Potential Game Convergence of Cognitive Radio Ad hoc Networks

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***Abstract*—Development of a system in which several unlicensed users strive to access the vacant licensed bands is a highly challenging task, especially in ad hoc networks where no centralized infrastructure exists. These unlicensed users, also called cognitive radios, must compete for the available frequency slots. The convergence of this system is difficult to achieve if nodes perform no information exchange and solely depend on their individual actions. In this paper, we analyze the cognitive radio networks for distributed channel allocation. It is observed that convergence is easier to establish in cooperative systems or systems where users perform information exchange. This kind of cooperative behavior can be enforced by providing incentives to nodes. The cooperation leads to an improvement in overall network performance as compared to the selfish users who are concerned only with their individual benefits.**

***Keywords*-cognitive radio; potential game; convergence;**

I. Introduction

Cognitive radio (CR) is a dynamic spectrum access technique which provides an appropriate way to access the licensed spectral bands by the unlicensed users. The wireless channel available for communication has limited bandwidth. All the available bandwidth resources are allocated to the licensed users and incorporating more users is becoming an increasingly challenging task. FCC has proposed some solutions to combat this issue, which include, spectrum sharing, spectrum leasing and spectrum reallocation [1]. While spectrum reallocation provides a long term solution for the spectrum resource problem, it requires a change in the current infrastructure. Spectrum leasing is another possible solution and is currently considered to be a static solution. The solution involving spectrum sharing has invoked a considerable amount of research [2]. These include proposing solutions from vast areas of study. Some of them consider a central authority or a control channel to monitor and manage the sharing [3-5]. This is easier to implement in case of centralized networks, but poses other challenges in ad hoc networks.

Ad hoc networks lack the controlling authority or a proper infrastructure. The nodes in ad hoc networks work independently to communicate and create a self governing network. These networks are more useful with less setup requirements, but they can be relatively complex to design [15-17]. Integrating cognitive radios in such networks is even more cumbersome as no centralized body is there to assist in the process. Cognitive ad hoc networks require an inbuilt mechanism to implement the spectrum access by unlicensed users. Two different techniques can be adopted to accomplish this: Spectrum Overlay, and Spectrum Underlay [3]. In spectrum overlay, the secondary users search for vacant channels or wait for the licensed users to complete their transmissions and vacate the channels. As soon as they find vacant channels, the unlicensed users begin their own transmissions. These transmissions must be terminated the moment licensed users (also called primary user or PU) reclaim the channel. The unlicensed users (secondary user or SU) may continue their transmissions over a different channel, or wait for the channel to be vacant again [4].

The spectrum underlay scheme, is however, different from the above approach. It involves simultaneous use of channel by the PUs and SUs and the signal of SU appears as noise to the licensed user. While both transmissions are performed concurrently, the unlicensed users must take into account the interference they create to the licensed counterpart, and must not exceed a certain limit. This approach appears more dynamic and it has higher complexity. Apparently, the amalgamation of these schemes in a network yielding a hybrid environment can result in better performance by combining the benefits of both techniques.

Game theory provides mathematical and analytical tools to formalize the best strategy under given conditions to optimize the outcome for the rational entities [5]. For the past several years, game theory is being applied to problems in communication and networking, especially in areas related to resource allocation, e.g., congestion control, topology control, trust management, routing and power control [8-14]. The inherent problem of cognitive networks is coexistence with the licensed users that own the spectrum. As stated earlier, due to the limited bandwidth resources, the unlicensed users compete for the available channels, and monitor the licensed users' activities. This scenario is modeled by game theoretic approach, where licensed and unlicensed users act as players [6]. These players can behave selfishly and choose their strategies for maximizing their individual performances, in non-cooperative games. If playing cooperatively, these players strive to optimize the overall network performance besides their individual benefits.

Recently, game theory is extensively used to model the

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collaboration among nodes in cognitive radio networks. In a two players game, there are four types of possible behaviors directly influencing the welfare of the players: selfishness, altruism, spite, and cooperation. Selfishness is harming someone else in order to benefit ones own self; altruism is harming oneself in order to provide profit to someone else; spite is harming oneself in order to harm someone else; cooperation is working for mutual benefit of players and their rivals. Here, we consider selfish and cooperative behaviors. These two branches of game theory formalized interdependence of players in different ways. In a selfish or non-cooperative approach, all available moves are given in a detailed model and no collaboration exists among players. On the contrary, the cooperative theory describes the outcomes that result when the players perform in different possible coalitions [7].

