locational volatilities. These are summarized in this section.

The objective here was to study all 2000-plus locations and to determine the volatility coefficient for the price at each location, to establish whether volatility is fairly constant across the system or whether it varies significantly by location. The results of this volatility analysis are summarized in the histogram illustrated in Figure 6.

It is clear from this histogram that most of the locations have more or less the same volatility in prices, but there are a few locations (83 to be exact) that exhibit significantly higher volatility in their prices. No location exhibited significantly lower volatility than the system volatility.

Figure 6: Histogram of observed volatilities at all locations. The first column contains almost all of the 2000 values. Thus, only a few locations exhibit high price volatility.

Conclusions

An empirical study of the price volatility in actual power markets has been conducted. This study suggests that frequency-domain methods are an outstanding tool to identify periodic components of price variability. The study also illustrates the features of a model that better represents volatility of random components as a combination of random walk and jump processes with mean reversion.

References

