

Multimode Competition in Lasers in the Light of Optical Feedback

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Abstract— The modes of a laser resonator experience optical amplification in the same gain medium, e.g. a laser crystal, in which they spatially overlap to a significant extent. This leads to the phenomenon of mode competition or gain competition, which can be studied extensively using the optical feedback mechanism. This paper analyses the existence of two modes in a laser system in the near infra red (NIR) range. Emphasis has been laid on the position and wavelengths of the two modes. Histogram analysis of the full width at half maximum (FWHM) of the modes mirrors remarkable upshots that have been elucidated in some detail.

Keywords- optical feedback; mode competition; full width at half maximum; semiconductor lasers

I. INTRODUCTION

The working of semiconductor lasers is profoundly affected by the mode-competition that is intrinsically caused by the nonlinear gain saturation effect [1]. This may in turn enhance the coupling of the side modes with the dominant mode so strongly that breeds mode hopping ([2]-[5]). Analysis of mode competition in semiconductor lasers is commonly performed by applying the approximate small-signal analysis to treat the rate equations that account for the intrinsic fluctuations in the modal photon number and electron number [6]. However, this analysis is applicable for cases of small fluctuations compared to the dc-intensity, and its treatment becomes intricate and complex if a large number of modes exist. The simultaneous existence of multiple modes in an instantaneous manner adds to the complication involved in finding a sustainable solution. Moreover, being a numerical method, the problem depends heavily on the selected parameters of the rate equations as well as the method and its internal parameters itself.

Distributed Feedback (DFB) lasers ([7], [8]) unveil a multi longitudinal mode behavior that is apparent from their output spectra. The output intensity of these longitudinal modes is a function of the input pumping current, whereas the competition among them is governed by numerous physical processes like spontaneous emission noise, spatial hole burning, spectral hole burning, and carrier diffusion [9]. This competition exacerbates in the form of venomous dynamic effects on the laser operation such out of which mode beating and mode hopping are worth mentioning. The former modulates the total output power in short time scales and the latter relates jumping of various modes and their overlap with each other. On account of their numerous applications in modern sensors [10], it is imperative to peruse the mode competition behavior in detail.

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Multimode semiconductor laser dynamics is normally described by the famous rate equations [11] framework, where a fixed number of modes is considered. These modes have different gain coefficients which mirror the frequency dependence of the gain curve. As remarked earlier, an explicit solution of the rate equations is very difficult to obtain, on account of ground realities. However, considering the physical significance of the aforementioned circumstances, it would be rational to reduce the number of modes and analyze the situation accordingly.

In this connection, the various optical feedback ([12], [13]) phenomena come into play and helped to formulate the said problem into a more understandable and realizable form. This paper focuses on the multimode phenomenon of such a laser in a two mode setup, under the esteemed influence of optical feedback jargon. Emphasis has been laid in the NIR because of two reasons. First, most of the optical communication systems are built to operate within this range. Second, abundant optical devices are available in the market in this regime. Therefore, it is imperative to analyze the modes existence in such an important scenario, and detail it.

The rest of this paper is organized as follows. The upcoming section pays a brief homage to the relevant scientific community and provides a status quo of this work. The mathematical model is presented in section III. Afterwards the simulation parameters for the said hypothesis are outlined in section IV. Section V specifies the results with in detail analysis of the interpretations. Finally the paper concludes by providing a short summary of the work in section VI and some proposals are laid down for future work in section VII.

II. BACKGROUND

Mode competition in semiconductor lasers is not a new topic. Considerable efforts have been made by groups like [3] and [14]. A detailed overview of rate equations and their applications in external cavity semiconductor lasers has been provided in [15]. The corner stone in the study of optical feedback in semiconductor lasers was laid by [16]. Research continued in this field by authors like [17]. The concept was further lengthened to transmission scheme in the 1.55 μm regime by [18]. For the interested readers, an astronomical list of relevant references can be found in this perspective exist in [13] and [17].

III. THEORETICAL FOUNDATION

A hypothetical one dimensional semiconductor laser model [17] is assumed with two external cavities as shown in Fig-1.

