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Robust Fault Detection Filter Design for Discrete Switched Linear Systems

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***Abstract*—A Robust fault detection filter(RFDF) is designed for discrete switched systems using H_-/H_∞ performance index. Average dwell time constraints have been used for switching and the fault detection filter is designed with the assumption that the switching sequence is already known. RFDF is designed using the multiple Lyapunov function (MLF) approach. Projection lemma has been used for the purpose of decoupling the system state space matrices from the Lyapunov matrices so that separate Lyapunov functions can be used for meeting the two objectives simultaneously i.e. minimizing the effect of unknown inputs on the residual and maximizing the effect of faults on the residual signal.**

***Index Terms*—Switched Systems, Fault Detection, Linear matrix inequalities, Average Dwell Time.**

I. Introduction

Hybrid systems are those systems that consist of both continuous and discrete dynamics. Hybrid systems have received considerable attention in the recent past as there are various chemical and mechanical processes that have a hybrid nature. Traffic control system and various power systems have a hybrid nature. Examples of various hybrid dynamical systems can be found in [1], [2].

Switched systems are a class of hybrid systems that consist of either continuous or discrete modes of operation and a switching law to switch from one mode to another. Stability of switched system is a major issue that has been analysed in much detail in [3], [4], [5]. Various techniques that are used for studying the stability of switched systems are switched Lyapunov function [6], common Lyapunov function and multiple Lyapunov function [3], [7]. A survey of different techniques for stability analysis of switched systems can be seen in [8]. Switching from one mode to another can be a function of time or states of the system [3], [9]. There are systems that become unstable if the switching is too fast because of the production of transients as switching takes place. However if the transients are allowed to die out i.e. the switching is sufficiently slow then these systems are stable. By allowing a certain dwell time between switching, the transients will die and the system will be stable [3], [10]. A necessary condition for stability of switched systems under slow switching is that each individual mode of the system should be stable. Average dwell time stability for slow switched systems is more flexible as compared to dwell time stability [11], [12].

The fault detection (FD) problem for switched systems has been considered in the literature both for state dependent switching [13], [14] and time dependent switching [15], [16], [17], [18]. In [15], [16] RFDF has been designed for continuous linear switched systems and uncertain discrete switched systems respectively using the H_-/H_∞ performance index and it has been assumed that the switching sequence is already known. FD for switched systems with state delays has been considered in [17]. In [18] fault detection filter has been designed for continuous switched systems using the H_∞ performance index. Design of fault detection observer for state switched discrete systems has been considered in [13] and the observer has been designed for the case when the active mode is unknown.

In this paper H_-/H_∞ index [19] will be used for designing a RFDF for discrete switched systems with average dwell time constraints. The objective is to minimize the effect of unknown inputs on the residual and maximize the sensitivity of faults. The robust fault detection filter designed for discrete switched systems in [15] gives conservative results. This is because the Lyapunov and system matrices are coupled due to which a single Lyapunov functiuon has to be used for minimizing the effect of unknown inputs on the residual and maximizing he effect of faults on the residual. In this paper projection lemma will be used to decouple the system matrices from Lyapunov matrices that will allow the use of separate Lyapunov function for minimizing the effect of unknown inputs on the residual and maximizing the effect of faults on the residual signal simultaneously. As a result of this less conservative results can be obtained as will be shown with the help of an example.

The remaining part of the paper is organized as follows: In Section (II), some preliminaries are given. Section (III) states the problem for discrete switched system and solution to the problem using projection lemma is given in Section (IV). Residual evaluation and threshold computation is then given in Section (V). In Section (VI) effectiveness of the proposed method is shown with the help of an example. Then conclusion is given in Section (VII).

II. Preliminaries

Lemma 1: [20] Given a symmetric matrix $Z \in S_m$ where S_m denotes a set of $m \times m$ symmetric matrices and two matrices U and V of column dimension m, there exists an unstructured matrix X that satisfies

$$U^T X V + V^T X^T U + Z < 0 \qquad (1)$$

if and only if the following projection inequalities with respect to X are satisfied:

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$$N_U^T Z N_U < 0 \quad (2)$$

$$N_V^T Z N_V < 0 \quad (3)$$

where N_U and N_V are arbitrary matrices whose columns form a basis of the nullspaces of U and V respectively.

