Testing Analog Circuits by Sensitivity Computation

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Abstract

An approach is presented for fault diagnosis, at component level of analog circuits, by using functional testing. It is based on the determination of the deviation of one or many components with respect to the value fixed by the designer. Components deviation is determined by measuring a number of output parameters and by sensitivity estimation. A solution of the test equations, based on the sensitivity matrix, gives the deviation of the defective components. Different types of measurements are combined to achieve an adequate test coverage with a minimum cost. Some experimental results are given to clarify our approach and to show its efficiency.

1. Introduction

In general, analog circuit testing has three objectives: detection, location and identification (diagnosis). Fault detection is a minimum requirement. Fault location is necessary for repairing faulty parts later. Fault identification is of interest for adjustment or tuning when it is necessary to know the exact value of the deviations. The problem of detecting and locating faults in analog circuits is different and takes a much longer time than digital circuits. The accuracy of prediction is also a serious problem where many defective components exist at the same time. On the other hand, not much research has been devoted to analog circuits and no general theory seems to have been established. The nature of faults and fault models in analog circuits make analog testing difficult [1].

The output parameters that characterize analog circuits are diverse and obtained by various measurement techniques. Three categories of commonly used measurements can be distinguished:

1-Harmonic measurement, which concerns the frequency response of the system under test. From this measurement, diverse parameters can be extracted, for example: the gain at a known frequency, the cut-off frequency, the Q factor.

2-Time domain measurement, which involves the use of pulse signals as the input stimuli of a circuit, including a square wave, step, or pulse train. The circuit's transient response at the outputs is then observed. Some of the output parameters observed are rise time, delay time and fall time.

3-Static measurement, which attempts to determine the parameters of the stable states of the circuit under test. This measurement includes the determination of the DC operating point. In our approach all of these types of measurement are used in order to increase the number of observed parameters and, consequently, improve the diagnostic resolution.

Our objective is to present an approach for fault diagnosis in analog circuits. It is based on the determination of the deviation of one or many elements (components) with respect to the value fixed by the designer. Component deviation is determined by measuring a number of output parameters (for example, frequency response, rise time and cut-off frequency) and by sensitivity estimation. Sensitivity can be estimated by calculation or by measurement.

2. Previous works

Up to now, the methods developed for analog circuit testing have been based on two techniques. One of them is the use of in-circuit testing technique and the other one is the functional test. The in-circuit technique is used for digital as well as for analog circuits and is designed to electrically isolate the components to be tested from all of the other components connected to them, and to test them individually to determine their functionality.

Lee [2] compared the approaches of using the operational amplifier configuration and the balanced bridge. He assumed that the balanced bridge approach, because of its non-real time nulling technique, offered operation at high frequencies. The in-circuit technique becomes very difficult for complex circuits, which require hundreds of test pins and driver-sensor amplifiers, and where access to the components' connections becomes increasingly difficult. Using this technique, the function of the complete
circuit is not tested. To avoid these difficulties, functional testing is often preferred.

Functional testing is based on the verification of circuit's functionality through applying stimuli signals at the input and verifying its outputs. This type of test is convenient for complex analog circuits. Therefore, its major drawback is the difficulty of detecting and identifying the defective elements, and also the complexity in writing the test programs. To date, many functional testing algorithms have been developed. Langley [3] used the complete analysis of filter transient response to a step input, which reduces the amount of testing time necessary, followed by diagnosis to the component level. Sloan [4] estimated the transfer function of a circuit by applying an input stimulus and by observing the response. Once the transfer function of a good circuit has been established, the circuit can then be tested by comparing its transfer function with that of the good circuit.

Dai [5] presents an approach for functional testing and parameter estimation of analog circuits in time domain. This approach extends the analysis of linear circuits to include nonlinear circuits. An algorithm has been presented by Milor [6] to automatically generate test sets for analog circuits that minimize testing time. The functional test sets are reduced to include only those sufficient to find out whether a circuit contains parametric faults. Chin [7] presents another approach, which combines examination of the transient response to step inputs with multi-variable discriminant analysis to distinguish a good circuit from a bad one. Unlike the approaches discussed previously, which are based on time domain analysis, McKeon [8] developed a model to test analog circuits by measuring DC voltage at different nodes. The relationship between the parameters, such as voltage and current, at the interconnection of the components are used to represent their behavior.