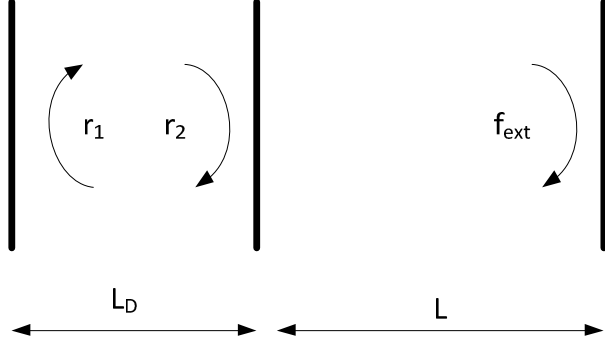


Figure 1: One dimensional model of a semiconductor laser external cavity laser diode [17]

The total reflected field after multiple reflections in the external cavity is given by:

$$E_r(t) = r_{\text{eff}}E(t) \quad (1)$$

where, $E(t) = \sqrt{S(t)} \exp j[\omega t + \varphi(t)]$ is the stationary field incoming to the right diode facet from the left in Fig. 1, $S(t) = S_1(t) + S_2(t)$ is the total photon intensity due to two modes, and $\varphi(t) = \varphi_1(t) + \varphi_2(t)$ is the total phase change due to the two modes. The effective reflection coefficient r_{eff} is defined as,

$$r_{\text{eff}} = r_2 * f \quad (2)$$

with the optical feedback factor f , defined in the text box below, and

$$f_{\text{ext}} = |f| * \exp(j \arg f) \quad (3)$$

We consider the round trip model [17] which is suitable for the analysis of our optical feedback setup.

$$G_r = \sqrt{R_1 R_2 |f|^2} * \exp(g - \alpha_s)L_D \times \exp(-2j\beta L_D + j \arg f) \quad (5)$$

A. Rate Equations

For the sake of feasibility, the Langevin noise sources have been avoided. The number of photons, electrons and phase corresponding to the two modes respectively can be written as,

$$\frac{dS_1}{dt} = \left[g(N, S_1) - \frac{1}{\tau_{p,1}} + \frac{\ln|f|^2}{\tau_D} \right] S_1 + R_{sp} \quad (6)$$

$$\frac{dS_2}{dt} = \left[g(N, S_2) - \frac{1}{\tau_{p,2}} + \frac{\ln|f|^2}{\tau_D} \right] S_2 + R_{sp} \quad (7)$$

$$\frac{dN}{dt} = \frac{I}{q} - \frac{N}{\tau_n} - g(N, S) \quad (8)$$

$$\frac{d\varphi_1}{dt} = \frac{\alpha \delta g_1}{2 \delta N} (N - N_{\text{th}}) + \frac{\arg f}{\tau_D} \quad (9)$$

$$\frac{d\varphi_2}{dt} = \frac{\alpha \delta g_2}{2 \delta N} (N - N_{\text{th}}) + \frac{\arg f}{\tau_D} \quad (10)$$

It is worth mentioning that the input pumping current is the same for both modes. That makes sense, since the input current is not varied and both modes get the same input power to oscillate. This is the reason that the number of electrons is the same for the laser system in this perspective. Finally, the electrical field inside the laser diode can be written as:

$$\frac{dE_0}{dt} = \frac{1}{\tau_D} \left[(j\varphi_G + \arg f) + (g - \alpha_s)L_D - \frac{1}{2} \ln \left(\frac{1}{R_1 R_2 |f|^2} \right) \right] E_0(t) \quad (11)$$

IV. SIMULATION SETUP

In this section, we apply the system of equations (6)-(11) to investigate mode competition in the said laser. The two modes are varied from 1537.4 nm to 1547.4 nm respectively, as explained in the following section. The threshold of an In GaAs laser is usually above 70 mA. For this purpose, it is imperative to analyze the existence of both modes above threshold current. We use a typical value of 100 mA at 25°C. For notational convenience, the symbols and their values are depicted in Table-1, which matches the standard literature.

The said system of differential equations can be solved via various numerical techniques. After checking the accuracy of some basic methods in MATLAB, we resort to the 4th order Runge Kutta method. This has two reasons: first the oscillations in the output intensity are prominent as compared to the contemporary methods, second this is in accordance with other relevant research works. Therefore, the selected method is implemented with a time interval of 20 ps over a time period of 10 μ s.

