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Low Complexity Bidirectional Equalization of Doubly Selective Channel using BEM

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Abstract—**Low complexity equalization of doubly selective channels is motivated for real time processing of received signals to establish duplex communications. Bidirectional equalization can improve the bit error rate performance. Bidirectional equalizer performs two equalizations, one on received block of information and other on its time reversed version. This can result in an increased complexity. The time varying nature of the doubly selective channel is also a cause for concern, from the perspective of computational complexity. Several low complexity equalization techniques for doubly selective channels have been reported in the literature. This work studies low complexity bidirectional equalization of doubly selective channels. The proposed scheme of bidirectional equalization offers low computational complexity in comparison with existing low complexity approaches of equalization of doubly dispersive channel.**

Index Terms: Bidirectional Equalization, Doubly Selective Channel, Low complexity, Basis Expansion Model.

I. INTRODUCTION

Low Complexity Equalization of Doubly Selective Channels using Basis Expansion Model (BEM) has been studied in [1]-[4]. In these references, the authors model the time varying equalizers by obtaining the BEM coefficients of time varying equalizer. In case of [2]-[4], these coefficients are directly obtained from the channel BEM coefficients. However this approach requires a large matrix inversion that entails significant complexity. To exploit the temporal structure of the problem [6] used block matrix inversion formula, while [7]-[9] used $\mathbf{LDL^T}$ factorization to exploit the banded structure of the time varying convolution matrix. However [1] obtained the BEM coefficients for the equalizer by a sampling and interpolation approach that significantly reduced complexity. This manuscript studies application of the sampling and interpolation based approach of [1] to bidirectional equalization. Similar to [1], the BEM coefficients of the time varying equalizer are found by sampling the equalizer. Proposed approach can offer a very flexible tradeoff between performance and complexity. We consider single carrier modulation like in [1].

II. DOUBLY SELECTIVE CHANNELS

Symbol spaced samples of received signal *y (n)* are

$$y(n) = \sum_{l=0}^{l=L} h(n,l)x(n-l) + w(n) \tag{1}$$

The transmit symbol at time *n* is *x(n)* and additive white Gaussian noise with variance σ^2_w at time n is *w(n)*. The channel response at time *n* to an impulse applied at time *n-l is h(n-l).* Due to the high data rates and vehicular motion the channel includes the effects of both ISI and time variation, and such a channel is a doubly selective channel. The doubly selective channel considered here belongs to the class of underspread channels [14], [16]. So by the basis expansion model the approximation of the channel is

$$h(n,l) = \sum_{q=-\frac{Q}{2}}^{\frac{Q}{2}} h_{ql}\, e^{\frac{-j2\pi qn}{N}} \tag{2}$$

The BEM coefficients h_{ql} model the channel variation block by block, while exponential basis $e^{\frac{-j2\pi qn}{N}}$ models variation of time within the block. Q/2 = $[f_d N]$ where f_d is normalized Doppler spread [16]. Doppler Spread determines Q and so Q determines time selectivity. Channel estimation for doubly selective channels as studied in [8], [12], [14] involve estimating the BEM coefficients. The time varying channel is then reconstructed from those BEM coefficients.

Assuming N symbols transmitted, a received block can be written as

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{w} \tag{3}$$

Where $\boldsymbol{H}$ is a N+L-1×N time varying convolution matrix, which is not Toeplitz but is banded; $\boldsymbol{x}$ is N transmitted symbols and $\boldsymbol{w}$ contains AWGN noise samples. As mentioned by [2], [3], the MMSE block linear equalizer (BLE) solution can be found as $(\boldsymbol{H}^H\boldsymbol{H}+\sigma^2_w\boldsymbol{I})^{-1}\boldsymbol{H}^H$ but the solution is computationally impractical [2], [3] due to the large size of $\boldsymbol{H}$.

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III. Low Complexity Equalization

MMSE BLE is practically complex to compute so different approaches with different complexity levels have been developed.