The remaining paper is arranged as follows: Section II explains the system model, Section III illustrates the formulations of the potential game for the given system. Sections IV and V discuss the simulations results and conclusion, respectively.

II. System Model

We consider N Cognitive radio users, and K channels or frequency bands, where $K < N$. Each frequency band has an individual capacity (all may be equal or not). The SUs may decide to stay or leave the channel based on available performance. When users change their choices, their payoff may vary. The network comprises of N transmitting/receiving cognitive nodes, which are uniformly distributed in a two-dimensional square region. We assume that the network topology remains fixed at least for the time required to establish strategies. The nodes sense the available spectrum bands and make decisions regarding the transmission channel. By distributively selecting transmission channel, CR can reduce co-channel interference. The Signal-to-Interference Ratio (SIR), γ_{ij}, from transmitter i at the jth receiver, over a certain channel can be expressed as:

$$\gamma_{ij} = \frac{p_i G_{ij}}{\sum\limits_{k=1, k\neq j}^{N} p_k G_{kj}} \tag{1}$$

where, p_i is the power transmitted by node i, G_{ij} is the link gain between ith transmitter and jth receiver. It is important to note that only the nodes choosing the same strategy (same channel) contribute to the interference.

We can model our problem of channel allocation in game theory such that G= $\{N, \{S_i\}, i \in K, \{U_i\}, i \in N\}$, where N is the finite number of players (cognitive radios). S_i is the set of strategy space $S = S_i$, $i \in K$, and actions are the channel selection. Preferences are the quality of channels based on SIR and utility function. For ith player in a game G, the utility function, depends on the strategy of player S_i, and that of its opponents: S_{-i}. A set of strategies form a Nash Equilibrium (NE) such that no player can benefit by deviating from its strategy.

The cognitive users can play the game in two different ways: simultaneous moves game, where all users choose their strategies and actions simultaneously, and sequential moves games, where users take turns in choosing their strategies. The simultaneous moves game rely only on the current information about the opponents and choose the next strategy based on this information. On the other hand, the sequential moves game chooses strategies based on the moves taken by the preceding players. This allows users to have knowledge of their opponents' strategies and decisions are made accordingly.

In game theory, a potential game is defined using a single global function called potential function. This potential function expresses the incentives of all players to alter their strategies. This is very convenient for analyzing the equilibrium properties of games, as it incorporates the incentives of all players into one function, and all possible Nash equilibriums can be found by optimizing the potential function.

Table I presents the cooperative game model for the cognitive radios. If channel k has m users and channel l has $N - m$ users, and a game is played among two players competing for a channel, then the interference each player must bear can be expressed as in Table I. If both players choose the same channel, they face the interference by m already existing users plus the interference created by their new entering opponent. On the other hand, if players opt for different strategies, they face the interference by the already existing users (m users in case of channel k and $N - m$ users in case of channel l). They, however, do not create interference for each other by cooperating and choosing different strategies. The players that choose same channel, create interference for other users. The utility function for each player depends on the number of users sharing the same channel. If more users are accessing a channel, larger interference is created over that channel and all users suffer from this high interference level.

The primary user is the sole proprietor of the channel. The SUs must identify vacant channels and transmit data over it. If several SUs strive to access the same channel, they must compete among each other. If they transmit simultaneously, they create interference for each other. If players are selfish, they do not cooperate. This non-cooperative behavior can be modeled as a game where players are concerned only with their own benefits regardless of the payoffs for their opponent. This kind of behavior may prove to be beneficial temporarily, but eventually deteriorates the network performance. If all nodes behave selfishly, they create more interference for others, which causes others to increase their transmit power resulting in higher interference for itself. Although the equilibrium point is reached, the benefit from persistently

