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Statistical Spectrum Sensing in Cognitive Radio

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***Abstract*—Statistical spectrum sensing is a promising method which can reliably detect the primary users while requiring little prior information in cognitive radio networks. In this paper, we present an overview of sensing methods based on Goodness-of-Fit tests. We discuss the performance of Energy Detector (ED) sensing, Anderson Darling (AD) sensing, Cramér Von Mises(CVM) sensing and Order Statistic (OS) sensing and we compare the results using Monte-Carlo simulations. It is shown that OS sensing outperforms ED sensing, CVM sensing and AD sensing. Next it is shown through simulations that the OS test statistic does not provide maximum probability of detection for a desired probability of false alarm and results are provided showing the regions of high probability of detection for desired probability of false alarm.**

I. Introduction

With the recent advancements in wireless communications, there is an increasing demand to support various wireless services, which require an efficient and an effective utilization of radio frequency spectrum. In classical spectrum regulatory framework, frequency spectrum is exclusively allocated to certain licensed services and strictly prohibits any violation from unlicensed user to utilize the licensed band. More importantly, reports [1] by Federal Communication Commission (FCC) reflects that frequency spectrum is grossly under utilized in vast temporal and geographical areas [1], which proves that existing frequency allocation schemes are incapable of providing an efficient utilization of frequency spectrum.

Cognitive Radio (CR) technology [2] seems to be a promising solution for an effective utilization of valuable radio resources and to overcome the problems of spectrum under utilization and spectrum scarcity. In CR, spectral congestion problem is solved by enabling unlicensed users (Secondary Users) to borrow the unused spectrum from licensed users (Primary Users) in an adaptive/opportunistic manner with change in radio operating environments and conditions.

The key task in CR technology is to perform *spectrum sensing*. Spectrum sensing is a task of obtaining awareness about spectrum usage and existence of primary user in certain geographical area [3]. Hence, by sensing and adapting to conditions, the CR technology enables secondary users to unlock the spectrum holes and thus can serve secondary users to borrow spectrum from primary users without causing any interference. To do so, secondary users have to detect the presence of primary users recurrently by spectrum sensing. Whenever, the primary user is present, the secondary user should detect the presence with high probability and cease to transmit its data.

A large number of spectrum sensing methods are available in literature [3],[4]. The most recognized sensing method is Energy Detector (ED) based sensing. Several sensing methods have also been proposed, including likelihood ratio test (LRT), matched filtering (MF) detection, cyclo-stationary detection (CSD), eigenvalue based sensing, wavelet based sensing, covariance based sensing and blindly combined energy detection [4].

ED sensing is widely recognized sensing method, however the performance of energy detector degrades drastically when the received SNR is low [3]. Based on widely used Anderson Darling Goodness of Fit (GoF) test, Wang *et al.* [5] proposed a sensing technique employing Anderson Darling test statistic known as Anderson Darling (AD) sensing. AD sensing technique requires the noise distribution to be known a-priori. Through analysis and numerical results the authors in [5] have shown that under same sensing conditions and channels environments, and considering an Additive White Gaussian Noise (AWGN) channel, the AD method outperforms the ED method particularly when the received SNR is low. On similar grounds, a spectrum sensing based on Cramér Von Mises (CVM) GoF test is presented in [6]. However, simulation results show that AD sensing also outperforms CVM sensing. Recently, Rostami *et al.* proposed Order Statistic (OS) based sensing method [7] based on the quantiles of each ordered observation. OS sensing method is based on Order Statistic GoF and also requires the noise distribution to be known a-priori like AD sensing and CVM sensing. Authors in [7], have shown that for an AWGN channel, the OS based sensing method performs ever better than AD sensing method particularly in low SNR regime. However, we have analyzed through simulation that the test statistic proposed in [7] is not optimized i.e. maximum probability of detection cannot be achieved using the test statistic proposed in [7]. Readers are referred to [3],[4] to get the understanding of spectrum sensing problem in cognitive radios. Unfortunately, [3] and [4] do not provide any review of spectrum sensing performance using GoF tests. [6] review of sensing methods based on AD and CVM GoF tests. However, in our paper we have presented a thorough review of sensing methods based on GoF tests available in literature.

Rest of the paper is organized as follows. The general system model for the general setup for sensing is given in Section II. A brief overview of ED sensing is provided in Section III, and that of CVM, AD and OS sensing is provided

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in Section IV, Section V and Section VI respectively. In Section VII we have presented comparison of various sensing methods. Our analysis regarding OS sensing is presented in Section VIII. Finally based on the analysis provided in Section VII and Section VIII, conclusion is drawn in Section IX.

II. System Model

In the system model, we consider that there is a single primary user and a single secondary user. It is further assumed that the channel is Additive White Gaussian Noise (AWGN) and no interfering elements present in the neighborhood of both the primary and secondary users. The observations at the cognitive radio can be represented as

$$x_i = \sqrt{\gamma}a + \eta_i,\ i = 1, 2, ..., N \tag{1}$$

where, i represents the total number of observations made at the cognitive radio during spectrum sensing process. x_i represents the i^{th} sample of the received signal, η_i represents the i^{th} sample of white gaussain noise with zero mean and unit variance $\mathcal{N}(0, 1)$, γ represents the received SNR (the variance of noise is unity, so average energy of signal is also equal to SNR) and a represents the signal from the primary user. Without loss of generality, we assume that $a = 1$, and that the received signal sample x_i is real valued.
Spectrum sensing in cognitive radio is the following binary hypothesis testing problem. $\mathcal{H}_0$ represents the absence of primary user and $\mathcal{H}_1$ represents the presence of primary user. It is important to note that the received signal in absence of the primary user will be Gaussian under $\mathcal{H}_0$ because of the presence of noise, and non-Gaussian under $\mathcal{H}_1$ because of the presence of mixture of noise and primary user signal. The binary hypothesis testing problem can thus represented as:

$$\begin{aligned} \mathcal{H}_0 &: x_i = \eta_i \\ \mathcal{H}_1 &: x_i = s_i + \eta_i \end{aligned} \tag{2}$$

where $s_i = \sqrt{\gamma}a$.
The objective of spectrum sensing is to make a decision on binary hypothesis testing (choose either $\mathcal{H}_0$ or $\mathcal{H}_1$), based on the nature of received signal. Stacking the received samples at the cognitive radio, yields the following vectors:

$$\begin{aligned} \mathbf{x} &= [x_1, x_2, \cdots, x_N]^{\mathrm{T}} \\ \mathbf{s} &= [s_1, s_2, \cdots, s_N]^{\mathrm{T}} \\ \boldsymbol{\eta} &= [\eta_1, \eta_2, \cdots, \eta_N]^{\mathrm{T}} \end{aligned} \tag{3}$$

