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Use of Radiative Transfer Equation (RTE) for estimating optical signal attenuation through inhomogeneous Clouds

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Abstract— **High data rate requirement has necessitated the use of optical wireless links from satellite-to-ground that invariably pass through the inhomogeneous cloud layers. It has thus become necessary to accurately model the scattering channel and estimate attenuation suffered by the information bearing optical signal traversing the multiple scattering inhomogeneous clouds. This paper introduces the utilization of three dimensional radiative transfer equation RTE for prediction of attenuation in the optical signal traversing the cloud layers. The three dimensional radiative transfer equation explicitly handles the spatial complexity and multiple scattering effects of inhomogeneous clouds and provides accurate propagation channel estimation without invoking mathematical assumptions.**

Cloud droplets mainly composed of liquid water attenuates the beam of light through absorption and scattering in forward direction. Therefore, it is necessary to generate three dimensional cloud fields that exhibit realistic spatial distribution of the cloud structure. This statistically generated cloud field from experimental data is used as an input to three dimensional radiative transfer model to calculate transmitted irradiance at the bottom of cloud to predict attenuation in cumulus cloud scene. This paper employs the numerical solution of three dimensional radiative transfer equation using Monte Carlo simulation that uses stochastic methods to simulate physical processes of scattering and absorption within inhomogeneous cloud layers. The open source I3RC Monte Carlo code has been chosen to perform simulations at commonly employed optical wavelengths of 850 nm and 1550 nm. The 10 µm has been specifically analyzed to gain insight of wavelength dependence in optical propagation through clouds.

Keywords *– Radiative Transfer Equation (RTE), multiple scattering, Monte Carlo simulations, Large Eddy simulations (LES), optical Propagation, inhomogeneous clouds.*

I. INTRODUCTION

The optical frequencies lying in the THz band promises very high data rates and have found applications in wireless propagation for terrestrial applications as well as inter-satellite and satellite-to-ground links. Use of optical frequencies results in very high gain antennas, more compact equipment, higher immunity to interference, enormous channel capacities and no licensing requirement. The main disadvantage of an optical wireless link is that a system's performance depends strongly on weather conditions. Clouds scatter and absorb optical signal thus reducing the available power at the receiver; these effects cause errors in transmitted data. To mitigate the transmission errors, system designers require accurate modeling of the propagation channel [1].

Energy can be added or removed for the laser beam propagation from satellite-to-ground due to its interaction with cloud droplets. The intensity of radiation beam will decrease because of absorption and scattering. Scattering and absorption of optical beam in cloud is strongly wavelength-dependent [2].

The studies of laser beam propagation through multiple scattering clouds are of importance for number of applied scientific and engineering problems including remote sensing, climatology target identification, imaging and optical communication problems [3]. This study is the subset of more general study of characterizing optical wave propagation through random media. The attenuation of beam depends on incident power of laser, its wavelength and the physical properties of random media. Random medium is one whose basic properties are random function of space and time. For the wavelength range of most interest the main atmospheric absorbers are water vapors, carbon dioxide and ozone.

Light propagation through cloud is basically a multiple scattering phenomena. Mie scattering theory, essential for characterizing single scattering event, is not capable of describing multiple scattering in a particular medium like the atmospheric channel containing clouds. Photons interaction with the atmospheric constituents result in beam spread and this limits the transmission bandwidth available for communication. For accurate prediction of scattering channel performance, accounting for multiple scattering enables accurate prediction of attenuation [4].

Clouds have been the subject of studies for centuries. There have been many attempts in the past to understand the effects that clouds impose on the communication channel between satellite and ground using laser-beam propagation. Clouds in the atmosphere are not infinite in extent and highly variable in time and space. Cloud attenuation estimation is difficult because of the diversity of the kind of cloud types and inhomogeneity of cloud microphysical characteristics [5].

This paper proposes the use of three dimensional radiative transfer equation for accurate estimation of optical attenuation through realistic inhomogeneous clouds in the satellite-to-ground links. Section 2 discusses the applicability of RTE to optical propagation scenario and

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summarizes the efforts made todate in this direction. Section 3 talks of the three dimensional RTE approach and the generation of realistic inhomogeneous cloud fields. Section 4 links the optical and microphysical properties of cloud to the solution of the 3-D RTE. Section 5 summarizes the Monte Carlo simulations performed and the results obtained thereof. The last section concludes the paper by emphasizing the fact that 3D RTE provide an accurate mathematical description for optical propagation through the atmosphere and that further research and experimentation is necessary to clearly establish the wavelength dependency of optical attenuation through the atmosphere.