Our research can be distinguished from these earlier works by our use of different types of input stimuli and the measurement of different kinds of output parameters. Each method of the previous works uses only one specific type of measurement or input stimuli to test analog circuits, which are based on: time domain, harmonic or static measurements. In our case we combine these three categories of measurements in order, to increase the amount of information necessary to predict defective elements in a circuit. Also, with our model, all errors from response deviation due to tolerance until catastrophic faults can be determined.

3. Sensitivity estimation

Terminology:

Circuit: a system containing a set of components (elements) connected together. In the rest of this paper, the terms element and component are used interchangeably.

Parameters: characteristics of a circuit obtained by measuring the output signals. Each parameter is described by a mathematical function in terms of some component. For example, the mathematical expression for the cut-off frequency is: \( f_c = \frac{1}{2\pi RC}. \)

Nominal value: the value of the component or the parameter of the good circuit.

Relative deviation: the deviation of the element or the parameter from its nominal value divided by its nominal value.

3.1 Sensitivity computation

Note that the term sensitivity has been used according to the following definition [9].

Definition: Sensitivity represents the effect of a change in the element \( x \) to the resulting change in the circuit performance parameters \( T \).

We must distinguish between differential sensitivity due to a small variation of elements and incremental sensitivity, due to a large variation of elements. The differential sensitivity reflects the change of the output parameters \( T \) due to an infinitesimal change \( (\Delta x \rightarrow 0) \) of the element \( x \). The following equation defines the differential sensitivity of parameter \( T_j \) with respect to component \( x_i \) [13]:

\[
S_{T_j}^{x_i} \triangleq \frac{\partial T_j}{\partial x_i} \triangleq \frac{\Delta T_j}{\Delta x_i \rightarrow 0}.
\]

On the other hand, when the element \( x \) is submitted to a large perturbation or deviation, the resulting change in the output parameter \( T \) is analyzed by the incremental sensitivity. The incremental sensitivity is defined by the following equation [10]:

\[
\Delta T_j \triangleq \frac{\partial T_j}{\partial x_i} \triangleq \frac{\Delta T_j}{\Delta x_i},
\]

where \( \Delta T_j \) is the change in parameter \( T_j \) resulting from an incremental change in component \( x_i \).

The equation giving the variation of one parameter \( T_j \) from its nominal value in terms of the deviation of one element \( x_i \) from its nominal value by using differential sensitivity has been studied by many researchers [11], [12] and [13]:

\[
\frac{\Delta T_j}{\Delta x_i} = S_{T_j}^{x_i}, \Delta x_i = \frac{\partial T_j}{\partial x_i}, \Delta x_i
\]

where:

\( \Delta T_j \): the deviation of the output parameter with respect to its nominal value \( T_j \).

\( \Delta x_i \): the deviation of the component \( x_i \) with respect to its nominal value \( x_i \).

\( T_j \): the nominal value of the \( j \)th parameter, \( j=1,2,\ldots,n \).

\( x_i \): the nominal value of the \( i \)th component, \( i=1,2,\ldots,k \).

Let us now adopt the notion of sensitivity to analog circuits testing. This is done by expressing the deviation of output parameters in terms of the deviation of components in a circuit. Where \( k \) refers to the number of components and \( n \) stands for the number of parameters,
The relative deviation of parameter $T_j$ in terms of the incremental sensitivity of the differential sensitivity is introduced \[ 10 \]. In analog circuits, the parameters are usually expressed as a function in fractional form. The exact relationship between the differential and incremental sensitivity of a rational function is calculated as follows:

If we have an output parameter in the form of a fractional function which is defined like this:

$$T(w, I) = \frac{N}{D(w, I)}$$

where: $\omega$ is frequency, $x$ is component value, $N$ is the nominator of the parameter $T$ and $D$ is the denominator of the parameter $T$.