It is assumed that the received samples are i.i.d. i.e. x_i and x_j are independent of each other $\forall i, j \in \mathcal{S}$, where $\mathcal{S} = \{1, 2, \cdots, N\}$. The hypothesis testing problem based in N samples is thus obtained as

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{x} = \boldsymbol{\eta} \\ \mathcal{H}_1 &: \mathbf{x} = \mathbf{s} + \boldsymbol{\eta} \end{aligned} \tag{4}$$

Let $\mathcal{F}_{\mathbf{x}}(x)$ represents the empirical cumulative density function (CDF) of the received samples. Mathematically, $\mathcal{F}_{\mathbf{x}}(x)$ can be written as:

$$\mathcal{F}_{\mathbf{x}}(x) = \frac{|\{i : x_i \leq x, 1 \leq i \leq N\}|}{N} \tag{5}$$

where for any set $\mathcal{S}$, $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$.
It is evident that in the absence of primary user, the received samples $x_1, x_2, \cdots, x_n$ are samples from noise and can be represented by independent and identically distributed sequence with a known noise distribution $\mathcal{F}_{\boldsymbol{\eta}}(x)$. It is reasonable to assume that noise samples come from a Gaussian distribution. So, $\mathcal{F}_{\boldsymbol{\eta}}(x)$ can be represented as:

$$\mathcal{F}_{\boldsymbol{\eta}}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-y^2/2} \mathrm{d}y \tag{6}$$

On the other hand, in the presence of primary user, the received samples may not be independent, even they are i.i.d, they do not come from the distribution function $\mathcal{F}_{\boldsymbol{\eta}}(x)$. So, the binary hypothesis testing problem can be revisited as:

$$\begin{aligned} \mathcal{H}_0 &: \mathcal{F}_{\mathbf{x}}(x) = \mathcal{F}_{\boldsymbol{\eta}}(x) \\ \mathcal{H}_1 &: \mathcal{F}_{\mathbf{x}}(x) \neq \mathcal{F}_{\boldsymbol{\eta}}(x) \end{aligned} \tag{7}$$

In order to test any sensing algorithm, first the respective test statistic is calculated i.e. $\mathcal{T}(x)$, afterwards the result of the test statistic is compared to the pre-determined threshold λ to decide about the presence or absence of the primary user. $\mathcal{T}(x) > \lambda$ indicates that the primary user is present and vice versa. The performance of the sensing algorithm is generally indicated by two metrics: probability of detection P_D, the probability of an algorithm to correctly detecting the presence of the primary user; and the probability of false alarm P_{FA}, the probability of an algorithm mistakenly declaring the presence of the primary user. A sensing method is called 'optimal' if it achieves the highest P_D for a given P_{FA} with given number of samples [4]. Mathematically P_{FA} for a given P_D can be represented as:

$$P_{FA} = \mathcal{P}\{\mathcal{T} > \lambda | \mathcal{H}_0\} \tag{8}$$

$$P_D = \mathcal{P}\{\mathcal{T} > \lambda | \mathcal{H}_1\} \tag{9}$$

where $\mathcal{P}\{.\}$ denotes the probability operator.
Moreover, Receiver Operating Characteristics (ROC) curves are important in determining the performance of sensing method. Generally in an ROC, P_{FA} is plotted on x-axis while P_D is plotted on y-axis.

III. Energy Detector Based Sensing

Energy detector based scheme is one of the most widely used sensing scheme because of its low computational and implementation complexities [3]. In this scheme, the presence of the primary user signal is detected by comparing the output of the energy detector with a pre-determined threshold λ_{ED} to make a binary decision. The performance of energy detector in low SNR regime degrades drastically and consequently leads to inability to differentiate between primary user signals and secondary user signals [3]. It is important to note that energy is a relative quantity and also require a reference noise energy to detect the presence of primary user signal in the mixture of signal and noise. This leads to poor performance of energy detector in the presence of noise uncertainty.

The decision metric for energy detector can be written as:

$$\mathcal{T}_{\mathsf{ED}} = \frac{1}{N}\sum_{i=1}^{N}|x_i|^2 \quad (10)$$

where N is the size of the observation vector. It is important to note that the performance of Energy Detector increases with increase in number of samples N as shown in Fig. 1 and Fig. 2. It is important to note that (10) follows chi-square distribution with $2N$ degrees of freedom χ^2_{2N} and hence it can be modeled as:

$$\mathcal{T}_{\mathsf{ED}} = \begin{cases} \chi^2_{2N} & \mathcal{H}_0 \\ \chi^2_{2N}(2\gamma) & \mathcal{H}_1 \end{cases} \quad (11)$$

where χ^2_{2N} denotes a chi-square distribution with $2N$ degrees of freedom and $\chi^2_{2N}(2\gamma)$ denotes a non-central chi-square distribution with $2N$ degrees of freedom with a non-centrality parameter 2γ.