II. RTE FOR OPTICAL PROPAGATION THROUGH CLOUDS

In modeling optical communication channel, the propagation channel is characterized by using two major theories: analytical theory and transport theory [6]. The analytical theory uses Maxwell's equations for the mathematical description of optical wave propagation while the radiative transfer equation based on the energy conservation does not require the full wave equation solution to describe the modulated optical intensity transmitted through inhomogeneous clouds [7].

For attenuation estimation in inhomogeneous clouds, intensity of light is the main consideration and the numerical solution of radiative transfer equation provides the accurate evaluation of optical propagation channel. The attenuation in inhomogeneous cloud depends on the optical and geometrical depth of cloud. Cloud optical depth is the most important parameter that depends on cloud cover and introduces randomness in the channel.

Application of the radiative transfer equation for modeling optical propagation through random media is finding firm grounds. Various researchers have employed radiative transfer equation to quantify the scattering and absorption effects of random media. Important research studies includes channel modeling in underwater wireless optical communication [8] , optical diffusion in tissues [9] and one dimensional radiative transfer equation for estimating attenuation in clouds [10]. Also in a recent contribution it has been shown that radiative transfer theory provides baseline for modeling multiple scattering atmospheric channel effects on propagation of optical beam and the results are in close agreement with the field data channel estimation [11].

III. THREE DIMENSIONAL RADIATIVE TRANSFER APPROACH

For detailed modeling of scattering media to perform radiation calculations at various wavelengths to estimate attenuation and influence of weather on optical wireless communication, it is necessary to choose cloud models that can provide accurate spatial details of cloud variability in time and space. The model shall account for the three-dimensional shape of clouds and their inhomogeneity in horizontal and vertical directions.

"The cloud horizontal inhomogeneity that can effect attenuation may be divided into two parts (1) optical depth variability in vertical direction, and (2) the transfer of optical beam energy in horizontal direction" [12].

Differences between 3D and 1D result are important in characterizing optical channel containing clouds [13]. The solution of three dimensional radiative transfer equation is obtained by the three dimensional structure of inhomogeneous cloud characterized by spatially varying optical properties such as volume extinction coefficient, single scattering albedo and phase function. This three dimensional cloud structure varies both vertically and horizontally to model the inhomogeneous characteristics of clouds.

The cloud microphysics such as liquid water content and effective radius is obtained from physical modeling using Large Eddy Simulations LES of cumulus cloud. Mie scattering theory is applied to convert these physical properties into optical properties. This three dimensional domain characterized by single scattering optical properties is used by the forward Monte Carlo radiative transfer code to get the irradiance at the bottom of the cloud field. The attenuation is estimated at various wavelengths using the incident source irradiance (known) at the top and the transmitted irradiance through the inhomogeneous cloud layers. This procedure is outlined in Figure 1.

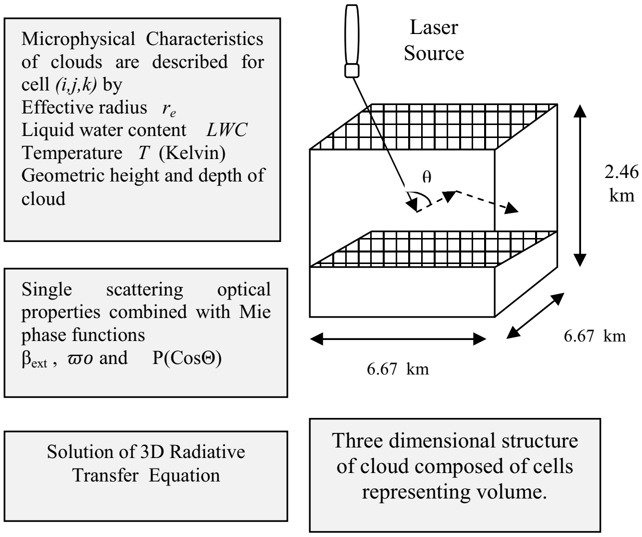


Figure 1. Three steps to obtain irradiance for estimating attenuation in inhomogeneous cloud field.