There are,

- [2]-[4] Barhumi et. al. proposed the used of Basis Expansion model, in which equalizer's BEM coefficient are calculated from the BEM coefficients of the channel. This scheme offered a significant reduction in complexity.
- P. Schniter in [5] designed serial equalizer. In this approach *N* small matrix inversions are calculated instead of taking inverse of one large block, which further reduces complexity.
- [6] et. al. obtained further reduction in complexity by making use of the overlap between matrices at successive instants.
- Rugini et. al. [7]-[9] used $\mathbf{LDL^T}$ factorization to give an efficient equalization scheme by exploiting the banded structure of the time varying convolution matrix.
- Low complexity equalization was approached in [1] by designing the equalizers at some point within the block and then using the BEM to interpolate for the equalizers in the remainder of the block.

IV. A Low Complexity Bidirectional Approach

In bidirectional equalization, two equalizations are performed. The equalizer operates on the received signal both in the forward and reverse directions.

A. Equalization in the forward direction :

The linear MMSE equalizer at time n for the forward direction is

$$\boldsymbol{m}_n = (\boldsymbol{H}_n \boldsymbol{H}_n^H + \sigma_w^2 \, \boldsymbol{I})^{-1} \boldsymbol{H}_n^H \mathbf{1} \qquad (4)$$

We see that this equalizer suppresses the ISI for all delays. So this equalizer performs very well. This requires the computations of a matrix inverse for all *n* which is computational intensive. Thus we adopt the approach of [1] which was to

"Design the equalizer at some points within the block and then use the BEM to interpolate for the equalizers in the remainder of the block"

In case of the MMSE decision feedback equalizer, the feedforward filter for the forward direction is

$$\boldsymbol{m_n} = (\boldsymbol{H_n}(\boldsymbol{I} - \boldsymbol{\Psi}_n \boldsymbol{\Psi}_n^H)\boldsymbol{H}_n^H + \sigma_w^2 \, \boldsymbol{I})^{-1} \boldsymbol{H}_n^H \, \mathbf{1} \qquad (5)$$

Where $\mathbf{1}$ is a $(\text{Leq} + \text{Leq} - 1) \times 1$ dimensional vector of all zeros and a one in position δ, where δ is the equalization cursor. $\boldsymbol{H_n}$ is a N+L-1×N time varying convolution matrix, which is not Toeplitz but is banded and $\boldsymbol{H}_n$ is the part of $\boldsymbol{H}$ that lies within the equalization window. Further $\boldsymbol{\Psi}_n = E\{x\hat{x}_n^H\}$, i.e. matrix $\boldsymbol{\Psi}_n$ is the correlation between the transmit signal and the detected signal in the feedback window. Now the basic idea is to not evaluate equation (5) for all *n*. Instead, like [1], [2], [3] we model the time varying equalizer using the BEM as

$$m(n,l) = \sum_{q=-\hat{Q}/2}^{\hat{Q}/2} m_{ql} \, e^{\frac{-j2\pi qn}{N}} \quad l = 0, \dots, \acute{L}-1 \qquad (6)$$

Where $\acute{L}$ is the length of the feedforward filter and $\hat{Q}$ determines the allowed temporal variations of the equalizer coefficients. An interpretation is that by modeling the time varying equalizer with BEM, the time varying equalizer taps are constrained to be within the subspace spanned by the Fourier basis $e^{\frac{-j2\pi qn}{N}}$ for $q = -\hat{Q}/2 \dots \hat{Q}/2$. The time varying feedback filter is obtained from the time varying feedforward filter as

$$\hat{\boldsymbol{b}}_n = -\boldsymbol{\Psi}_n^H \boldsymbol{H}_n^H \boldsymbol{m}_n \qquad (7)$$

Like [1], equation (5) is solved for enough values of n so that BEM coefficients m_{ql} may be estimated. Once they are obtained, equation (6) can be used to obtain time varying feed forward filter for all values of n. Equation (5) is evaluated at uniformly spaced intervals n=0, Δ, 2Δ, …, *K*Δ where *K*= [N/Δ] in the block.

$$\begin{bmatrix} \boldsymbol{m}_0(l) \\ \vdots \\ \boldsymbol{m}_{(K-1)\Delta}(l) \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} m_{-\hat{Q}/2l} \\ \vdots \\ m_{\hat{Q}/2l} \end{bmatrix} \qquad (8)$$

Where $\boldsymbol{m}_n(l)$ represents the l^{th} element of the equalizer at time *n*. Also

$$\boldsymbol{T} = \begin{bmatrix} e^{\frac{j2\pi(\hat{Q}/2)(0)}{N}} & \cdots & e^{\frac{-j2\pi(\hat{Q}/2)(0)}{N}} \\ \vdots & \ddots & \vdots \\ e^{\frac{j2\pi(\hat{Q}/2)(K\Delta)}{N}} & \cdots & e^{\frac{-j2\pi(\hat{Q}/2)(K\Delta)}{N}} \end{bmatrix} \qquad (9)$$