For ED the probabilities P_{FA} and P_D can be calculated as:

$$P_{FA} = \mathcal{P}\{\mathcal{T}_{\mathsf{ED}} > \lambda_{\mathsf{ED}}|\mathcal{H}_0\} = \frac{\Gamma(N, \lambda_{\mathsf{ED}}/2)}{\Gamma(N)} \quad (12)$$

$$P_D = \mathcal{P}\{\mathcal{T}_{\mathsf{ED}} > \lambda_{\mathsf{ED}}|\mathcal{H}_1\} = \mathcal{Q}_m(\sqrt{2N\gamma}, \sqrt{\lambda_{\mathsf{ED}}}) \quad (13)$$

In the above equations, λ_{ED} and γ denotes the energy detector threshold and the instantaneous received SNR and N represents the number of samples. $\Gamma(a, x)$ is incomplete gamma function given by $\Gamma(a, x) = \int_x^\infty t^{a-1}e^{-t}\mathrm{d}t$, $\Gamma(a)$ is the gamma function and $\mathcal{Q}_m$ is the generalized Marcum Q-function given by $\mathcal{Q}_m(a, x) = \frac{1}{a^{m-1}}\int_x^\infty t^m e^{-\frac{t^2+a^2}{2}} I_{m-1}(at)\mathrm{d}t$, with $I_{m-1}()$ is the modified Bessel function of the first kind and order $m-1$.

A brief overview of all the steps involved in ED sensing are given below:

Step 01: Determine the threshold λ_{ED}.

Step 02: Calculate the test statistic $\mathcal{T}_{\mathsf{ED}}$ using (10).

Step 03: The decision regarding the occupancy of the channel can be made by comparing the decision metric $\mathcal{T}_{\mathsf{ED}}$ against the threshold λ_{ED}. If $\mathcal{T}_{\mathsf{ED}} < \lambda_{\mathsf{ED}}$, $\mathcal{H}_0$ is true showing that the channel is idle, else if $\mathcal{T}_{\mathsf{ED}} > \lambda_{\mathsf{ED}}$, $\mathcal{H}_1$ is true and we consider that the channel is busy.

IV. Cramér Von Mises (CVM) Based Sensing

[6] proposed a spectrum sensing scheme which uses Cramér Von Mises GoF test. Authors considered spectrum sensing problem as GoF problem and applied the CVM statistic to check whether the received samples came from a known noise distribution. The most important fact is that the distribution should be known a-priori. It is important to note that the performance of CVM Sensing scheme is much better than ED Sensing scheme, even at low SNRs. The performance of CVM Sensing also increases with increase in number of samples N as shown in Fig. 3 and Fig. 4. Fig. 3 shows the impact of SNR on P_D while Fig. 4 shows ROC curves for CVM sensing with different samples N. The Cramér Von Mises statistic is defined by:

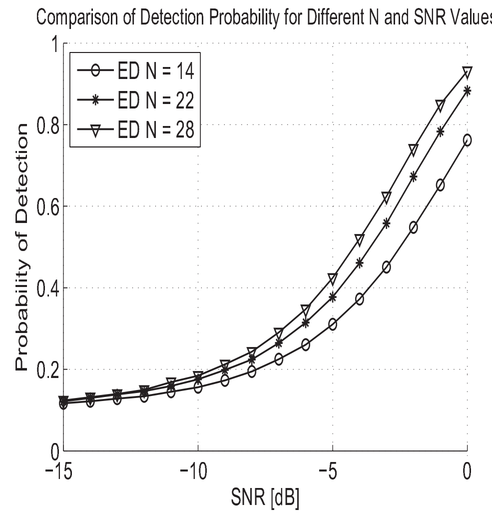

Fig. 1. Comparison of detection probability for ED sensing with different SNR values and different N and $P_{FA} = 0.1$

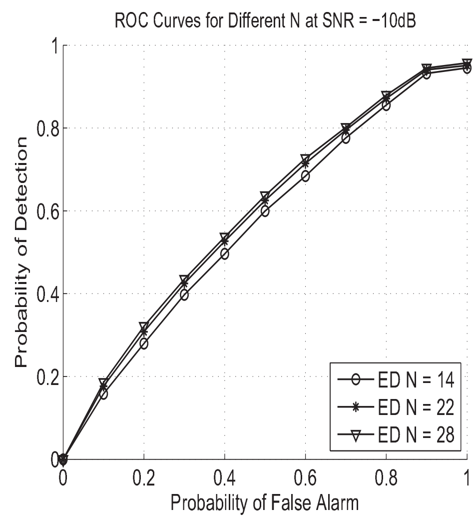

Fig. 2. ROC curves for ED sensing with different N

$$\mathcal{T}_{\mathsf{CVM}} \triangleq N\int_{-\infty}^{\infty}(\mathcal{F}_{\mathbf{x}}(x) - \mathcal{F}_{\boldsymbol{\eta}}(x))^2\mathrm{d}\mathcal{F}_{\boldsymbol{\eta}}(\mathrm{x}) \quad (14)$$

From [8], $\mathcal{T}_{\mathsf{CVM}}$ can be written as

$$\mathcal{T}_{\mathsf{CVM}} = \frac{1}{12N} + \sum_{i=1}^{N}\left[\mathcal{F}_{\boldsymbol{\eta}}(x_i) - \frac{2i-1}{2N}\right]^2 \quad (15)$$

For CVM the probabilities P_{FA} can be calculated as:

$$P_{FA} = \mathcal{P}\{\mathcal{T}_{\mathsf{CVM}} > \lambda_{\mathsf{CVM}}|\mathcal{H}_0\} \quad (16)$$

A brief overview of all the steps involved in CVM sensing are given:

Step 01: Determine the threshold λ_{CVM} according to a given P_{FA}. A table listing values of λ_{CVM} to different false alarm probabilities P_{FA} is given in [8].