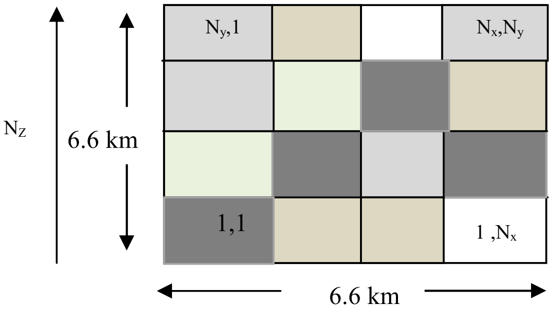


Figure 2. Three dimensional cloud scene.

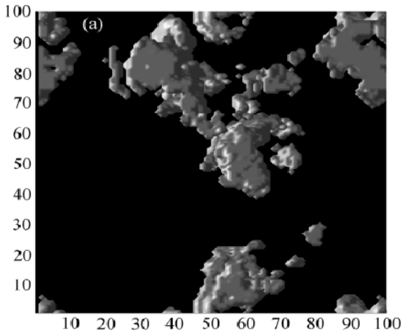


Figure 3. Geometry of Cu Field [17]

A. The 3D Cloud Model

Cloud structure varies in all three-dimensions and its inhomogeneity causes problems in predicting attenuation to model atmospheric channel [14]. It is challenging to specify three-dimensional realistic structure of a cloud to be used as an input to the radiative transfer calculations. To predict attenuation in 3-D cloud fields, its realistic physical characteristics are required as an input to perform radiative transfer calculations. It can be obtained from either physical or stochastic cloud generation model. Inhomogeneous cloud field is divided into 3D grid of cloud cells (grid size depends on resolution stored in cloud input file). Each grid box is treated as homogeneous relative to its optical properties.

The new approach adopted here is to generate liquid water content (**LWC**) and effective radius (**r$_e$**) in three dimensions derived from in situ aircraft measurements during field programs using Large Eddy simulations (LES) [15][16]. LES model is generally physically consistent, accounts inhomogeneity and provide accurate spatial resolution but is computationally expensive.

B. Three dimensional Radiative Transfer Equation

Radiative transfer equation calculates the distribution of radiation (as a function of direction and space) from the optical properties and the boundary.

*"**The radiative transfer equation specifies the change in radiance and/or irradiance along a beam of electromagnetic energy at a point in the atmosphere"[18]**.*

The intensity of laser beam attenuated in inhomogeneous cloudy medium satisfies the radiative transfer equation. RTE is a well balanced equation describing the transfer of radiative energy in the atmospheric channel that includes energy loss due to absorption and scattering and gain due to multiple forward scatter. Therefore this equation has found much applicability in various problems of engineering and atmospheric science [19].

$$-\frac{dI(s,\Omega)}{\beta_{ext}(s)ds} = I(s,\Omega) - J(s,\Omega) \qquad (1)$$

Where I is the intensity (or radiance), β_{ext} is the extinction coefficient for cloud particles, which is a function of spatial grid s (x, y, z) and the spatial dimensions $\Omega(\mu, \phi)$.

C. Numerical Solution of Three dimensional RTE

The exact analytical solution for the problem of radiation transfer in an inhomogeneous cloud medium is not possible. Numerical methods that are capable of solving three dimensional radiative transfer problems are divided into two groups. (1) Deterministic methods (2) Statistical methods [20].

Spherical harmonics discrete ordinate method SHDOM is the only explicit or deterministic model that can account for three dimensional radiation transport in the atmosphere. This method is accurate and widely used in atmospheric research, but it is very difficult to modify and is not a suitable tool for research in optical communication. The second method is Monte Carlo simulations applicable to three dimensional radiative transfer problem and provides estimation of radiation field by using stochastic modelling.

The Monte Carlo method is employed in solving various mathematical, sciences and engineering problems by simulating uniform random variables. Monte Carlo is a numerical technique and can be applied to any physical process using a stochastic model as shown in Figure 4.

Monte Carlo Method solves radiative transfer problems for 3-D geometries as easily as one dimensional because all that is needed to solve any scattering problem is to describe the photons initial position and direction, optical properties of scattering medium and in which direction light scatters after scattering as well as the photons exit.