The value of this parameter, which results from a deviation in element $x$ by a quantity $\Delta x$, is calculated as:

$$T(w, x + \Delta x) = \frac{N' + N' \Delta x}{D + D \Delta x}$$

where : $N' = \frac{DN'}{Dx}$, $D' = \frac{DN}{Dx}$

Now, we use a first order approximation to determine the deviation $\Delta T$ of the output parameter $T$. This approximation is limited to the first derivative of the parameter with respect to element $x$. Equation (3) shows this development.

$$\Delta T(w, x) = T(w, x + \Delta x) - T(w, x) = \frac{N + N' \Delta x}{D + D \Delta x}$$

Now, the relative deviation of the parameter is expressed as a function of the differential sensitivity of the parameter and the differential sensitivity of its denominator:

$$\Delta T = \frac{(\frac{\partial T}{\partial x}) \Delta x}{1 + \frac{\partial T}{\partial x}}$$

$$= \frac{\Delta T}{\Delta x} = \frac{\frac{\partial T}{\partial x} \Delta x}{1 + \frac{\partial T}{\partial x}}$$

This development yields equation (5), which gives the relationship between the incremental and differential sensitivity of the rational form parameters.

$$\rho_{\Delta x} = \frac{\frac{\partial T}{\partial x}}{1 + \frac{\partial T}{\partial x}} = \frac{S^T}{\Delta x}$$

where:

$\rho_{\Delta x}$ : differential sensitivity of denominator D.


$\rho_{\Delta x}$ : incremental sensitivity of rational form parameter.

The use of incremental sensitivity in equation (5) can be generalized as the use of differential sensitivity for many defective components. In the case of the deviation of $k$ components $\{x_1, x_2, \ldots, x_k\}$ and one output parameter from their nominal values, the relative deviation $\Delta T$ of the output parameter by using the first order incremental sensitivity is given by:

$$\Delta T = \frac{\sum_{i=1}^{k} \frac{S^T}{\Delta x_i}}{1 + \sum_{i=1}^{k} \frac{\partial T}{\partial x_i}}$$

Equation (5) reveals the interesting constraint that if a component has a zero differential sensitivity, the incremental sensitivity will also be zero. We note from this equation that the sensitivity of the denominator is an important factor in determining the incremental sensitivity of a rational function.

3.2 Sensitivity estimation by measurement

As we have mentioned, the differential sensitivities $S^T$ of the parameter $T$ and of its denominator $S^D$ can be determined by mathematical computation by calculating the quantities $\frac{\partial T}{\partial x}$ and $\frac{\partial D}{\partial x}$. In more complex circuits, the sensitivity calculation represents a hard task. To avoid this difficulty, we can measure the sensitivity from the good circuit under test. To estimate the sensitivities $S^T$ and $S^D$ necessary to determine the incremental sensitivity $\rho_{\Delta x}$, by measurement, we submitted the component $x$ to two deviations, $\Delta x_a$ and $\Delta x_b$, and then we measured the corresponding deviation values, $\Delta T_a$ and $\Delta T_b$ of the parameter $T$, at the output. From equation (5) we obtain:

$$\Delta T_a = \frac{S^T}{\Delta x_a} \frac{\Delta x_a}{x}$$

$$\Delta T_b = \frac{S^T}{\Delta x_b} \frac{\Delta x_b}{x}$$

From these two equations and after mathematical development, we obtain:

$$S^T = \frac{x}{T} \left[ \frac{\Delta T_a \Delta T_b (\Delta x_a - \Delta x_b)}{\Delta x_a \Delta x_b (\Delta T_a - \Delta T_b)} \right]$$

$$S^D = \frac{x}{T} \left[ \frac{\Delta T_a \Delta T_b (\Delta x_a - \Delta x_b)}{\Delta x_a \Delta x_b (\Delta T_a - \Delta T_b)} \right]$$

After the sensitivity computation or measurement, we have to measure the output parameters in order to be able to determine the values of the component deviations. These parameters take different forms. In section 4, some groups of output parameters will be given.
4. Measured parameters

A sufficient number of measured output parameters is required for the computation of the deviations of many defective components in the circuit, as we will see in section 5. The output parameters chosen for testing, must constitute a necessary and sufficient set to ensure that faults are detected. The set of parameters should also be a minimal set, in the sense that the number of nodes that must be accessible is minimal.