Table I: Interference Game Model for Cooperative SUs

Player $-i$ / Player i	Channel k	Channel l
Channel k	$\left(\sum_{j=1,j\neq -i}^{m+2} p_jG_{ji}\ ,\ \sum_{j=1,j\neq i}^{m+2} p_jG_{ji}\right)$	$\left(\sum_{j=1,j\neq -i}^{m} p_jG_{ji}\ ,\ \sum_{j=1,j\neq i}^{N-m} p_jG_{ji}\right)$
Channel l	$\left(\sum_{j=1,j\neq -i}^{N-m} p_jG_{ji}\ ,\ \sum_{j=1,j\neq i}^{m} p_jG_{ji}\right)$	$\left(\sum_{j=1,j\neq -i}^{N-m+2} p_jG_{ji}\ ,\ \sum_{j=1,j\neq i}^{N-m+2} p_jG_{ji}\right)$

selfish behavior is relatively reduced.

On the other hand, if SUs compete for the channel in a way so as to cause minimum possible interference for their competitors, more users can benefit. This behavior formulates a cooperative game and takes into account the benefits of individual users and the performance degradation they can cause for the network. The cooperative game considers individual benefits and the overall network performance can be expressed in terms of a potential function.

We define an interference function β_i for the cooperative scenario, which takes into account the interference observed and created by the ith cognitive node as:

$$\beta_i = \frac{p_iG_i}{\sum_{j=1,j\neq i}^{N} p_jG_{ji} + \sum_{j=1,j\neq i}^{N} p_iG_{ij}} \tag{2}$$

This function includes the interference suffered by the ith SU (first term in the denominator) and the interference created by that node for all other nodes in the network (second term in the denominator).

Another important issue in implementation of cognitive radios is the PU detection. The radios are assumed to be equipped with the sensing mechanism to monitor PU activity. There is always a probability that SU makes an error in correctly detecting the presence of PU. This probability of incorrect detection is given by α and affects the individual utilities and the overall network performance. In this case, the SIR function β_i is modified as:

$$\beta_i = \frac{p_iG_i}{(1-\alpha)(\sum_{j=1,j\neq i}^{N} p_jG_{ji} + \sum_{j=1,j\neq i}^{N} p_iG_{ij}) + \alpha(p_iG_{io} + p_oG_{oi})} \tag{3}$$

where, p_o is the transmitted power of primary user, G_{io} and G_{oi} are the link gains of ith SU from PU and vice versa respectively.

III. Potential Game for Power Control

We formulate the cognitive radio game as a potential game. The potential function defined for this case must take into account the power control requirements. In order to optimize the utility function with respect to power, the second derivative must exist. The power control in this case, is defined by the utility function, given as:

$$U_i = \beta_i - f(c_i) \tag{4}$$

where, $f(c_i)$ is the cost function for the ith SU to access one of the available channels. This problem can be addressed in two ways: the nodes choose the channel that provides the best utility, or nodes attempt to access the lowest cost channel and evaluate their utility for that channel (which may not be the highest but it has the lowest cost). The cost function depends on the channels and interference. The higher the total number of available channels, lower is the cost of each channel. Similarly, if a node that is trying to access the channel, creates more interference over that channel, it has to pay a higher cost. On the other hand, if a channel has higher interference over it, its cost is lower and vice versa. Thus, cost increases as the interference created is increased, and decreases as interference over a channel is increased or the number of available channels is increased. If greater number of channels are available, the SUs have more strategies to choose from, hence the cost is lowered. Thus, we can write the cost function as:

$$f(c_i) = \frac{\sum_{j=1,j\neq i}^{N} p_iG_{ij}}{K\sum_{j=1,j\neq i}^{N} p_jG_{ji}} \tag{5}$$

where, K is the number of available channels. The utility function, thus becomes:

$$U_i = \frac{p_iG_i}{\sum_{j=1,j\neq i}^{N} p_jG_{ji} + \sum_{j=1,j\neq i}^{N} p_iG_{ij}} - \frac{\sum_{j=1,j\neq i}^{N} p_iG_{ij}}{K\sum_{j=1,j\neq i}^{N} p_jG_{ji}} \tag{6}$$