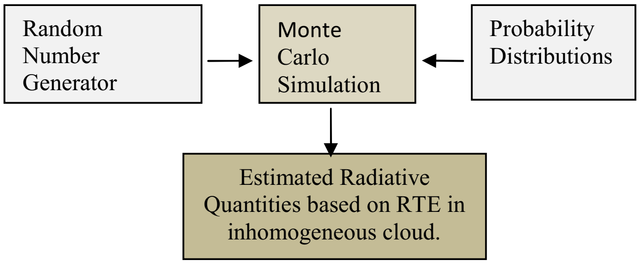


Figure 4. Monte Carlo simulation to estimate radiance in inhomogeneous cloud

IV. OPTICAL AND MICROPHYSIACAL PROPERTIES OF CLOUDS

Cloud droplets are spheres of liquid water with average radius of droplets around 10 μm. Real clouds in nature occur with droplets of variable size and kind over time and space. To obtain cloud optical properties from physical properties, the spherical droplets of cloud are characterized by gamma distribution or lognormal distribution with specific variance [21].

$$n(r)=\frac{N}{\beta\sqrt{2\pi r}}\exp[-(\log r-\log r_m)^2/2\beta^2] \qquad (2)$$

N is the droplet concentration (number of droplets per unit volume), β is the logarithmic width of the distribution and r_m is the drop radius at the maximum of the distribution. The number of droplets per unit volume with radii between r and $r+dr$ is denoted by $n(r)dr$.

Mie theory is applicable when laser beam propagating through clouds and fog interacts with spherical particulate with dimensions comparable to the operating wavelength of the laser beam. These optical properties are determined by first applying Mie scattering theory to calculate the single scattering properties of individual droplets as functions of droplet radius r and then by knowing drop size distribution $n(r)$, the optical properties of grids are calculated by integrating over $n(r)$ as shown in Figure 5 [22].

For spherical particles, the spectral optical parameters are computed using Mie-theory on the basis of given particle size distributions and spectral complex refractive indices.

A. *Parameterization of the optical properties of clouds*

The scattering and extinction by small volume within a cloud is determined by obtaining the optical properties such as volume extinction coefficient k, the single scattering albedo ϖo, and scattering direction described by scattering phase function $P(\cos\Theta)$ [23].

$$k=\beta_{ext}=\int_0^\infty \sigma_{ext}(r)\,n(r)\,dr$$

$$=\int_0^\infty \pi r^2 Q_{ext}(r)\,n(r)\,dr \qquad (3)$$

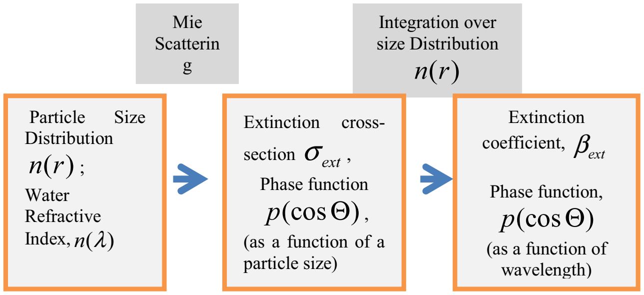


Figure 5. Strategy to compute optics of cloud water particles.

The single scattering albedo ϖo determines the absorption in the cloud. There is more absorption for smaller $\varpi 0$ and less radiation is transmitted, thus irradiance decreases. Single scattering albedo of $\varpi o = 1$ means non-absorbing cloud and $\varpi o = 0$ means negligible scattering [23].

$$\varpi_0=\frac{1}{k}\int_0^\infty \sigma_{sca}(r)\,n(r)\,dr \qquad (4)$$

The scattering phase function characterizes the angular distribution of the scattered radiation field. The combined scattering phase function is the scattering cross section weighted average of the individual phase functions [23]

$$P(\cos\Theta)=\frac{1}{\varpi_0 k}\int_0^\infty \sigma_{sca}(r)P(\cos\Theta;r)\;n(r)dr \qquad (5)$$

Where Θ is the angle of scattering relative to the incident radiation.For spherical droplets, this function exhibits a strong peak in the forward direction. For most applications the phase function can be easily approximated based on asymmetry parameter given by Henyey and Greenstein function [23].

$$HG(\cos\Theta)=P(\cos\Theta)=\frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}} \qquad (6)$$

The parameter g is called the asymmetry factor, which describes energy scattered in the various directions. For symmetric scattering, which puts equal amounts of energy in the forward and backward direction, g is zero. As more energy is scattered in the forward direction as in clouds, g increases toward unity [24].