Like [1], by solving this system of linear equations, the $\hat{Q}$ coefficients of the time varying feedforward filter BEM representation associated with lag *l* are obtained. By solving this for all *l=0,…, Ĺ-1*, all the required BEM coefficients are obtained. It is important to observe here that the matrix in this system of equations is the DFT matrix, this could potentially allow for the use of efficient algorithms such as the fast Fourier transform. However even if these efficient algorithm are not exploited, the matrix is unitary and its inverse is simply its conjugate transpose thus the complexity associated with solving this system of linear equations is not more than K^2. Once the BEM coefficient are obtained for all $l=0,\ldots,\acute{L}-1$, the time varying equalizer is then evaluated for all time n using (6) without requirement of any more matrix inversions.

The algorithm for low complexity forward direction equalization is summarized below:

1. Solve equation (5) for $n = 0, \Delta,\ldots,(K-1)\Delta$. This requires matrix inversions.
2. Solve for the feedforward BEM coefficients using equation (8) for $l = 0,\ldots, \acute{L}-1$. These equations involve unitary matrices and thus the inversions are trivial.
3. With the obtained BEM coefficients use equation (6) to obtain the time varying equalizer for all n.
4. Solve for the time varying feedback filter coefficients using equation (7).

B. Equalization in reverse direction:

For the equalization in reverse direction, the same technique can be used. Then the feedforward filter for reverse direction is given as

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = (\widehat{\boldsymbol{H}}_n(\boldsymbol{I} - \boldsymbol{\Psi}_n\boldsymbol{\Psi}_n^H)\widehat{\boldsymbol{H}}_{\boldsymbol{n}}^{\boldsymbol{H}} + \sigma_w^2\boldsymbol{I})^{-1}\widehat{\boldsymbol{H}}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{1} \qquad (10)$$

Here $\widehat{\boldsymbol{H}}_n = \boldsymbol{P}_1\boldsymbol{H}_n\boldsymbol{P}_2$ and $\boldsymbol{P}_1$ has dimension $Leq \times Leq$ and $\boldsymbol{P}_2$ has dimension $(Leq+L-1)\times(Leq+L-1)$. $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ have the form

$$\boldsymbol{P}_1 \cong \boldsymbol{P}_2 = \begin{bmatrix} 0 & \cdots & 0\ 1 \\ & & 1\ 0 \\ \vdots & \iddots & \vdots \\ 0\ 1 & & \\ 1\ 0 & \cdots & 0 \end{bmatrix} \qquad (11)$$

These are permutation matrices and the inverse of the permutation matrix is its Hermitian i.e. $\boldsymbol{P}_1\boldsymbol{P}_1^H=\boldsymbol{I}$ *and* $\boldsymbol{P}_2\boldsymbol{P}_2^H=\boldsymbol{I}$. The feedforward filter can thus be expressed as

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = (\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{P}_1^H(\boldsymbol{I}-\boldsymbol{\Psi}_n\boldsymbol{\Psi}_n^H)\boldsymbol{P}_1\boldsymbol{H}_n\boldsymbol{P}_2 + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{H}}\boldsymbol{1} \quad (12)$$

Further we can write the above equation as

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = \boldsymbol{P}_2(\boldsymbol{H}_n^H\boldsymbol{P}_1^H(\boldsymbol{I}-\boldsymbol{\Psi}_n\boldsymbol{\Psi}_n^H)\boldsymbol{P}_1\boldsymbol{H}_n + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{P}_2^H\boldsymbol{P}_2^H\boldsymbol{H}_n^H\boldsymbol{P}_1^H\boldsymbol{1} \quad (13)$$

The permutation matrix is a special case of unitary matrix so that $\boldsymbol{P}_1\boldsymbol{P}_1^H=\boldsymbol{I}$. It can be seen that $\boldsymbol{P}_2^H = \boldsymbol{P}_2$, so in addition to being unitary, this permutation matrix is also conjugate symmetric. As a result it has the interesting property of being its own inverse. So equation (13) can be written as