$$|f|^2 = \frac{1 + \frac{2f_{\text{ext}}}{r_2} \sqrt{\frac{S(t-\tau)}{S(t)}} \cos(\omega\tau + \varphi(t) - \varphi(t-\tau)) + \frac{f_{\text{ext}}^2}{r_2} \frac{S(t-\tau)}{S(t)}}{1 + 2r_2 f_{\text{ext}} \sqrt{\frac{S(t-\tau)}{S(t)}} \cos(\omega\tau + \varphi(t) - \varphi(t-\tau)) + (r_2 f_{\text{ext}})^2 \frac{S(t-\tau)}{S(t)}} \quad (4)$$

$$\arg f = \tan^{-1} \left\{ -f_{\text{ext}} \left(\frac{1}{r_2} - r_2 \right) \sqrt{\frac{S(t-\tau)}{S(t)}} \sin(\omega\tau + \varphi(t) - \varphi(t-\tau)) + f_{\text{ext}}^2 \frac{S(t-\tau)}{S(t)} \right\}$$

TABLE 1: Various Abbreviations and Their Values

Acron ym	Description	Value
α_s	Scattering loss in the laser's active medium	3600 m^{-1}
\bar{n}_e	Group refractive index	4.5
R_1/R_2	Reflectivity of the front/ rear mirror	0.32
n_{sp}	Inversion factor	2
α	Line width enhancement factor	4
k	Gain saturation factor	3.5 W^{-1}
τ_n	Carrier lifetime	2 ns
N_0	Numbers of carriers to reach zero gain	2×10^8
L_D	Laser diode length	400 μm
L_1	External cavity length, corresponding to mode 1	1 m
L_1	External cavity length, corresponding to mode 2	2 m
n_c	Coupling efficiency of the fibre lens	0.5
R_1	Reflectivity of the external mirror	0.7
n_e	Effective index of the single mode fibre	1.5

Next, since it is a numerical method, it is imperative not to rely on a single result. From our practical experience, it was found that running the program more number of times improves a result by only $10^{-4} \%$ with considerable additional time. Therefore, the computer program was run three times and the average value was taken. For instance, the output in the form of two modes looks like in figure-2 for $M_1=1540 \text{ nm}$ and $M_2=1542 \text{ nm}$.

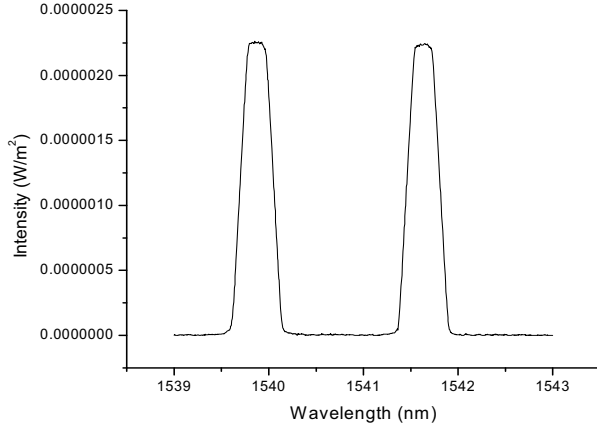


Figure 2: Two mode spectrum originated from the solution of equations (6)-(11) for $M_1=1540 \text{ nm}$ and $M_2=1542 \text{ nm}$.

V. RESULTS AND DISCUSSION

To start with, the intensities of both modes are observed at various wavelength ratios. This is done to check the existence of the modes at various positions, shown in Fig-3.

Next, to prove this stance, we proceeded as follows: The wavelength of the 1st mode is kept at 1537.4 nm, and that of the second mode is varied from 1537.4 nm to 1547.4 nm. Each of the graphs is fitted with the help of Gaussian curve fitting tool, and then the full width at half maximum (FWHM) of the two modes is measured accordingly. The histogram plot for the values of the FWHM of the two

modes corresponds to the existence of two modes, shown in figure-4. On account of the limitation of space, several various aspects such as change in phase and the photon density have been examined but not presented in this paper. The emphasis is laid on the mode competition in the light of feedback effects.