We have mentioned that the characteristics of analog systems are quite varied. Most of them are based on harmonic, static and time-domain measurements. In our case, we use a diverse group of parameters, including: frequency response, cut-off frequency, rise time, fall time, measurement of the phase \( \phi(w) \), delay time and Q factor.

By knowing the nominal values of the parameters of the good circuit and by measuring the real values of the bad circuit, we can compute the relative deviations of these parameters:

\[
\Delta T = \left[ \frac{\Delta T_1}{T_1}, \frac{\Delta T_2}{T_2}, \ldots, \frac{\Delta T_n}{T_n} \right]
\]

where: \( \Delta T_i = \frac{T_{out}-T_{in}}{T_{nom}} \)
and
\( T_{out} \): the value of the \( j \)th parameter of the bad circuit, which was noted before as the deviated value of the parameter.
\( T_{nom} \): the value of the \( j \)th parameter of the good circuit, which was noted before as the nominal value of the parameter.

5. An approach

After measuring the different parameters of the bad circuit and comparing them to the nominal values of the good circuit, we can compute the relative deviations of one or more defective components. Now, let us see how the defective components are identified and how the values of their relative deviation \( \Delta T_j \) are determined with respect to the nominal values. This is done by measuring the output parameters and estimating the sensitivity.

The basic equations in our algorithm that are necessary to determine the defective components are given below. By using the incremental sensitivities found in equation (5), the circuit under test can be characterized by the following system of equations:

\[
\begin{align*}
\frac{S_{ij}^1 \Delta x_1 + S_{ij}^2 \Delta x_2 + \ldots + S_{ij}^k \Delta x_k}{1 + S_{ij}^1 \Delta x_1 + S_{ij}^2 \Delta x_2 + \ldots + S_{ij}^k \Delta x_k} &= \Delta T_i \\
\frac{S_{ij}^1 \Delta x_1 + S_{ij}^2 \Delta x_2 + \ldots + S_{ij}^k \Delta x_k}{1 + S_{ij}^1 \Delta x_1 + S_{ij}^2 \Delta x_2 + \ldots + S_{ij}^k \Delta x_k} &= \Delta T_j
\end{align*}
\]

where: the sensitivities \( S_{ij}^1, S_{ij}^2, \ldots, S_{ij}^k \) are constant values, \( i=1,2,\ldots,k \) and \( j=1,2,\ldots,n \), and \( \Delta x_i, \Delta x_j \) are the unknown variables.

To solve this system of equations (and this is the basic part of the proposed approach), the number of parameters has to be equal to the number of components \( n \geq k \). The system of equation must also be linearly independent. If the number of parameters is insufficient to solve the equation system (6), we have to increase the number of internal test points in order to obtain an additional number of parameters.

The unknown variables \( \Delta x_i \) in equation (6) represent the relative deviation of the components with respect to their nominal values. Because the deviation calculation \( \Delta T_j \) of the parameters in equation (3) is an approximation of the first order, the solutions \( \Delta x_i \) obtained by the system of equation (6) are approximate and not exact. Often, a number of iterations is required in order to approach the exact solution and to determine the most realistic deviation of components.

6. Experimental study

6.1 Fault model

To evaluate the results obtained by the algorithm, the detected faults are classified into several categories. Figure 1 illustrates the limits between the different categories of detected faults.

![Figure 1: Classification of faults.](image)

Errors less than 5% are acceptable and are considered to be in the tolerance box and in the acceptable region. Faults greater than 5% are considered out of specification and are unacceptable. In this case, the circuit still works but represents a failure. Catastrophic faults are caused by large variation of components or by structural deformation like short and open circuit. These faults are manifested in a totally malfunctioning circuit.