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = \boldsymbol{P}_2(\boldsymbol{H}_n^H\boldsymbol{P}_1^H(\boldsymbol{I} - \boldsymbol{\Psi}_n\boldsymbol{\Psi}_n^H)\boldsymbol{P}_1\boldsymbol{H}_n + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{H}_n^H\boldsymbol{P}_1^H\boldsymbol{1} \quad (14)$$

Note $\boldsymbol{P}_1^H\boldsymbol{1} = \widetilde{\boldsymbol{1}}$ .Where $\widetilde{\boldsymbol{1}}$ is $(Leq+L-1)\times 1$ dimensional vector of all zeros and 1 in position $(Leq+L-\delta)$.Further we can write the equation (14) as

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = \boldsymbol{P}_2(\boldsymbol{H}_n^H\boldsymbol{P}_1^H(\boldsymbol{I} - \boldsymbol{\Psi}_n\boldsymbol{\Psi}_n^H)\boldsymbol{P}_1\boldsymbol{H}_n + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{H}_n^H\,\widetilde{\boldsymbol{1}} \qquad (15)$$

For the special case of linear equalization, the reverse equalizations simplify considerably as they do not require any new matrix inversions.

We know that linear equalizer in forward direction:

$$\boldsymbol{m}_{\boldsymbol{n}} = (\boldsymbol{H}_n\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}} + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{1} \qquad (16)$$

And equalizer in reverse direction for linear approach will be:

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = (\widehat{\boldsymbol{H}}_n\widehat{\boldsymbol{H}}_{\boldsymbol{n}}^{\boldsymbol{H}} + \sigma_w^2\boldsymbol{I})^{-1}\widehat{\boldsymbol{H}}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{1} \qquad (17)$$

Here $\widehat{\boldsymbol{H}}_n = \boldsymbol{P}_1\boldsymbol{H}_n\boldsymbol{P}_2$ and $\boldsymbol{P}_1$ has dimension $Leq \times Leq$ and $\boldsymbol{P}_2$has dimension $(Leq+L-1)\times(Leq+L-1)$.

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = (\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{P}_1^H\boldsymbol{P}_1\boldsymbol{H}_n\boldsymbol{P}_2 + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{n}}^{\boldsymbol{H}}\boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{H}}\boldsymbol{1} \quad (18)$$

By using the special properties of the matrices $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ already discussed equations (18) becomes

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = \boldsymbol{P}_2(\boldsymbol{H}_n^H\boldsymbol{H}_n + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{H}_n^H\boldsymbol{P}_1^H\boldsymbol{1} \qquad (19)$$

Also $\boldsymbol{P}_1^H\boldsymbol{1} = \widetilde{\boldsymbol{1}}$ .Where $\widetilde{\boldsymbol{1}}$ is $(Leq+L-1)\times 1$ dimensional vector of all zeros and 1 in position $(Leq+L-\delta)$.

$$\widehat{\boldsymbol{m}}_{\boldsymbol{n}} = \underbrace{\boldsymbol{P}_2}_{\text{Permutation}}\ \underbrace{(\boldsymbol{H}_n^H\boldsymbol{H}_n + \sigma_w^2\boldsymbol{I})^{-1}\boldsymbol{H}_n^H\,\widetilde{\boldsymbol{1}}}_{\text{forward equalization}} \qquad (20)$$

So the time reversed version of equalization does not use the equation (17) at uniform instants but instead uses (20). But (20) can be recognized as a permutation applied to a forward equalization applied at a different lag. In this method complexity for the reverse equalization is reduced. The inversions involved in the equation (20) have already been calculated in the forward equalization. After solving the equalizers at uniform instants in reverse direction in (20) the BEM coefficients are obtained and the equalizer is obtained at all times by interpolation.

The algorithm for reverse direction equalization is summarized below:

1. Solve equation (20) for $n = 0, \Delta, \ldots (K-1)\Delta$.
2. Solve for the feedforward BEM coefficients using equation (8) for $l = 0,\ldots, \acute{L}-1$. The matrix inversions in this step are trivial because the matrices are unitary.
3. With the obtained BEM coefficients use equation (6) to obtain the time varying equalizer for all n.
4. Solve for the time varying feedback filter coefficients using equation (7).