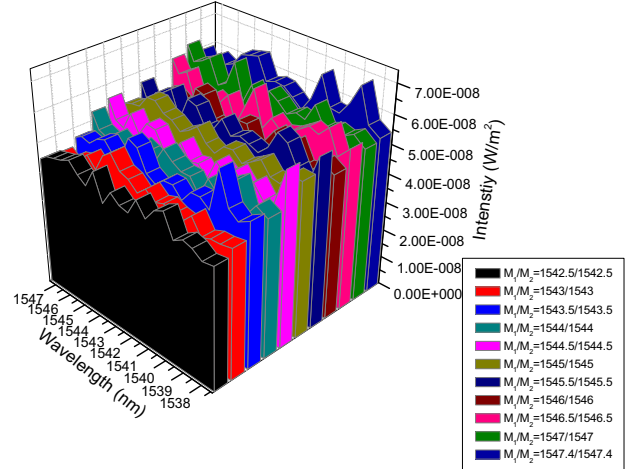
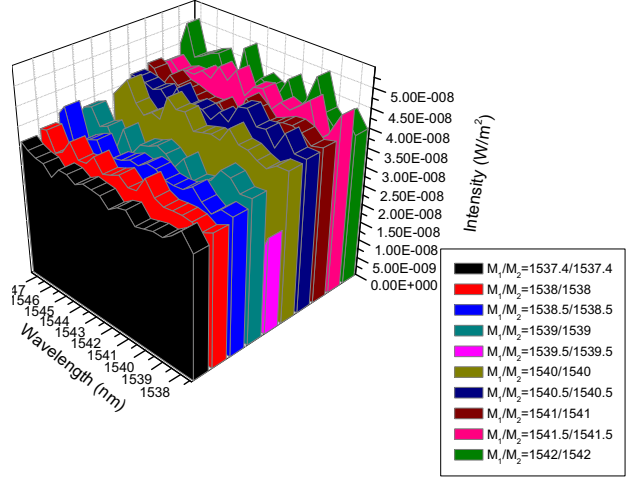


Figure 3: The intensities of both modes at wavelengths 1537.4-1542 nm (above) and 1542.5-1547.4 nm (below), respectively.

1) It is observed from the graph of various mode intensities that both modes exist at most occasions. This means that both modes participate in a healthy mode competition scenario, and tend to gain control over each other.

2) The only situation where the intensities drop over is the wavelength of 1539.5 nm. A possible reason might be the different gain curve at this end, which supports the values intensity output on the whole. However, the complete argument at this point remains unknown.

3) The first mode shows signs of stable existence as compared to the second mode. This is because of numerous reasons, primarily because this is close to the laser's output.