$Average\ Dwell\ Time$: [11] For the switching signal $\sigma(k)$ and for any $k_f > 0$ and $k_i > 0$, let $N_\sigma(k_i, k_f)$ be the number of switching variations of $\sigma(k)$ over the interval$[k_i, k_f]$. If for any given $N_0 > 0$ and $\tau_a > 0$ we have

$$N_\sigma(k_i, k_f) \leq N_0 + \frac{(k_f - k_i)}{\tau_a}$$

then τ_a and N_0 are called the average dwell time and chatter bound respectively.

Lemma 2: [21] For the discrete switched system $x_{k+1} = A_i x_k$, suppose there exists a candidate Lyapunov function $V_i(x_k)$ that satisfies the following two properties

$$\triangle V_i(x_k) = V_i(x_{k+1}) - V_i(x_k) \leq -\zeta V_i(x_k)$$

and

$$V_j(x_k) \leq \mu V_i(x_k)$$

then the system is globally asymptotically stable for any switching law with average dwell time given by

$$\tau_a \geq \tau_a^* = \frac{ln\mu}{ln(1-\zeta)}$$

Where $\mu > 0$ and $0 < \zeta < 1$. The value of τ_a should be rounded to the nearest integer value as the system is discrete here.

III. Problem Formulation

Consider a discrete switched system with state space model given as

$$x_{k+1} = A_i x_k + B_i u_k + E_{di} d_k + E_{fi} f_k \quad (4)$$

$$y_k = C_i x_k + D_i u_k + F_{di} d_k + F_{fi} f_k \quad (5)$$

Where $i \in \{1, 2, ...N\}$ represents the active mode of operation and N represents the total modes of the system. The fault detection filter for the discrete switched system (4)-(5) is given by

$$\hat{x}_{k+1} = A_i \hat{x}_k + B_i u_k + L_i(y_k - \hat{y}_k) \quad (6)$$

$$\hat{y}_k = C_i \hat{x}_k + D_i u_k \quad (7)$$

$$r_{ki} = G_i(y_k - \hat{y}_k) \quad (8)$$

L_i is the observer gain matrix and G_i is the post filter. We can write the dynamics of the fault detection filter as

$$e_{k+1} = \bar{A}_i e_k + \bar{E}_{di} d_k + \bar{E}_{fi} f_k \quad (9)$$

$$r_{ki} = \bar{C}_i e_k + \bar{F}_{di} d_k + \bar{F}_{fi} f_k \quad (10)$$

Where $e_k = x_k - \hat{x}_k$

$\bar{A}_i = A_i - L_i C_i$, $\bar{E}_{di} = E_{di} - L_i F_{di}$, $\bar{E}_{fi} = E_{fi} - L_i F_{fi}$. $\bar{C}_i = G_i C_i$, $\bar{F}_{di} = G_i F_{di}$ and $\bar{F}_{fi} = G_i F_{fi}$.

The two objectives that need to be satisfied simultaneously are

- Minimizing the effect of disturbances on residual signal

$$\|r_{di}\|_{2,f=0} < \gamma_i \|d\|_2$$

- Maximizing the effect of faults on residual signal

$$\|r_{fi}\|_{2,d=0} > \beta_i \|f\|_2$$

IV. RFDF Design for Discrete Switched System

By using the method presented in [15] the fault detection filter can be designed by using the following lemma

Lemma 3: [15] The system (9)-(10) is said to be globally asymptotically stable and satisfies

$$\|r_{di}\|_2 < \gamma_i \|d\|_2$$

$$\|r_{fi}\|_2 > \beta_i \|f\|_2$$

for any switching signal with average dwell time

$$\tau_a \geq \tau_a^* = \frac{ln\mu}{ln(1-\zeta)}$$

if there exist some matrices $P_i = P_i^T > 0$ and $Q_i = Q_i^T > 0$ and scalars $\gamma_i > 0$, $\beta_i > 0$ so that

$$\begin{bmatrix} -P_j & P_j \bar{A}_j & P_j \bar{E}_{dj} \\ * & -(1-\zeta)P_i + \bar{C}_j^T \bar{C}_j & \bar{C}_j^T \bar{F}_{dj} \\ * & * & \bar{F}_{dj}^T \bar{F}_{dj} - \gamma_j^2 I \end{bmatrix} < 0 \quad (11)$$