Step 02: Arrange the received signal samples $\mathbf{x} = [x_1, x_2, \cdots, x_N]^{\mathrm{T}}$, in the ascending order of magnitude as:

$$x_{(1)} \leq x_{(2)}, \cdots \leq x_{(N)}$$

Let

$$\widetilde{\mathbf{x}} = \mathsf{sort}(\mathbf{x}) = [x_{(1)}, x_{(2)}, \cdots, x_{(N)}]^{\mathrm{T}}$$

Step 03: The sorted signal samples are transformed by the known noise CDF $\mathcal{F}_{\boldsymbol{\eta}}(x)$ given in (6) as,

$$z_i = \mathcal{F}_{\boldsymbol{\eta}}(x_{(i)}), \quad i \in \mathcal{S} \tag{17}$$

Step 04: Calculate the test statistic $\mathcal{T}_{\mathsf{CVM}}$ using (15). It is important to note that after the transformation $z_i = \mathcal{F}_{\boldsymbol{\eta}}(x_{(i)})$ (15) can be written as:

$$\mathcal{T}_{\mathsf{CVM}} = \frac{1}{12N} + \sum_{i=1}^{N} \left[z_i - \frac{2i-1}{2N} \right]^2 \tag{18}$$

Step 05: The decision regarding the occupancy of the channel can be made by comparing the decision metric $\mathcal{T}_{\mathsf{CVM}}$ against the threshold λ_{CVM}. If $\mathcal{T}_{\mathsf{CVM}} < \lambda_{\mathsf{CVM}}$, $\mathcal{H}_0$ is true showing that the channel is idle, else if $\mathcal{T}_{\mathsf{CVM}} > \lambda_{\mathsf{CVM}}$, $\mathcal{H}_1$ is true and we consider that the channel is busy.

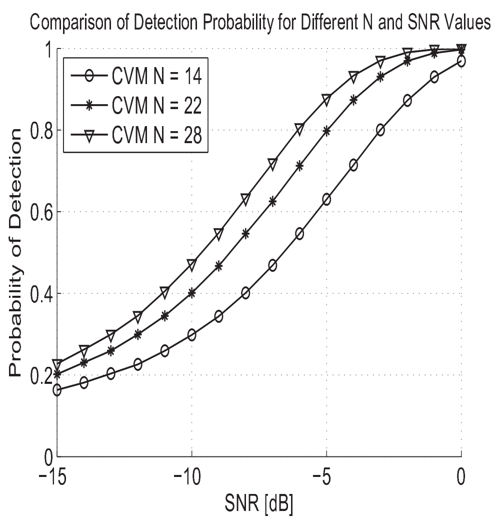

Fig. 3. Comparison of detection probability for CVM sensing with different SNR values and different N and $P_{FA} = 0.1$

V. Anderson Darling (AD) Based Sensing

In [5], the authors proposed a spectrum sensing technique based on Anderson-Darling (AD) GoF Test. The authors use the spectrum sensing problem as a GoF problem and used AD statistic to check whether the received samples are drawn from a known noise distribution. Similar to CVM sensing the noise distribution to perform the sensing should be known a-priori. It is stated in [5] that the CVM statistic presented in (15), does not give enough weights to the tails of the distribution $\mathcal{F}_{\boldsymbol{\eta}}$ and hence to add flexibility and generality the CVM statistic

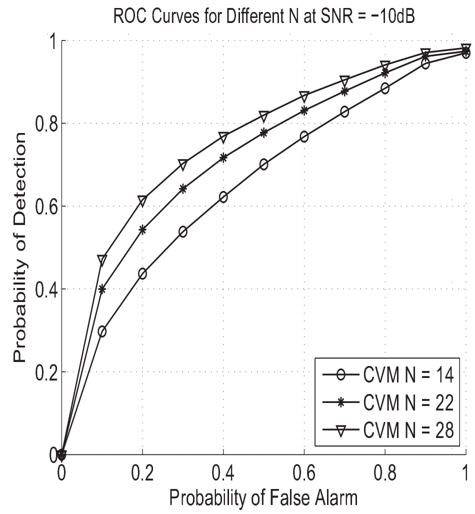

Fig. 4. ROC curves for CVM sensing with different N

is generalized by introducing the weight function, Anderson and Darling proposed the following statistic:

$$\mathcal{T}_{\mathsf{AD}} = \int_{-\infty}^{\infty} (\mathcal{F}_{\mathbf{x}}(x) - \mathcal{F}_{\boldsymbol{\eta}}(x))^2 \psi(\mathcal{F}_{\boldsymbol{\eta}}(x)) \mathrm{d}\mathcal{F}_{\boldsymbol{\eta}}(\mathrm{x}) \tag{19}$$

where $\psi(t)$ defines the non-negative weight function given by $\psi(t) = (t(1-t))^{-1}$ and is defined over $0 \leq t < 1$.
According to [8], above equation can be rewritten as:

$$\mathcal{T}_{\mathsf{AD}} = -\frac{\sum_{i=1}^{N} (2i-1)(\ln(\mathcal{F}_{\boldsymbol{\eta}}(x_i)) - \ln(\mathcal{F}_{\boldsymbol{\eta}}(x_{N+1-i})))}{N} - N \tag{20}$$

The asymptotic distribution of $\mathcal{T}_{\mathsf{AD}}$ under $\mathcal{H}_0$ can be given as:

$$\mathcal{F}(\mathcal{T}_{\mathsf{AD}}|\mathcal{H}_0; x) = \frac{\sqrt{2\pi}}{\lambda_{\mathsf{AD}}} \sum_{i=0}^{\infty} a_i(4i+1) \exp\left(-\frac{(4i+1)^2\pi^2}{8\lambda_{\mathsf{AD}}} \right) \times \int_0^{\infty} \exp\left(\frac{\lambda_{\mathsf{AD}}}{8(w^2+1)} - \frac{(4i+1)^2\pi^2 w^2}{8\lambda_{\mathsf{AD}}} \right) \mathrm{d}w \tag{21}$$

where, $a_i = (-1)^i \Gamma(i+0.5) \big/ (\Gamma(0.5)i!)$ and Γ is a Gamma function [ref AD paper]. Thus P_{FA} can be calculated as:

$$P_{FA} = 1 - \mathcal{F}(\mathcal{T}_{\mathsf{AD}}|\mathcal{H}_0; x) \tag{22}$$

A brief overview all of the steps involved in AD sensing are given:

Step 01: Determine the threshold λ_{AD} according to a given P_{FA}. This can be done by using the (22). However, a table listing values of λ_{AD} to different false alarm probabilities P_{FA} is given in [8].