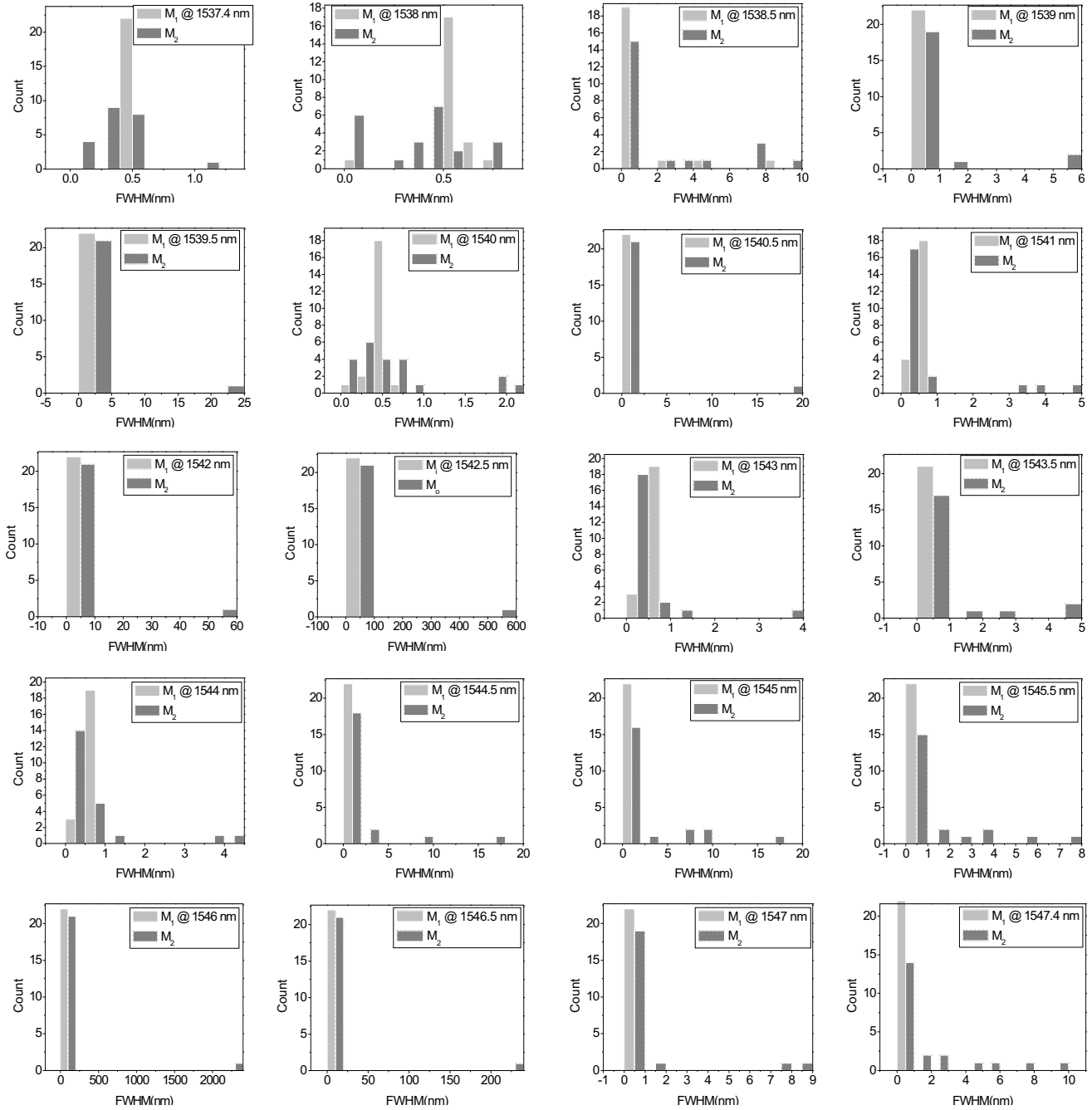


Figure 4: Histogram analysis of the FWHM of both modes at various wavelengths

This means that it has a shorter cavity length and less phase changes, which is also in accordance with the rate equations. Thus the output intensity is more stable as compared to the second mode, which is at more distance from the laser output and experiences more phase changes in comparison. This is evident from the histogram count in most plots.

4) In typical areas there was an intense mode competition. For instance if we have a look at $M_1=1539.5$ nm, both modes struggle extensively for their co-existence. This mirrors

another important factor: mode spacing accounts for as a crucial parameter in mode competition.

5) The histogram analysis also reveals that the second mode completely disappears under certain circumstances. For instance, at $M_1=1540$ nm, the first mode exists as per normal, but the remarkable variation in the FWHM of the second mode amply reflects the latter's wobbly presence. A prominent issue is the existence of side modes due to parasitic cavities that

substantially hinder the realization of the second mode. Other reasons for this might be the line broadening effect in lasers.

6) Interestingly, the phenomenon explained above does not affect the first mode's existence, and it continues to thrive even under pronounced mode competition.

7) There are some wavelengths at which the simultaneous existence of both modes seems dubious. The intensity profile is not accurate enough to verify or deny this fact, and the abscissa in the histogram plots is broad enough to peruse this fact. Therefore, it still seems a mystery at this moment as to whether mode competition in this perspective has some influence on the laser's operation or not.

VI. CONCLUSIONS

Mode competition in lasers is one of the burning questions of the time and is hotly debated in sensor design and applications. In this work, optical feedback model is used to understand this phenomenon in semiconductor lasers. To keep things simple, a two mode hypothetical model was presented and investigated under the optical feedback effects. On account of the various applications, the analysis was performed in the NIR regime. Strong mode competition was observed at certain wavelengths, and is found to be a function of the mode spacing and FWHM. Besides, the second mode seems to be unstable comparatively and was found to disappear at few occasions. As remarked in [16] and [17], the gain profile of the laser has a distinct impact on the longitudinal modes of a laser and the overall operation of the device. The current work can be regarded as a first step in this connection; the outcomes are encouraging and motivate us to go ahead in this direction in the future.

VII. EXTENSIONS

As remarked earlier, many key facts have been left unscathed in this connection. Therefore, numerous barriers need to be surmounted before arriving at a concrete and consolidated outcome. The first question that comes to the mind is the behavior of the laser at other wavelengths. Another topic completely untouched is the operation of the laser at other values of current and temperature, since they are vital factors that help determine the transfer function of any laser. An interesting area to be explored can be the investigation carried out very close to the threshold current in order that a resilient mode competition can be probed that is a function of the input current. We plan to do the analysis at other wavelengths as well as other temperature and current values to have an in depth scrutiny of the said dispute.

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