$$P_j - \mu P_i < 0 \quad (12)$$

$$\begin{bmatrix} Q_j & Q_j \bar{A}_j & Q_j \bar{E}_{fj} \\ * & (1-\zeta)Q_i + \bar{C}_j^T \bar{C}_j & \bar{C}_j^T \bar{F}_{fj} \\ * & * & \bar{F}_{fj}^T \bar{F}_{fj} - \beta_j^2 I \end{bmatrix} > 0 \quad (13)$$

$$Q_j - \mu Q_i < 0 \quad (14)$$

for all $i, j \epsilon \{1, 2, ..., N\}$ and $i \neq j$ where $\mu > 1$ and $0 < \zeta < 1$ are given constants. The matrix inequalites (MIs) (11)-(14) are actually nonlinear. So following substitutions are made in order to transform them into linear matrix inequalities (LMIs)

$$P_i L_i = X_i \quad \Rightarrow \quad L_i = (P_i)^{-1} X_i \quad (15)$$

$$Q_i L_i = Y_i \quad \Rightarrow \quad L_i = (Q_i)^{-1} Y_i \quad (16)$$

$$M_i = G_i^T G_i \quad (17)$$

The equations (15)-(16) give two values of the observer gain for each mode of the system. The solution proposed in [15] is that we set $P_i = Q_i$ and in this way we can get a single value of observer gain L_i for each mode of operation. This will give conservative results as a single Lyapunov function is being used to meet two objectives simultaneously. Moreover the switching constraints have been imposed on the Lyapunov function $V_i(e_k)$ in MI (12) and (14) as well as on $\triangle V(e_k)$ when switching takes place from mode i to mode j i.e. $\triangle V(e_k) = V_j(e_{k+1}) - V_i(e_k) < -\zeta V_i(e_k)$. This additional constraint on $\triangle V(e_k)$ at the switching instant is not needed as for stability of slowly switched systems with average dwell time $\tau_a \geq \tau_a^* = \frac{ln\mu}{ln(1-\zeta)}$ we only need

- $V_j(e_k) < \mu V_i(e_k)$ and
- $\triangle V_j(e_k) = V_j(e_{k+1}) - V_j(e_k) < -\zeta V_j(e_k)$

So by removing the additional constraint on $\triangle V(e_k)$ at the time of switching in lemma 3 the new results obtained are given in the following theorem

Theorem 1: The system (9)-(10) is said to be globally asymptotically stable and satisfies

$$\|r_{di}\|_2 < \gamma_i \|d\|_2$$

$$\|r_{fi}\|_2 > \beta_i \|f\|_2$$

for any switching signal with average dwell time

$$\tau_a \geq \tau_a^* = \frac{ln\mu}{ln(1-\zeta)}$$

if there exist some matrices $P_i = P_i^T > 0$ and scalars $\gamma_i > 0$, $\beta_i > 0$ so that

$$\begin{bmatrix} -P_i & P_i\bar{A}_i & P_i\bar{E}_{di} \\ * & -(1-\zeta)P_i + \bar{C}_i^T\bar{C}_i & \bar{C}_i^T\bar{F}_{di} \\ * & * & \bar{F}_{di}^T\bar{F}_{di} - \gamma_i^2 I \end{bmatrix} < 0 \quad (18)$$

$$\begin{bmatrix} P_i & P_i\bar{A}_i & P_i\bar{E}_{fi} \\ * & (1-\zeta)P_i + \bar{C}_i^T\bar{C}_i & \bar{C}_i^T\bar{F}_{fi} \\ * & * & \bar{F}_{fi}^T\bar{F}_{fi} - \beta_i^2 I \end{bmatrix} > 0 \quad (19)$$

$$P_i - \mu P_j < 0 \quad (20)$$

for all $i, j \epsilon \{1, 2, ..., N\}$ and $i \neq j$ where $\mu > 1$ and $0 < \zeta < 1$ are given constants. Then by setting $P_iL_i = X_i$ and $G_i^TG_i = M_i$ as in theorem 2 the MIs (18)-(20) can be transformed into LMIs.