For this case, we can write the potential function as:

$$V = \sum_{i=1}^{N}\left[\frac{p_iG_i}{\sum_{j=1,j\neq i}^{N} p_jG_{ji} + \sum_{j=1,j\neq i}^{N} p_iG_{ij}} - \frac{\sum_{j=1,j\neq i}^{N} p_iG_{ij}}{K\sum_{j=1,j\neq i}^{N} p_jG_{ji}}\right] \tag{7}$$

The objective in this paper is to implement a power control algorithm for cognitive users. The aim is to access channels which offer best utility for optimum power. In order to determine the optimum power level, which maximizes this potential function, we take the first derivative of the above equation of potential function, which can be written as:

$$\frac{\partial V}{\partial p_i} = \frac{G_i \sum_{j=1,j\neq i}^{N} p_j G_{ij}}{\left(\sum_{j=1,j\neq i}^{N} p_i G_{ji} + \sum_{j=1,j\neq i}^{N} p_j G_{ij}\right)^2} - \frac{\sum_{j=1,j\neq i}^{N} G_{ji}}{K \sum_{j=1,j\neq i}^{N} p_j G_{ij}}$$

Equating this derivative to zero, we obtain the resulting optimum power level p_i^* as:

$$p_i^* = \frac{\sum_{j=1,j\neq i}^{N} p_j G_{ij} \left[\sqrt{K G_i \sum_{j=1,j\neq i}^{N} G_{ji}} - \sum_{j=1,j\neq i}^{N} G_{ji}\right]}{\left(\sum_{j=1,j\neq i}^{N} G_{ji}\right)^2} \tag{8}$$

This equation provides the optimum power level p^* required by the CRs to access the available channels.

IV. Simulations and Results

The simulation environment considers a two dimensional network of dimension D=200, in which nodes are uniformly distributed. The total number of nodes considered for these simulations is $N = 36$ and total number of available channels are $K = 4$. We observe the convergence and transmit powers for the proposed potential game. The results are shown in Figures 1-4. Figure1 represents the strategies of cognitive radios based on minimum cost. Here, the secondary users converge to strategies that offer lower price for channel access.

Figure 2 shows the chosen strategies of CRs according to the transmit power. Here, the secondary users converge to actions according to the adopted power control scheme. Figure 3 depicts the transmitted power of each user when minimum cost scheme is adopted, while Figure 4 shows the transmitted power when proposed power control scheme is implemented. We can observe from these two figures that more power is consumed when minimum cost scheme is adopted as compared to the power consumption for the power control game. Hence, power control game improves the power consumption by the CRs.

When choosing channels according to minimum cost, convergence is observed for the number of iterations equal to the number of sequential moves. In this case, most of the CRs end up choosing the same channel due to lower cost offered at higher interference. The transmitted power is lower, therefore, the nodes on a channel cause less interference for each other to minimize the cost. For power control game, nodes choose their best strategy by maximizing their utilities based on the potential function. The convergence occurs relatively early, distributing the network load almost uniformly between all channels. At convergence, the power level at each channel also converges to the same value. This value, however, is different for different channels.

When all nodes make their decisions simultaneously, convergence is difficult to accomplish. Each node strives to access the channel which offers it highest utility, regardless of the actions taken by their opponents. But the actions of opponents affect the utility of every user, as the opponent nodes also choose their best strategy and may end up choosing the same strategy. This deteriorates the performance of both nodes, as they become the source of interference for each other. In order to avoid choosing same strategy, it is rather difficult in simultaneous moves game. There must be some additional information that assists the node in avoiding conflicting strategies. Learning can be adopted to improve