Step 02: Arrange the received signal samples $\mathbf{x} = [x_1, x_2, \cdots, x_N]^{\mathrm{T}}$, in the ascending order of magnitude as:

$$x_{(1)} \leq x_{(2)} \cdots \leq x_{(N)}$$

Let

$$\widetilde{\mathbf{x}} = \mathsf{sort}(\mathbf{x}) = [x_{(1)}, x_{(2)}, \cdots, x_{(N)}]^{\mathrm{T}}$$

Step 03: The sorted signal samples are transformed by the know noise CDF $\mathcal{F}_{\boldsymbol{\eta}}(x)$ as,

$$z_i = \mathcal{F}_{\boldsymbol{\eta}}(x_{(i)}), \quad i \in \mathcal{S} \tag{23}$$

Step 04: Calculate the test statistic $\mathcal{T}_{\mathsf{AD}}$ using (15). It is important to note that after the transformation $z_i = \mathcal{F}_{\boldsymbol{\eta}}(x_{(i)})$ (**??**) can be written as:

$$\mathcal{T}_{\mathsf{AD}} = -\frac{\sum_{i=1}^{N} (2i-1)(\ln(z_i)) - \ln(z_{N+1-i}))}{N} - N \tag{24}$$

Step 05: The decision regarding the occupancy of the channel can be made by comparing the decision metric $\mathcal{T}_{\mathsf{AD}}$ against the threshold λ_{AD} to check if $\mathcal{H}_0$ can be rejected. If $\mathcal{T}_{\mathsf{AD}} > \lambda_{\mathsf{AD}}$, $\mathcal{H}_0$ is rejected and the channel is considered occupied, else if $\mathcal{T}_{\mathsf{AD}} < \lambda_{\mathsf{AD}}$, $\mathcal{H}_0$ is accepted and the channel is considered idle.

The performance of AD sensing increases with increase in number of samples N as shown in Fig. 5 and Fig. 6. Fig. 5 examines the impact of SNR of P_D while Fig. 6 compares the ROC curves achieved by AD sensing for different number of samples N.

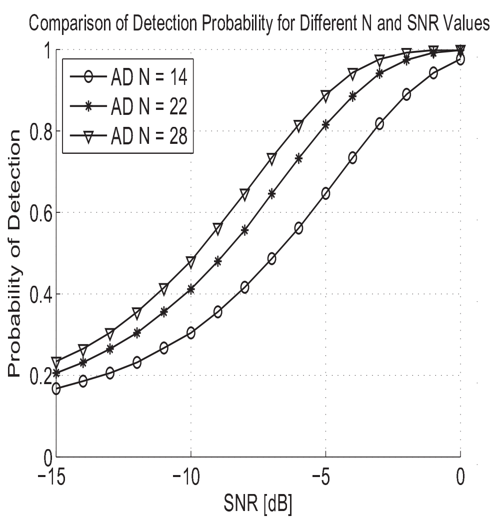

Fig. 5. Comparison of detection probability for AD sensing with different SNR values and different N and $P_{FA} = 0.1$

VI. Order Statistics (OS) Based Sensing

Recently, Rostami *et. al* in [7] have proposed Order Statistics based sensing method. OS sensing method is also based on GoF testing. Authors in [7] applied the modified GoF testing problem based in Order Statistics to propose a novel sensing method. Similar to CVM and AD sensing method, this method can be applied to any noise distribution but that should be known a-priori. OS sensing is based on quantiles of all ordered observations in the distribution. Authors [7], referred these quantiles as $\boldsymbol{\rho}$ vector and applied the similar vector in the test

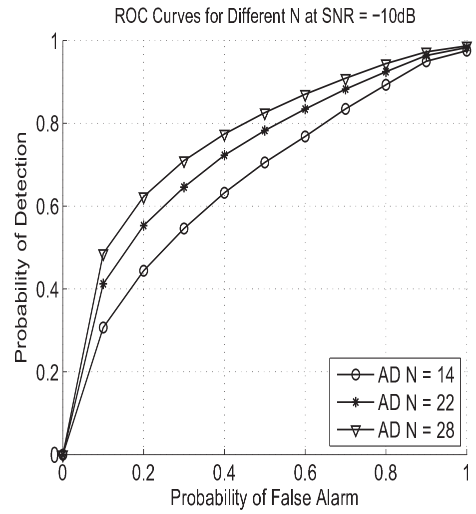

Fig. 6. ROC curves for AD sensing with different N

statistic. The authors also argued that the extreme components of the $\boldsymbol{\rho}$ vector indicates a poor fit with $\mathcal{F}_{\boldsymbol{\eta}}$. The results in [7] show that OS sensing method outperforms both AD and CVM sensing methods in an AWGN channel particularly at low SNR values.

A brief overview all of the steps involved in OS sensing are given:

Step 01: Calculate the threshold λ_{OS} for a given number of samples and desired P_{FA} using the following equation:

$$\begin{aligned}\lambda_{\mathsf{OS}} =& 2.599 + 0.8228N - 30.79P_{FA} + \\ & 73.79{P_{FA}}^2 - 49.08{P_{FA}}^3 - 0.6466P_{FA}N\end{aligned} \tag{25}$$

It is also reported in [7] that it is mathematically intractable to derive a closed form expression for λ_{OS} [7]. The authors in [7] performed extensive simulations to approximate the λ_{OS}. It is also important to note the the decision threshold is a function of number of samples N and the probability of false alarm P_{FA}. Simulations were performed by varying N such that $(20 \leq N \leq 100)$ and range the range of values of false alarm probability $(0.05 \leq P_{FA} \leq 0.95)$. The Root Mean Square (RMS) error between the simulated threshold and approximated threshold λ_{OS} is about 0.35. It is also important to note that if large number of samples are chosen e.g. $N = 1000$, (25) completely fails to determine the correct threshold for the test statistic. (25) is only valid for samples sizes up to 100. If the sample size is increased correspondingly the calculated threshold λ_{OS} is also increased, which in turn results in wrong computation of the P_D.