Now since a single Lyapunov function is being used in lemma 3 and Theorem 1 for simultaneously minimizing the effect of unknown inputs on residual and maximizing the sensitivity level which gives conservative results. So in order to use different Lyapunov functions for the multiobjective problem and obtain less conservative results following theorem is proposed

Theorem 2: The RFDF given in (9)-(10) is stable and satisfies the performance index

$$\|r_{di}\|_{2,f=0} < \gamma_i \|d\|_2$$

$$\|r_{fi}\|_{2,d=0} > \beta_i \|f\|_2$$

for any switching law with average dwell time

$$\tau_a \geq \tau_a^* = \frac{ln\mu}{ln(1-\zeta)}$$

if there exist some matrices $P_i = P_i^T > 0$, $Q_i = Q_i^T > 0$, X_i, Y_i and scalars $\gamma_i > 0$ and $\beta_i > 0$ such that following LMIs are satisfied

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & \Phi_{23} \\ * & * & \Phi_{33} \end{bmatrix} < 0 \quad (21)$$

$\Phi_{11} = -(1-\zeta)P_i + C_i^TM_iC_i + He(A_i^TX_i - C_i^TY_i)$
$\Phi_{12} = \lambda(A_i^TX_i - C_i^TY_i) - X_i^T$
$\Phi_{13} = C_i^TM_iF_{di} + X_i^TE_{di} - Y_i^TF_{di}$
$\Phi_{22} = P_i - \lambda He(X_i)$
$\Phi_{23} = \lambda(X_i^TE_{di} - Y_i^TF_{di})$
$\Phi_{33} = F_{di}^TM_iF_{di} - \gamma_i^2 I$

$$P_i - \mu P_j < 0 \quad (22)$$

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0 \quad (23)$$

$\Theta_{11} = -(1-\zeta)Q_i - C_i^TM_iC_i + He(A_i^TX_i - C_i^TY_i)$
$\Theta_{12} = \lambda(A_i^TX_i - C_i^TY_i) - X_i^T$
$\Theta_{13} = -C_i^TM_iF_{fi} + X_i^TE_{fi} - Y_i^TF_{fi}$
$\Theta_{22} = Q_i - \lambda He(X_i)$
$\Theta_{23} = \lambda(X_i^TE_{fi} - Y_i^TF_{fi})$
$\Theta_{33} = -F_{fi}^TM_iF_{fi} + \beta_i^2 I$

$$Q_i - \mu Q_j < 0 \quad (24)$$

for all $i, j \epsilon \{1, 2, ..., N\}$, and $i \neq j$ and $\mu > 1$ and $0 < \zeta < 1$ are given constants. Observer gain $L_i = (Y_iX_i^{-1})^T$ and post filter $G_i = (M_i)^{1/2}$.

$Proof$:

As the problem is multiobjective in which we have to meet the two objectives simulataneously so it is a conservative problem. In order to achieve less conservative results separate Lyapunov functions have to be defined for meeting each of the two objectives simultaneously. So in order to meet the two objectives, we can set two Lyapunov functions , one to minimze the effect of disturbances and other to increase the effect of faults on the residual signal. Considering the following Lyapunov functions for each mode of the switched system

$$V_i(e_k) = e_k^TP_ie_k$$

$$V_i(e_k) = e_k^TQ_ie_k$$

The objective of minimizing the effect of unknown inputs on the residual is obtained as

$$r_d^Tr_d + \triangle V_i(e_k) < \gamma_i^2 d^Td - \zeta V_i(e_k) \quad (25)$$

Here

$$\triangle V_i(e_k) = V_i(e_{k+1}) - V_i(e_k)$$

$$\triangle V_i(e_k) = (\bar{A}_ie_k + \bar{E}_{di}d_k)^TP_i(\bar{A}_ie_k + \bar{E}_{di}d_k) - e_k^TP_ie_k$$

Now

$$\begin{aligned}\triangle V_i(e_k)+\zeta V_i(e_k) &= (\bar{A}_i e_k+\bar{E}_{di}d_k)^T P_i(\bar{A}_i e_k+\bar{E}_{di}d_k) \\ &\quad -e_k^T P_i e_k+\zeta e_k^T P_i e_k \\ &= \begin{bmatrix} e_k^T & d_k^T \end{bmatrix} R_1 \begin{bmatrix} e_k \\ d_k \end{bmatrix} \end{aligned} \quad (26)$$

where $R_1=\begin{bmatrix} \bar{A}_i^T P_i \bar{A}_i-(1-\zeta)P_i & \bar{A}_i^T P_i \bar{E}_{di} \\ * & \bar{E}_{di}^T P_i \bar{E}_{di} \end{bmatrix}$
We can now write R_1 in the following form

$$R_1 = \begin{bmatrix} I & 0 \\ \bar{A}_i & \bar{E}_{di} \end{bmatrix}^T (\phi_d \otimes P_i) \begin{bmatrix} I & 0 \\ \bar{A}_i & \bar{E}_{di} \end{bmatrix} \quad (27)$$