B. *Connection of microphysical parameters and optical properties of clouds*

Measurements showed a strong dependence of droplet effective radius as a function of height in the cloud. There exists a correlation in optical $\tau(x,y)$ and geometrical depths $h(x,y)$ and both contribute in the change of radiation field in the atmosphere [25].

All the physical properties are the moments of droplet size distributions. Effective radius r_e is defined as the ratio of the third to second moments of the droplet size distribution. Optical thickness is related to liquid water content and effective radius via [25]

$$\tau(x,y)=\frac{3}{2\rho}\int_0^{h(x,y)}\frac{LWC(z;x,y)}{r_e(z;x,y)}dz \qquad (7)$$

The extinction coefficient of the cloud may be approximated by [23].

$$\beta_{ext}\approx\frac{3}{2}\frac{LWC}{r_e} \qquad (8)$$

If the scatterers are spheres such as cloud particles, phase function can be calculated from Mie theory. When Mie phase functions are used then the effective radius of the particles and their refractive indices are required parameters.

V. SIMULATION RESULTS

To evaluate the technique presented in previous sections for the estimation of attenuation in inhomogeneous 3D cloud fields. The I3RC simulator has been used to solve the three dimensional radiative transfer equation in inhomogeneous cloud field. The I3RC code framework is provided to support a wide range of radiative transfer problems. For attenuation estimation there is need to set up the radiative transfer problem by giving the 3D cloud field taken from LES according to the procedure outlined in Figure 6.

Multiple scattering redistributes the photons based on the optical properties of clouds field until they escape or get absorbed. The model not only counts but also derive estimates of various radiative quantities using photon counting. The irradiance is defined as the flux of energy incident from one hemisphere which is transmitted through surface of area (W m^{-2} μm^{-2}). It is simulated by counting the number of incident photons per unit area [29].

The prototype cloud field is used from I3RC Phase II: cumulus field (cu) with physical characteristics shown in Figure 7. Simulations are performed at a non absorbing wavelength of 850 nm, at near-infrared wavelength of 1550 nm and at an absorbing wavelength of 10 µm.

To set up the optical properties of the atmosphere, the 3D cloud field with three-dimensionally varying physical properties is chosen in the form of a 50× 50 × 18 cell 3D field, extending over 6.67 × 6.67 km^2 area (horizontally) with specified grid spacing.

The vertical gridding (layering) was equally spaced in the z range [1km, 2.46 km] of the cloud field. The top of the atmosphere corresponds to 2.46 km altitude as there is nothing above this level in the cumulus cloud field used.

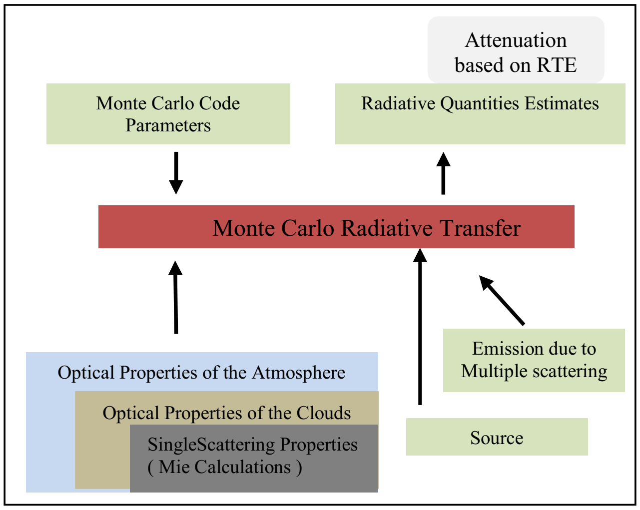


Figure 6. I3RC various software modules for the description of radiative transfer in three-dimensionally varying Atmosphere containing clouds [26].

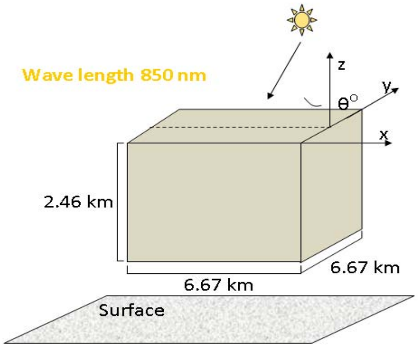


Figure 7: Geometry of Problem

TABLE I. SUMMARY OF CLOUD CASE STUDIED

Cumulus Cloud Scene	
Number of Vertical grids	18
Number of Horizontal grids	50
Vertical grid spacing	40m
Horizontal grid spacing	134m
Horizontal Domain	6.67Km
Cloud type	Cumulus
Cloud droplet number	$100cm^{-3}$
Average cloud optical depth	10
Average cloud effective radius	10µm

For the calculation of attenuation in dB, the decrease in source irradiance is considered at the bottom of the clouds. It can be formulated in Equation 9 [10].