V. Complexity Analysis

First the complexity of proposed scheme is analyzed in forward direction and in reverse direction separately. After that combined complexity for two equalizations (Bidirectional equalization) is analyzed. Then complexity of proposed scheme with the complexity of other equalization techniques is compared. The complexity is calculated in terms of number of complex multiply add (MA) operations.

A. Complexity of equalization in forward direction:

1. In the first step, inversions of K matrices of size $\acute{L}\times\acute{L}$ has complexity of approximately $K\acute{L}^3/3$ MA operations and the multiplication cost is $(L+\acute{L}-1)\acute{L}^2$ MA operations.

2. Step two requires solution of systems of equations which have a complexity of K^2 MA operations each. The complexity in step two can be reduced to $K \log (K)$ if fast Fourier Transform is used as **T** is a sparse DFT matrix. However here we are considering complexity K^2 for step two.

3. In step three, the time varying feedforward filter can be obtained directly from the BEM coefficients with complexity $N\acute{Q}\acute{L}$ MA operations but it can be efficiently obtained by using fast Fourier transform algorithm and its complexity is $\acute{Q}\ \acute{L}\ \log_2 N$.

4. In step four, the time varying feedback filter coefficients are obtained. Equation (8) is a matrix vector multiply because n is simply a rectangular windowing operation which extracts the relevant L_f rows from $\boldsymbol{H}_n^H$. This has a complexity of $N\acute{L}L_f$ MA operations.

Total complexity for forward direction equalization is $\acute{Q}\acute{L}\ \log_2(N) + N\acute{L}L_f + \acute{L}K^2 + K\acute{L}^3/3 + K(L+\acute{L}-1)\acute{L}^2$ MA operations. If a time varying linear equalizer is to be constructed by this algorithm, the complexity requirement can be obtained as $\acute{Q}\acute{L}\ \log_2(N) + \acute{L}K^2 + K\acute{L}^3/3 + K(L+\acute{L}-1)\acute{L}^2$MA operations by setting $L_f = 0$. Indeed the predominant computational effort is in the third step where the time varying equalizer is reconstructed from the BEM coefficients.

Notice that this is the design complexity and the implementation complexity is separate. When an equalizer of length $\acute{L}$ is applied for detection of N symbols, this will require $N\acute{L}$ MA operations. So the overall complexity associated with this approach is $\acute{Q}\acute{L}\log_2(N) + N\acute{L}(1+L_f) + \acute{L}K^2 + K\acute{L}^3/3 + K(L+\acute{L}-1)\acute{L}^2$ MA operations. This complexity is significantly less than that associated with the BEM based techniques in [2], [3] and to order of magnitude it is comparable to the techniques of [5]–[7]. The complexity of all these techniques as well as ours is $\mathcal{O}(N)$ as compared to the $\mathcal{O}(N^3)$ required for the BLE.

B. Complexity of equalizaion in reverse direction:

1. In first step of equalization in reverse direction, equation (10) can be used to design feedforward filter and complexity for this approach is approximately $K\acute{L}^3/3$ MA operations and the multiplication cost is $(L+\acute{L}-1)\acute{L}^2$ MA operations. Further complexity of first step in reverse direction can be reduced by approaching linear equalization. In this method complexity is reduced because the inversions involved in this approach have already been calculated in the forward direction equalization. Complexity for linear approach is $K\acute{L}^2$ .Complexity associated in first step of reverse direction linear equalization is 7 times less than that of complexity of first step of forward direction equalization.

2. Step two in reverse direction equalization also have same complexity as in step two of forward direction equalization and that requires solution of systems of equations which have a complexity of K^2 MA operations each.

3. In step three, the time varying feedforward filter can be obtained directly from the BEM coefficients with complexity $N\acute{Q}\acute{L}$ MA operations but it can be efficiently obtained by using fast Fourier transform algorithm and its complexity is $\acute{Q}\ \acute{L}\ \log_2 N$

4. Step four is also same like step four of first equalization in forward direction, the time varying feedback filter coefficients are obtained. Equation (8) is a matrix vector multiply because n is simply a rectangular windowing operation which extracts the relevant L_f rows from $\boldsymbol{H}_n^H$. This has a complexity of $N\acute{L}L_f$ MA operations.