Here $\phi_d=\begin{bmatrix} -(1-\zeta) & 0 \\ 0 & 1 \end{bmatrix}$

$$r_d^T r_d-\gamma_i^2 d^T d = \begin{bmatrix} e_k^T & d_k^T \end{bmatrix} R_2 \begin{bmatrix} e_k \\ d_k \end{bmatrix} \quad (28)$$

where $R_2=\begin{bmatrix} C_i^T G_i^T G_i C_i & C_i^T G_i^T G_i F_{di} \\ * & F_{di}^T G_i^T G_i F_{di}-\gamma_i^2 I \end{bmatrix}$
Now rewriting R_2 in the form

$$R_2 = \begin{bmatrix} 0 & I \\ G_i C_i & G_i F_{di} \end{bmatrix}^T (S) \begin{bmatrix} 0 & I \\ G_i C_i & G_i F_{di} \end{bmatrix} \quad (29)$$

and $S=\begin{bmatrix} -\gamma_i^2 I & 0 \\ 0 & I \end{bmatrix}$.
Now we can write (25) as

$$r_d^T r_d+\triangle V_i(e_k)-\gamma_i^2 d^T d+\zeta V_i(e_k)<0$$

or we can say that

$$\begin{bmatrix} I & 0 \\ \bar{A}_i & \bar{E}_{di} \end{bmatrix}^T (\phi_d \otimes P_i) \begin{bmatrix} I & 0 \\ \bar{A}_i & \bar{E}_{di} \end{bmatrix} + \begin{bmatrix} 0 & I \\ G_i C_i & G_i F_{di} \end{bmatrix}^T (S) \begin{bmatrix} 0 & I \\ G_i C_i & G_i F_{di} \end{bmatrix} < 0 \quad (30)$$

It is clearly of the form

$$N_U^T Z N_U<0$$

where $N_U=\begin{bmatrix} I & 0 \\ \bar{A}_i & \bar{E}_{di} \\ 0 & I \end{bmatrix}$, $Z=\begin{bmatrix} z_{11} & 0 & z_{13} \\ * & z_{22} & 0 \\ * & * & z_{33} \end{bmatrix}$

$z_{11}=-(1-\zeta)P_i+C_i^T G_i^T G_i C_i$
$z_{13}=C_i^T G_i^T G_i F_{di}$
$z_{22}=P_i$
$z_{33}=F_{di}^T G_i^T G_i F_{di}-\gamma_i^2 I$

Then U is obtained as

$$U=\begin{bmatrix} \bar{A}_i & -I & \bar{E}_{di} \end{bmatrix}$$

Now the MI (30) can be written in the form

$$Z+U^T X+X^T U<0 \quad (31)$$

where $X=\begin{bmatrix} X_{1i} & X_{2i} & X_{3i} \end{bmatrix}$

Now by choosing $X_{1i}=X_i$, $X_{2i}=\lambda X_i$ and $X_{3i}=0$ we get

$$Z + \begin{bmatrix} \bar{A}_i^T \\ -I \\ \bar{E}_{di}^T \end{bmatrix} \begin{bmatrix} X_i & \lambda X_i & 0 \end{bmatrix} + \begin{bmatrix} X_i^T \\ \lambda X_i^T \\ 0 \end{bmatrix} \begin{bmatrix} \bar{A}_i & -I & \bar{E}_{di} \end{bmatrix} < 0 \quad (32)$$

Now the MI (30) is equivalent to MI (32). (32) after simplification is given by

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} & \bar{\Phi}_{13} \\ * & \bar{\Phi}_{22} & \bar{\Phi}_{23} \\ * & * & \bar{\Phi}_{33} \end{bmatrix} < 0 \quad (33)$$