Step 02: Arrange the received signal samples $\mathbf{x} = [x_1, x_2, \cdots, x_N]^{\mathrm{T}}$, in the ascending order of magnitude as:

$$x_{(1)} \leq x_{(2)} \cdots \leq x_{(N)}$$

Let

$$\widetilde{\mathbf{x}} = \mathsf{sort}(\mathbf{x}) = [x_{(1)}, x_{(2)}, \cdots, x_{(N)}]^{\mathrm{T}}$$

Step 03: The sorted signal samples are transformed by the know noise CDF $\mathcal{F}_{\boldsymbol{\eta}}(x)$ as,

$$z_i = \mathcal{F}_{\boldsymbol{\eta}}(x_{(i)}), \quad i \in \mathcal{S} \tag{26}$$

Define,

$$\mathbf{z} = [z_1, z_2, \cdots, z_N]^{\mathrm{T}}$$

Step 04: Elements of $\mathbf{z}$ are sorted in ascending order of magnitude as:

$$z_{(1)} \leq z_{(2)} \cdots \leq z_{(N)}$$

Let, $\widetilde{\mathbf{z}}$ be the sorted vector obtained by arranging the elements of $\mathbf{z}$ in ascending order of magnitude. So, we have:

$$\widetilde{\mathbf{z}} = \mathsf{sort}(\mathbf{z}) = [z_{(1)}, z_{(2)}, \cdots, z_{(N)}]^{\mathrm{T}}$$

Step 05: ρ vector is obtained by transforming the elements of $\widetilde{\mathbf{z}}$ by using beta CDF [7]. The transformation can be represented as:

$$\rho_i = \beta\left(z_{(i)}; i, N-i+1\right), \quad i \in \mathcal{S} \tag{27}$$

So, $\boldsymbol{\rho}$ can be represented as:

$$\boldsymbol{\rho} = [\rho_1, \rho_2, \cdots, \rho_N]^{\mathrm{T}} \tag{28}$$

where $\beta(y; \alpha, \beta)$ denotes the beta CDF, with $\alpha = i$ and $\beta = N - i + 1$ represents the shape parameters of the distribution. Expression presented in (27) can be simplified as:

$$\rho_i = \sum_{j=1}^{N} \frac{N!}{j!(N-j)!} {z_{(i)}}^j \left(1 - z_{(i)}\right)^{N-j}, \quad i \in \mathcal{S} \tag{29}$$

Step 06: The elements of $\boldsymbol{\rho}$ vector are then arranged in the ascending order of magnitude as:

$$\rho_{(1)} \leq \rho_{(2)} \cdots \leq \rho_{(N)}$$

$$\widetilde{\boldsymbol{\rho}} = \mathsf{sort}(\boldsymbol{\rho}) = [\rho_{(1)}, \rho_{(2)}, \cdots, \rho_{(N)}]^{\mathrm{T}} \tag{30}$$

Step 07: The elements of the $\widetilde{\boldsymbol{\rho}}$ vector are then applied to the following text statistic:

$$\mathcal{T}_{\mathsf{OS}} = \sum_{i \in \mathcal{S}} \left| \rho_{(i)} - \frac{i}{(N+1)^2} \right| \tag{31}$$

Step 08: The decision regarding the occupancy of the channel can be made by comparing the decision metric $\mathcal{T}_{\mathsf{OS}}$ against the threshold λ_{OS}. If $\mathcal{T}_{\mathsf{OS}} < \lambda_{\mathsf{OS}}$, $\mathcal{H}_0$ is true showing that the channel is idle, else if $\mathcal{T}_{\mathsf{OS}} > \lambda_{\mathsf{OS}}$, $\mathcal{H}_1$ is true and we consider that the channel is busy.

VII. Comparison of Sensing Methods

In this section, simulation results are presented to evaluate the performance of sensing methods presented in section III, section IV, section V and section VI. We have compared the performance of ED sensing, CVM sensing, AD sensing and OS sensing using 100000 Monte Carlo runs and results are compared using ROC curves and also results have been provided which show the impact of SNR on the sensing method.

In [6], it is shown through simulations that CVM sensing

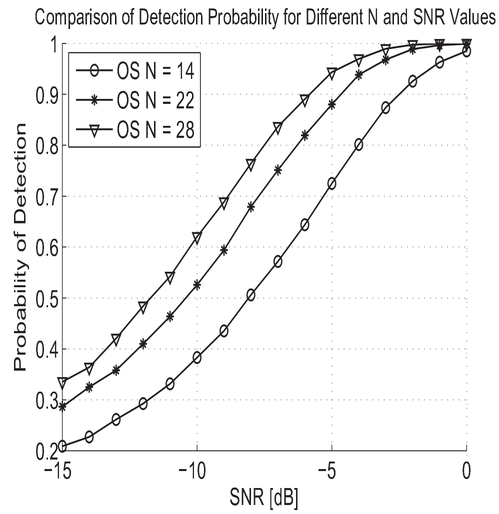

Fig. 7. Comparison of detection probability for OS sensing with different SNR values and different N and $P_{FA} = 0.1$