Total complexity for reverse direction equalization is $K\acute{L}^2 + \acute{Q}\acute{L}\ \log_2(N) + N\acute{L}L_f + \acute{L}K^2$ MA operations. If a time varying linear equalizer is to be constructed by this algorithm, the complexity requirement can be obtained as $K\acute{L}^2 + \acute{Q}\acute{L}\ \log_2 N + \acute{L}K^2$ MA operations by setting $L_f = 0$. Indeed the predominant computational effort is in the third step where the time varying equalizer is reconstructed from the BEM coefficients.

Notice that this is the design complexity for reverse direction and the implementation complexity is separate. When an equalizer of length $\acute{L}$ is applied for detection of N symbols, this will require $N\acute{L}$ MA operations. So the overall complexity associated with this approach is $\acute{Q}\acute{L}\ \log_2(N) + N\acute{L}(1+L_f) + K\acute{L}^2 + \acute{L}K^2$ MA operations.

C. Overall Complexity for bidirectional equalization:

The overall complexity associated with forward direction equalization and reverse direction equalization is $2(\acute{Q}\,\acute{L}\,\log_2 N) + 2(N\acute{L}L_f) + 2(\acute{L}K^2) + K\acute{L}^2 + K\acute{L}^3/3 + K(L + \acute{L} - 1)\acute{L}^2$. It is design complexity for bidirectional equalization, and design complexity for our proposed bidirectional technique is $2(\acute{Q}\,\acute{L}\,\log_2(N)) + 2(N\acute{L}(1 + L_f)) + 2(\acute{L}K^2) + K\acute{L}^2 + K\acute{L}^3/3 + K(L + \acute{L} - 1)\acute{L}^2$

D. Comparison with Existing Low Complexity techniques:

In our approach the dominant computational effort is in third step of both forward and reverse direction where the time varying equalizer is reconstructed from the BEM coefficients. The complexity of time varying equalizer reconstructed from the BEM coefficients is eclipsed by the complexity of obtaining the equalizer BEM coefficients in [2]-[4]. Because in approach of [2]-[4] requires inversion of a matrix of dimension $P = (L + \acute{L} + 1)(Q + \acute{Q} + 1)$.This matrix inversion entails $P^3/3$ operations which is dominant computational effort in the design of equalizer [2], [3]. Like [1] in addition to lower complexity our scheme has other advantage. Approach in [2]-[4] approximate both the channel and the equalizer using the BEM, But our approach like [1] approximate only the feedforward filter in both forward direction and reverse direction using the BEM.

For OFDM transmission the complexity associated with serial equalizers is $N(\acute{L}^3/3 + (L + \acute{L} - 1)$ MA operations in [4], and that is an efficient saving over the use of a block equalizer. Here $\acute{L}^3/3$ MA operations are required for matrix inversion while $(L + \acute{L} - 1)\acute{L}^2$ MA operations are required for the matrix multiplications to redesign the serial equalizer for each of the N transmit symbols in the block.

However like [1] there will be some performance loss in our scheme due to the approximation of the feedforward filter with the BEM. This loss depends upon the choice of $\acute{Q}$ and hence on the complexity we are willing to accept (when *K*=*N* is chosen then there will be no loss).

The complexity associated with [5] does not take into consideration the temporal structure of linear time varying convolution that can be exploited to reduce the computational complexity further as noted by [6]. The matrices H_n have significant overlap for adjacent n and this overlap may be exploited derive recursive algorithm for both the multiplications and inversions involved in the design of the time varying equalizer. So [6] proposed the block matrix inversion formula to exploit the temporal structure provided by the doubly selective channel. The idea in [6] is that the *N* serial equalizer design problems in a block can be efficiently solved if the matrix multiplications and inversions are recursively solved. The overall complexity of the recursions to obtain the time varying decision feedback equalizer including both the matrix multiplication and inversions work out to be $(N - 1)(3\acute{L}^2 + 2\acute{L}) + 6\acute{L}^2 + (L + \acute{L} - 1)\acute{L} + 8 + (L + \acute{L} - 1)\acute{L} + \acute{L}^2)$ MA operations while $\acute{L}^3/3 + L + \acute{L} - 1)\acute{L}^2$ MA operations are needed to bootstrap the algorithm. While this is improvement over the complexity associated with [5] due to the efficiency of the recursions, the complexity was reduced even further by [7], [8]. The work in [7], [8] exploited the banded structure of the time varying convolution matrix with a $\mathbf{LDL}^T$ factorization to obtain an equalization complexity of $N(4Q^2 + 12Q + 2)$ MA operations which provide a lower complexity.