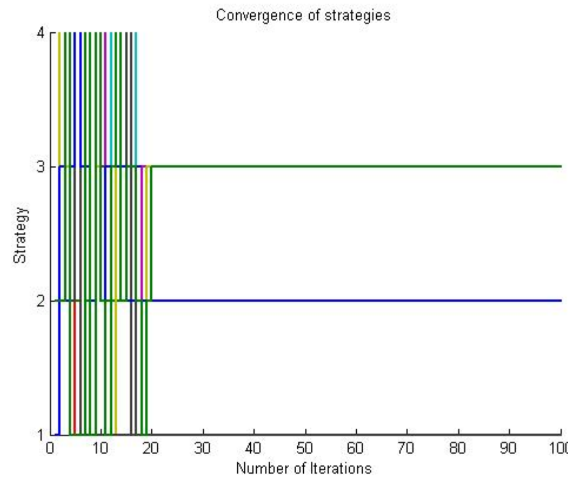

Figure 1: Plot of Convergence of Minimum Cost Strategies

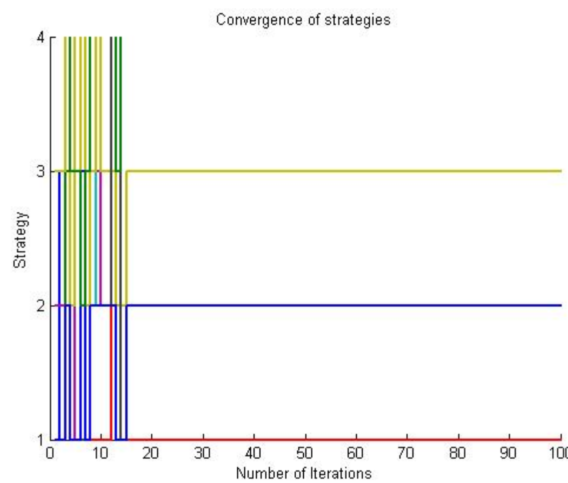

Figure 2: Plot of Convergence of Power Control Strategies

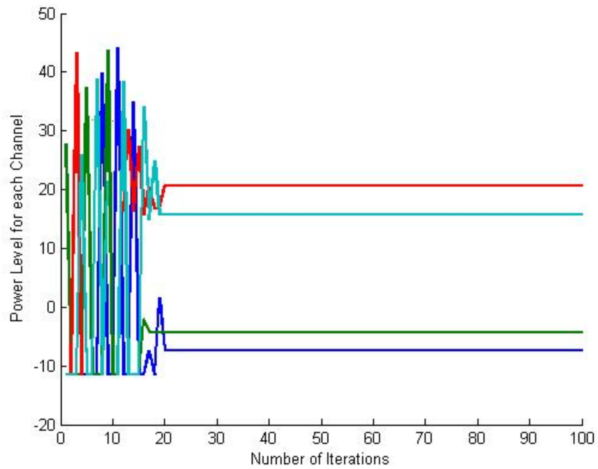

Figure 3: Plot of the Transmit Power for Minimum Cost game

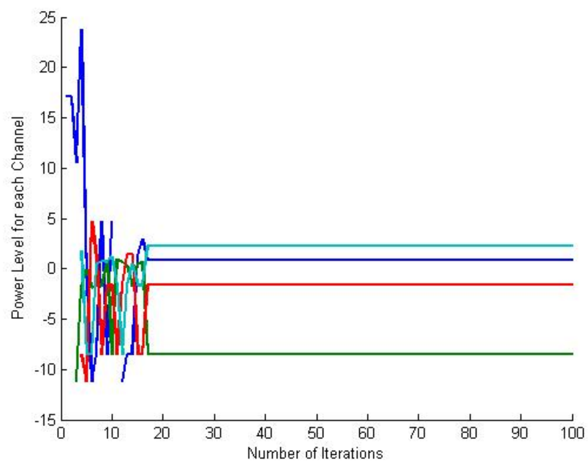

Figure 4: Plot of the Transmit Power for Power Control game

this knowledge and induce a cooperation-like mechanism.

V. Conclusion

This paper observes the convergence of cognitive radio network for minimum cost and power control games. The minimum cost game allows nodes to choose the channels which offer lowest cost so that utility may be maximized but the transmit power may not be optimum. The power control game aims to optimize power, which allows to select channels based on maximum utility at the optimized power. The chosen strategies in this case may not be the lowest in terms of cost. As the players choose their strategies sequentially, they have prior knowledge of their opponents' strategies. This allows to maintain cooperation among the nodes and convergence is established in this case. For minimum cost strategies, nodes settle down to different power levels for different channels, which are higher as compared to the power control strategies which converge to lower power levels.

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