$\bar{\Phi}_{11}=-(1-\zeta)P_i+C_i^T G_i^T G_i C_i+He(\bar{A}_i^T X_i)$
$\bar{\Phi}_{12}=\lambda \bar{A}_i^T X_i-X_i^T$
$\bar{\Phi}_{13}=C_i^T G_i^T G_i F_{di}+X_i^T \bar{E}_{di}$
$\bar{\Phi}_{22}=P_i-\lambda He(X_i)$
$\bar{\Phi}_{23}=\lambda X_i^T \bar{E}_{di}$
$\bar{\Phi}_{33}=F_{di}^T G_i^T G_i F_{di}-\gamma_i^2 I$

Then by choosing $Y_i=L_i^T X_i$ and $M_i=G_i^T G_i$ in MI (33) we can get the LMI (21). The objective of maximizing the effect of faults on the residual is obtained as

$$-r_f^T r_f+\triangle V_i(e_k)<-\beta_i^2 f^T f-\zeta V_i(e_k) \quad (34)$$

By following the similar procedure as before, the effect of faults can be maximized on the residual signal. The resulting matrix inequality that is obtained is as follows

$$\begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & \bar{\Theta}_{13} \\ * & \bar{\Theta}_{22} & \bar{\Theta}_{23} \\ * & * & \bar{\Theta}_{33} \end{bmatrix} < 0 \quad (35)$$

$\bar{\Theta}_{11}=-(1-\zeta)Q_i-C_i^T G_i^T G_i C_i+He(\bar{A}_i^T X_i)$
$\bar{\Theta}_{12}=\lambda \bar{A}_i^T X_i-X_i^T$
$\bar{\Theta}_{13}=-C_i^T G_i^T G_i F_{fi}+X_i^T \bar{E}_{fi}$
$\bar{\Theta}_{22}=Q_i-\lambda He(X_i)$
$\bar{\Theta}_{23}=\lambda X_i^T \bar{E}_{fi}$
$\bar{\Theta}_{33}=-F_{fi}^T G_i^T G_i F_{fi}+\beta_i^2 I$

Then by choosing $Y_i=L_i^T X_i$ and $M_i=G_i^T G_i$ in MI (35) we can get the LMI (23).
For arbitrary switching under average dwell time it is also required that following LMIs should be satisfied

$$P_i-\mu P_j<0$$

$$Q_i-\mu Q_j<0$$

This completes the proof.

V. Residual Evaluation

In order to detect a fault first a residual evaluation function needs to be defined which is then compared with some threshold value to detect the presence of fault in the system. There can be different types of evaluation functions that can be used. Different types of evaluation functions can be found in [19]. The residual evaluation function used here is J_{RMS} which is given by

$$J_{RMS} = \|r(k)\|_{RMS} = (\frac{1}{N}\sum_{j=1}^{N}\|r(j+k)\|^2)^{\frac{1}{2}}$$

The next step is to define a certain threshold value for fault detection. The threshold can be either fixed or it can be adaptive [15], [22], [19], [23]. Fixed threshold will be used in this paper which will be norm based. Now if

- $J_{RMS} > J_{th,RMS} \Rightarrow$ Alarm, Fault is detected
- $J_{RMS} < J_{th,RMS} \Rightarrow$ No Alarm, No Fault

Where $J_{th,RMS} = \sup_{f=0,\|d\|\leq\delta_{d,2}} J_{RMS}$
and $\delta_{d,2}$ is the peak norm of the disturbance.

For norm based threshold computation it is assumed that the norm of disturbance is bounded by some value say for example $\|d\| < \delta_{2,d}$ then the threshold for RMS energy based evaluation function can be obtained as

$$J_{th,RMS,2} = \frac{\gamma_i}{\sqrt{N}}(\delta_{d,2})$$

VI. Simulation Results

Consider the following discrete switched system consisting of three modes of operation.