6.2 Example

This example demonstrates the application of our method in a practical case, where we show how the test nodes are chosen and which parameters are to be measured. Another example was given in a previous work, which was designed to show the mathematical aspect of our algorithm [14]. In the present example, the analog circuit to be tested is a biomedical circuit, which is used as an interface between the patient and the computer. We have developed and implemented this circuit using 3 Micron CMOS technology, as described...
in a previous work [15]. The function of the biomedical circuit is the amplification, transformation and processing of the myoelectrical signal generated by the patient's physiological systems. The block diagram is shown in Figure 2.

![Figure 2 Block diagram of the biomedical circuit.](image)

Let us now look at the amplification block and the test methodology based on the sensitivity computation adopted for this block. Figure 3 presents the amplifier's circuit, which is used to amplify the myoelectrical signal and to eliminate noise by using a low-pass filter and a high-pass filter. The selected nodes for the test voltage measurement are 1, 2, 3 and 4.

![Figure 3 Amplification circuit.](image)

The method for determining the deviation of the components will now be described. Three different parts of the circuit are considered and their component deviations are estimated.

1) Determination of the functionality of R1 and R2 in part A: to estimate the deviation of these two components, two parameters need to be measured:
   a) The gain \( G_1 = \frac{V_2}{V_1} = -\frac{R_2}{R_1} \)
   b) In order to add another parameter in part A, we put a capacitance \( C_1 \) with a known value (only during testing) in parallel with \( R_2 \). Then, the cut-off frequency \( f_{c1} = \frac{1}{2\pi R_2 C_1} \) is measured.
   After the sensitivities of the gain and the cut-off frequency with respect to \( R_1 \) and \( R_2 \) are calculated, we use equation (6) to estimate the relative deviation \( \Delta R_1/R_1 \) and \( \Delta R_2/R_2 \).

2) Determination of the functionality of C1 and R3 in part B: in this case, two parameters are also needed to estimate the relative deviation of the components in part B:
   a) The gain \( |G_{0w}| = \frac{V_3}{V_1} \) is measured by fixing a known frequency.
   b) The cut-off frequency of the high-pass filter \( f_{c2} = \frac{1}{2\pi R_3 C_2} \) is determined.

<table>
<thead>
<tr>
<th>Components</th>
<th>Good circuit components values</th>
<th>Bad circuit components values</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>10k</td>
<td>5k</td>
</tr>
<tr>
<td>R2</td>
<td>330k</td>
<td>200k</td>
</tr>
<tr>
<td>R3</td>
<td>1M</td>
<td>800k</td>
</tr>
<tr>
<td>R4</td>
<td>10k</td>
<td>8k</td>
</tr>
<tr>
<td>R5</td>
<td>33k</td>
<td>390k</td>
</tr>
<tr>
<td>C1</td>
<td>0.33uf</td>
<td>0.1uf</td>
</tr>
<tr>
<td>C2</td>
<td>4.7uf</td>
<td>5.60uf</td>
</tr>
<tr>
<td>C3</td>
<td>0.1uf</td>
<td>0.1uf</td>
</tr>
</tbody>
</table>

Table 2 Measured parameters.

<table>
<thead>
<tr>
<th>Part</th>
<th>Measured parameters</th>
<th>Good circuit</th>
<th>Bad circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cut-off frequency</td>
<td>4.8Hz</td>
<td>7.95Hz</td>
</tr>
<tr>
<td></td>
<td>DC - Gain voltage</td>
<td>33</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>Gain at known frequency ( f = 0.7Hz )</td>
<td>0.825</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Cut-off frequency</td>
<td>0.48Hz</td>
<td>1.99Hz</td>
</tr>
<tr>
<td></td>
<td>DC - Gain voltage</td>
<td>34</td>
<td>50.3</td>
</tr>
<tr>
<td>C</td>
<td>Gain at known frequency ( f = 100Hz )</td>
<td>24.6</td>
<td>29.725</td>
</tr>
<tr>
<td></td>
<td>Cut-off frequency</td>
<td>102.6Hz</td>
<td>71.25Hz</td>
</tr>
</tbody>
</table>

The relative deviations \( \Delta R_1/R_1 \) and \( \Delta C_1/C_1 \) are estimated by equation (6), as was done previously.