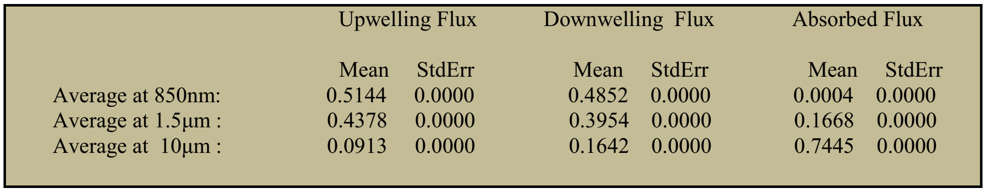
$$\text{Attenuation (dB)}= 10\log_{10}\left(\frac{\text{irradiance at the top of atmosphere}}{\text{irradiance at the bottom of clouds}}\right) \tag{9}$$

The upwelling and downwelling irradiance is used to compute attenuation. For the calculation of attenuation in dB, the decrease in source irradiance is considered from the top of the atmosphere to the bottom of the clouds as shown in Figure 7.

The estimated attenuation at 850nm, 1550 nm and 10µm is 13.14, 14.02 and 17.84 dB.

The penetration of optical beam through clouds depends on the small angle scattering approximations, as more and more scattering events occur in forward direction and there is a chance that optical beam remains in the field of view of the receiver.

Mid-infrared wavelengths, such as 10 µm are highly absorbed in clouds, and so there is little scattering as the single scattering albedo decreases because of thermal emission in clouds at this wavelength, thus no communication signal will penetrate to be above the large amount of thermal emission in the mid-infrared. Wavelengths less than about 900 nm are not absorbed by clouds and through much multiple scattering, a significant fraction of the light can penetrate.



	Upwelling Flux		Downwelling Flux		Absorbed Flux	
	Mean	StdErr	Mean	StdErr	Mean	StdErr
Average at 850nm:	0.5144	0.0000	0.4852	0.0000	0.0004	0.0000
Average at 1.5µm :	0.4378	0.0000	0.3954	0.0000	0.1668	0.0000
Average at 10µm :	0.0913	0.0000	0.1642	0.0000	0.7445	0.0000

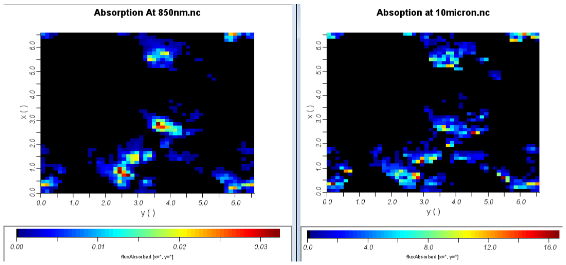


Figure 8. Absorption profile at (a) 850nm and (b) 10µm

VI. CONCLUSIONS

The application of radiative transfer equation for optical propagation through atmosphere is finding firm grounds [11]. The computational cost of considering three dimensional cloud field is higher than one dimensional approximation but leads to accurate modeling.

The results obtain suggest that 850 nm is actually a preferred wavelength for satellite-to-ground links and moving to mid infrared region of wavelength around 10 µm does not provide any significant advantage in terms of optical signal penetration through the clouds.

The wavelength of 850 nm is just outside the visible range of light and cloud drops have very high single scattering albedo so absorption is quite small. Therefore, it has a strong forward peak caused by multiple scattering in the cloud and the small scattering angle approximations work quite well. The probability that the light remains in receiver field of view (FOV) remains high. Whereas at 10 µm the single-scattering albedo of drops reduces because of substantial emission by cloud particles and beam gets attenuated due to absorption.

Having larger droplets for given liquid water content implies lower optical thickness which reduces multiple scattering. Further experimental results for satellite-to-ground optical links at various wavelengths are necessary to develop a comprehensive understanding of optical propagation through clouds.

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