Like [1] the complexity of our scheme is linear in both the channel delay and Doppler spreads which allows the complexity to scale proportional to the channel spread in both the dimensions.

VI. Simulation Result

Bidirectional equalization of doubly selective channels has been numerically simulated using Monte Carlo simulations. A doubly selective channel with delay spread *L*=6 and Doppler spread f_d=0.005 has been simulated. Simulation results in Fig.1 show the performance of a linear equalizer with $\acute{L}$=10. Provided in Fig.1 is the BER performance of the proposed low complexity bidirectional equalizer as well as the benchmark of the bidirectional equalizer that does not use any complexity reducing measures. As can be seen in the figure, performance of the proposed low complexity equalizer is not significantly degraded. Although the BER performance of the proposed approached is slightly inferior, the resulting savings in the computational complexity are staggering. The benchmark performance requires 1,878,416 MA operations; the proposed low complexity scheme requires 117,360 MA operations.

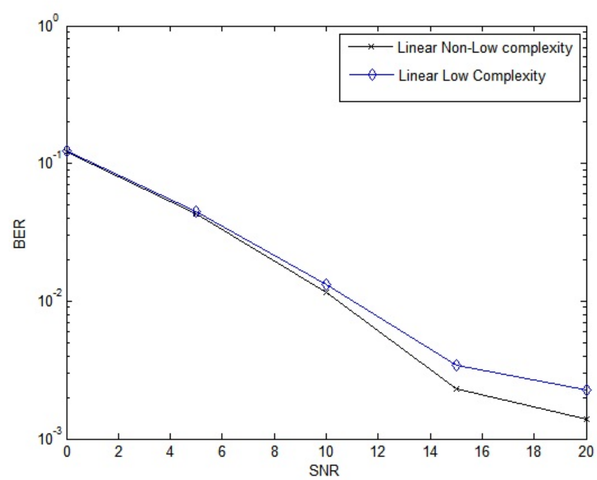

Fig. 1 The BER Performance of low complexity linear bidirectional design and comparison with non-low complexity technique.

Numerical simulations of the proposed low complexity bidirectional DFE are shown in Fig.2. The DFE has feedforward filter length $\acute{L}$=10 and feedback filter length L_f=5. Also provided is a benchmark of the bidirectional DFE without using any low complexity approach. Again the performance loss is very little when viewed in the context of the staggering reduction of the complexity. While the benchmark performance requires 1,980,416 MA operations, the proposed low complexity scheme requires 219,760 MA operations.

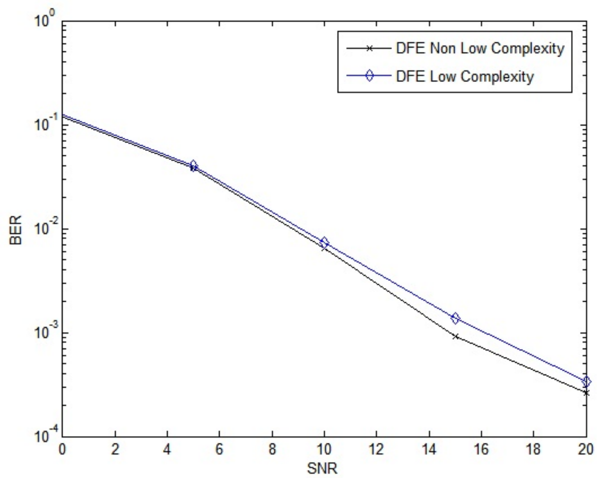

Fig. 2 The BER Performance of proposed low complexity DFE bidirectional design and comparison with non-low complexity technique.

VII. CONCLUSION RESULT

This manuscript focuses on the design of very low complexity bidirectional equalizer for doubly selective channels. Like [1] our proposed algorithm for low complexity bidirectional equalization of doubly selective channels is based on exploiting small matrix inversions and the BEM for complexity reduction has been discussed and requirements for computations have been studied. This algorithm provides significant lower complexity than the existing BEM based approaches of [2]-[4]. Our proposed scheme provides low complexity using BEM and better performance because of two equalizations; forward and revere direction equalizations.BEM based approaches of very low complexity to equalization of doubly selective channels provide a very flexible tradeoff between performance and complexity.

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