3) Determination of the functionality of \( R_4, R_5 \), and \( C_2 \) in part C: three parameters are needed to determine the deviation of these components:
   a) The gain in DC voltage, \( G_3 = \frac{V_4}{V_3} = (1 + \frac{R_4}{R_3}) \)
   b) The gain \( |G_{0w}| \) in a known value of frequency.
   c) The cut-off frequency of the low-pass filter \( f_{c3} = \frac{1}{2\pi R_5 C_2} \).

In the same manner as before, by using equation (8), the relative deviations \( \Delta R_4/R_4, \Delta R_5/R_5 \) and \( \Delta C_2/C_2 \) are estimated.

In this example, we designed a bad circuit with components whose values are known in advance and which are different from the values of the good circuit's components.

Table 1 shows the values of the components of the good and bad circuits in the amplification block given in Figure 4. Our objective is to be able to determine the values of the bad circuit's components given in Table 1 just by using the algorithm discussed in section 5 and by measuring the output parameters mentioned previously.

Table 2 shows the values of the measured parameters of the good and bad circuits. Output parameters measurement is obtained by using HSPICE simulator [16] installed on SUN workstations. Table 3 shows the relative deviations between the components of the good circuit and the bad circuit. The results are obtained from the measured parameters of Table 2 and by using our algorithm.
Table 3  Computed values of the bad circuit’s components.

<table>
<thead>
<tr>
<th>Component</th>
<th>Part</th>
<th>Relative deviation of components</th>
<th>The computed values of the bad circuit components</th>
<th>Relative error between the computed and assumed values</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>A</td>
<td>( \frac{R_1}{R_1'} = -0.0049 )</td>
<td>85.305k</td>
<td>1.02%</td>
</tr>
<tr>
<td>R2</td>
<td>A</td>
<td>( \frac{R_2}{R_2'} = -0.9339 )</td>
<td>200.013k</td>
<td>0.0065%</td>
</tr>
<tr>
<td>R3</td>
<td>B</td>
<td>( \frac{R_3}{R_3'} = -0.14 )</td>
<td>120.920k</td>
<td>2.5%</td>
</tr>
<tr>
<td>C1</td>
<td>B</td>
<td>( \frac{C_1}{C_1'} = -0.58 )</td>
<td>0.1186uf</td>
<td>3.86%</td>
</tr>
<tr>
<td>R4</td>
<td>C</td>
<td>( \frac{R_4}{R_4'} = 0.0854 )</td>
<td>7.1615k</td>
<td>0.8%</td>
</tr>
<tr>
<td>R5</td>
<td>C</td>
<td>( \frac{R_5}{R_5'} = 0.0854 )</td>
<td>351.53k</td>
<td>0.8232%</td>
</tr>
<tr>
<td>C2</td>
<td>C</td>
<td>( \frac{C_2}{C_2'} = 0.3745 )</td>
<td>6.46uf</td>
<td>0.178%</td>
</tr>
</tbody>
</table>

The computed values of the bad circuit’s components are supposed to be very close to the values of the bad circuit given in Table 2. The resulting errors are given in the last column of Table 3. Note that the use of only one iteration gives good results for parts A and B of the amplification circuit. On the other hand, the use of one iteration for part C is insufficient and the results is far from the desired degree of precision. After a second iteration, the calculated values of components \( R_4, R_5 \) and \( C_2 \) of the bad circuit are very close to the realistic values given in Table 1. In part C, the number of equations to be solved is greater than the number of equations in parts A and B. For this reason, the number of iterations required to obtain an acceptable degree of precision is also greater. We can see that all of these faults are considered to be out of specification and are not acceptable.

7. Conclusion

A diagnosis of the faulty components and an estimation of their relative deviation from nominals fixed by the designer are presented. The determination of the components' deviation is based on the sensitivity approach. The use of incremental sensitivity gives good results in the presence of the parameters expressed in fractional form and also in polynomial form. Our approach supports both a small and large variation in the components. The incremental sensitivity in this approach is obtained from first order development. In future work, we will explore the possibility of using a higher order sensitivity in order to obtain more accurate results with fewer number of iterations, and for other diagnostic purposes.

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References