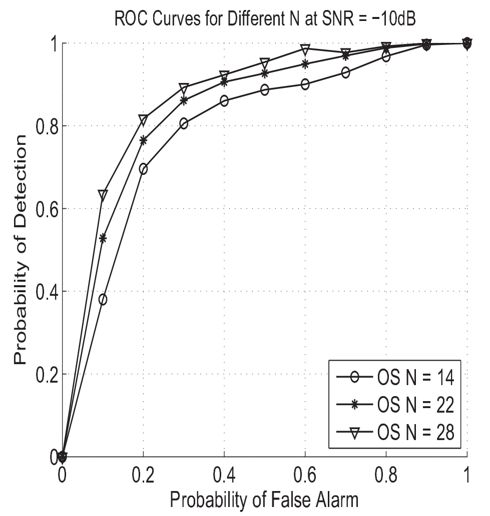

Fig. 8. ROC curves for OS sensing with different N

outperforms ED sensing in low SNR regime. Similarly authors in [5] and [6] have shown through simulations that considering an AWGN channel, the performance of AD sensing is better than both CVM sensing and ED sensing even at low SNR. However, Rostami *et al.* [7] have shown through Monte Carlo Simulations that the performance of OS sensing is better than ED sensing, CVM sensing and also AD sensing. Fig. 9, Fig. 10 and Fig. 11 compares the impact of different SNR for ED sensing, CVM sensing, AD sensing and OS sensing using $N = 14$, $N = 22$ and $N = 28$ respectively and for $P_{FA} = 0.1$. It is clear that no matter what is the received SNR, the performance of OS sensing is better than all the methods compared in this paper. Similarly, Fig. 12, Fig. 13 and Fig. 14, compares the ROC curves achieved by ED sensing, CVM sensing, AD sensing and OS sensing for $N = 14$, $N = 22$ and $N = 28$ respectively and SNR = -10dB. These results also

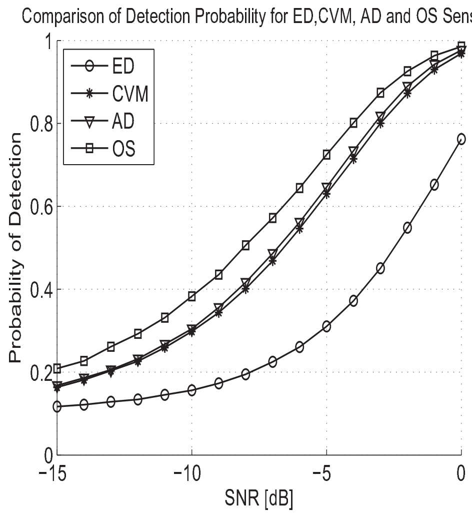

Fig. 9. Comparison of detection probability for ED sensing, CVM sensing, AD sensing and OS sensing with different SNR values, $N = 14$ and $P_{FA} = 0.1$

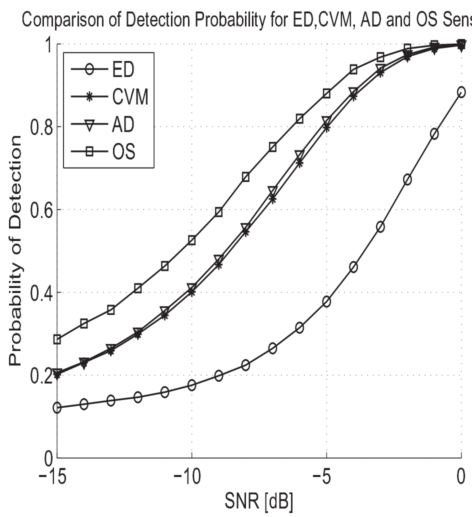

Fig. 10. Comparison of detection probability for ED sensing, CVM sensing, AD sensing and OS sensing with different SNR values, $N = 22$ and $P_{FA} = 0.1$

show that OS sensing is the most powerful technique and ED sensing is least powerful sensing methods, no matter what the number of samples N are.

VIII. Analysis of Order Statistic (OS) Sensing

The analysis provided in [9], motivated us to analyze the OS test statistic. So, instead of using (31), we considered the following statistic for deciding the presence of primary user is given by

$$\mathcal{T}_{\text{analysis}} = \sum_{i \in \mathcal{S}} \left| \rho_{(i)} - \frac{i + p_1}{(N + p_2)^2} \right| \quad (32)$$

It can be seen from (32), that if $p_1 = 0$ and $p_2 = 1$, $\mathcal{T}_{\text{analysis}}$ reduces to (31) which represents the original OS sensing statistic. However, we performed simulations using $p_1 = (0)(0.1)(1)$ and $p_2 = (0)(0.1)(1)$. $p = (0)(0.1)(1)$ means that the starting value of p is 0 and with an increment of 0.1 the ending value is 1. A total of 121 different test statistics were compared for $P_{FA} = 0.1$ and $N = 32$. The probability of detection for all the tests were calculated using Monte Carlo simulations Fig. 15 is plotted. It is important to note the red areas represent high probability of detection points while on the other hand blue areas represent the points with low probability of detection. From this analysis we concluded that the test statistic provided in [7] does not achieve maximum probability of detection. As shown in Fig. **??**, there are certain regions where the probability of detection for is better than OS sensing probability of detection which lie in a moderate range.

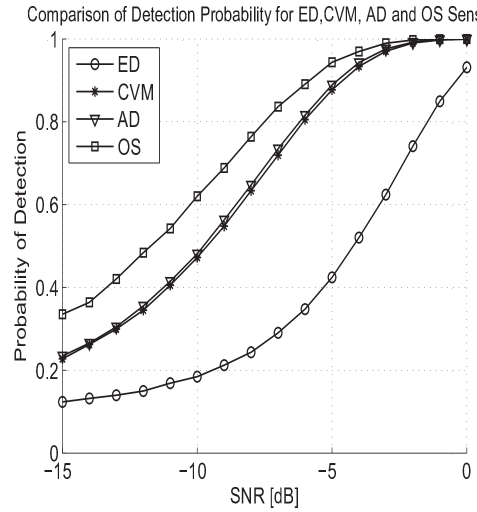

Fig. 11. Comparison of detection probability for ED sensing, CVM sensing, AD sensing and OS sensing with different SNR values, $N = 28$ and $P_{FA} = 0.1$