$$A_1 = \begin{bmatrix} -1 & 1 \\ -.02 & -.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} .1 \\ 1 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & .4 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$
$$Ed_1 = \begin{bmatrix} -.02 & .1 \\ .1 & -.2 \end{bmatrix}, \quad Ef_1 = \begin{bmatrix} 1 & -.3 \\ 2.3 & 1 \end{bmatrix}$$
$$Fd_1 = \begin{bmatrix} .05 & 0.015 \\ 0 & -.193 \\ -.012 & 0 \end{bmatrix}, \quad Ff_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -.5 & .1 \\ -1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} .2 \\ .7 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1.7 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ -.7 \\ 1 \end{bmatrix}$$
$$Ed_2 = \begin{bmatrix} -.1 & 0 \\ 0 & -.43 \end{bmatrix}, \quad Ef_2 = \begin{bmatrix} .313 & 0 \\ 0 & .78 \end{bmatrix}$$
$$Fd_2 = \begin{bmatrix} .041 & -.015 \\ .09 & -0.5 \\ 0 & .06 \end{bmatrix}, \quad Ff_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -.3 & .2 \\ -1 & -.01 \end{bmatrix}, \quad B_3 = \begin{bmatrix} .3 \\ .891 \end{bmatrix}$$
$$C_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ .413 & .365 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 1 \\ -.8 \\ 1 \end{bmatrix}$$
$$Ed_3 = \begin{bmatrix} -.2 & 0 \\ 0 & -.1 \end{bmatrix}, \quad Ef_3 = \begin{bmatrix} .271 & -.789 \\ .4 & 1 \end{bmatrix}$$
$$Fd_3 = \begin{bmatrix} .02 & 0.2 \\ 0 & -.1 \\ -0.01 & .06 \end{bmatrix}, \quad Ff_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Now considering the two cases :

1) First the worst case performance index $\gamma_{max} \geq \gamma_i$ is considered for each mode. With $\mu = 1.5$, $\zeta = .5$, $\beta_{max} = 1.414$, γ_{max1} obtained using Theorem (1) is 0.9202 and γ_{max2} obtained using Theorem (2) with $\lambda = 7$ is 0.8530. It is clearly seen that $\gamma_{max2} < \gamma_{max1}$.
2) Individual mode performance index is considered that is, γ_i for all $i \in \{1,2,3\}$. Now using Theorem (1) with $\mu = 1.5$, $\zeta = .5$, $\beta_{max} = 1.414$ we get $\gamma_{11} = 0.6570, \gamma_{21} = 0.7975, \gamma_{31} = 1.0235$ and by using Theorem (2) with $\lambda = 7$ we get $\gamma_{12} = 0.5876, \gamma_{22} = 0.8008, \gamma_{32} = 0.9617$. Clearly the results obtained from Theorem (2) are better than with Theorem (1).

Fig. 1 shows the residual evaluation functions and the corresponding threshold for each mode with Fault detection filter designed using Theorem (2). Local performance index γ_i for all $i \in \{1,2,3\}$ has been used. Mode1 is active from k=0 to 1000, Mode2 is active from k=1001 to 2000 and Mode3 is active from k=2001 to 3000. A step fault was introduced in the first output in Mode1 at k=500 and in the second output in Mode3 at k=2500.

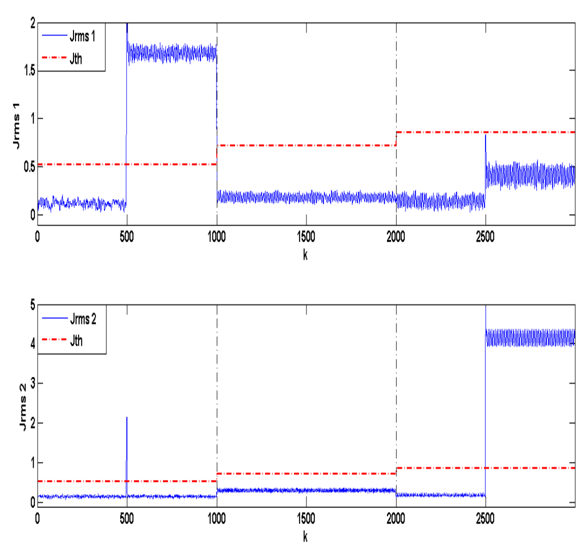

Fig. 1. Residual Evaluation Functions for first two outputs

VII. Conclusion

RFDF has been designed for linear switched systems with the use of projection lemma to decouple the system matrices from Lyapunov matrices and hence reduce the conservativeness in the design given in [15]. The multiobjective problem can then be solved by a separate Lyapunov function for each objective as the system matrices are no longer coupled with the Lyapunov matrices. By the application of projection lemma, extra variables were introduced that also help in reducing the conservativeness. The proposed method can be extended to the case of uncertain discrete switched systems.

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