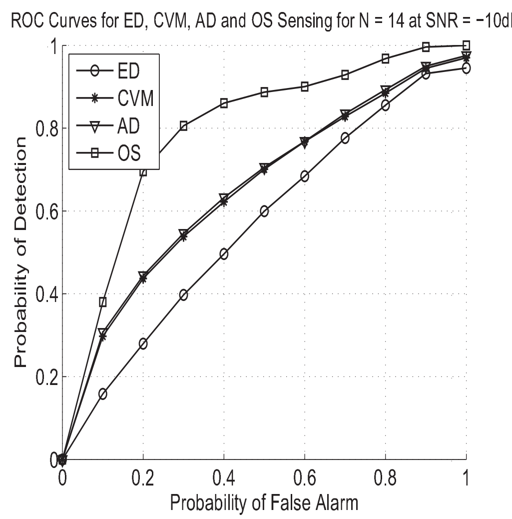

Fig. 12. ROC curves for ED sensing, CVM sensing, AD sensing and OS sensing with SNR = -10dB and $N = 14$

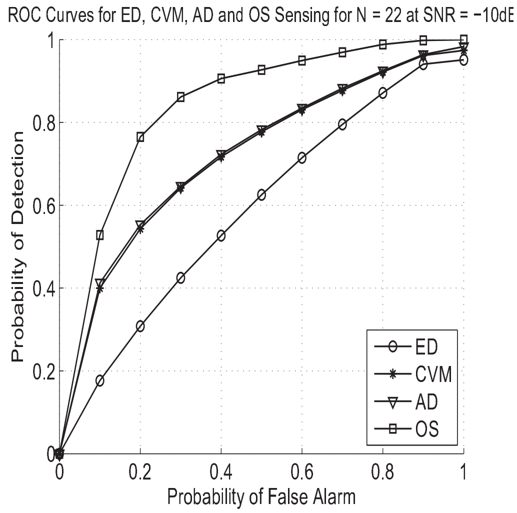

Fig. 13. ROC curves for ED sensing, CVM sensing, AD sensing and OS sensing with SNR = -10dB and $N = 22$

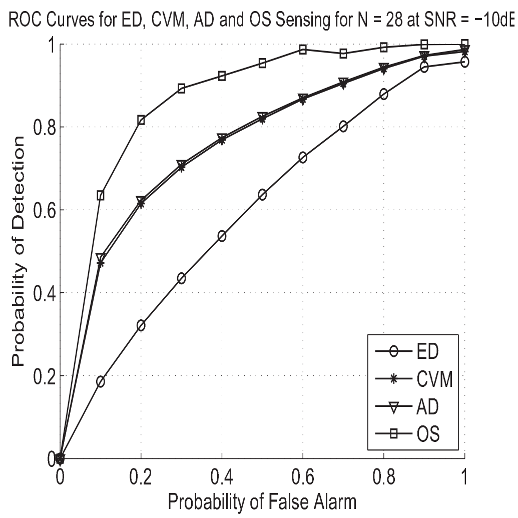

Fig. 14. ROC curves for ED sensing, CVM sensing, AD sensing and OS sensing with SNR = -10dB and $N = 28$

IX. Conclusion

In this paper, we have discussed different sensing methods primarily based on GoF tests and also analyzed the performance of OS sensing method. Simulation results show that the performance of OS sensing method is superior as compared to ED sensing, CVM sensing and AD sensing method. It is also analyzed the ED sensing method is least powerful among all presented in this paper. We further emphasized on the fact that the OS sensing test statistic does not give maximum probability of detection for certain P_{FA}. Thus, from the results it can be concluded that an optimized OS sensing method with high probability of detection can be achieved by changing the OS sensing method test statistic.

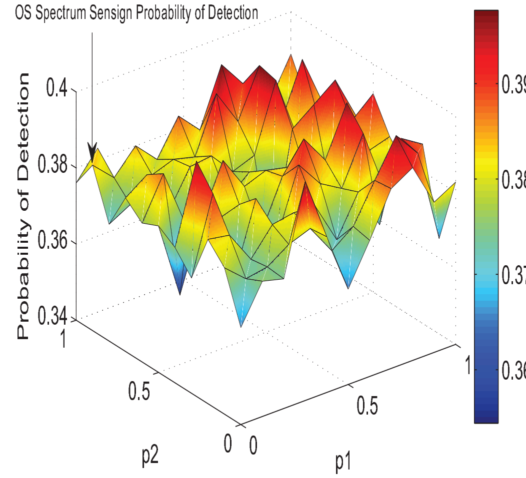

Fig. 15. Probability of Detection Analysis

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References

[1] Federal Communications Commission, Spectrum Policy Task Force," Rep. ET docket no. 02-135, Nov. 2002.
[2] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13-18, 1999.
[3] T. Yücek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys & Tutorials*, vol. 11, pp. 116-130, 2009.
[4] Y. Zeng, Y. C. Liang, A. T. Hoang and R. Zhang, "A review on spectrum sensing for cognitive radio: challanges and solutions," *EURASIP Journal on Advances in Signal Processing*, Artical ID 381465, 15 pages, 2010.
[5] H. Wang, E.-H. Yang, Z. Zhao and W. Zhang, "Spectrum sensing in cognitive radio using goodness of fit testing," *IEEE Trans. on Wireless Commun.*, vol. 8, pp. 5427-5430, 2009.
[6] S. Lei, H. Wang and L. Shen, "Spectrum sensing based on goodness of fit tests," *IEEE International Conference on Electronics, Communications and Control (ICECC)*, pp. 485-489, 2011.
[7] S. Rostami, K. Arshad and K. Moessner, "Order-Statistic based spectrum sensing for cognitive radio," *IEEE Commun. Letters*, vol. 16, no. 5, pp. 592-595, 2012.
[8] M. A. Stephens, "EDF Statistics for goodness of fit and some comparisons", *Journal of American Statistic Association*, vol. 69, no. 347, pp. 730-737, 1974.
[9] D. A. Gwinn, "Modified Anderson-Darling and Cramér-Von Mises goodness of fit tests for normal distribution", *Masters Dissertation submitted to Air Force Institute of Technology*, 1993.