Abstract

A fuzzy logic based approach dealing with temporal uncertainty is presented in this paper. Both imprecision of temporal measurements and uncertain temporal knowledge are taken into consideration. Various causes of temporal uncertainty or fuzziness are discussed. Based on fuzzy set and possibility theory, each temporal uncertainty is represented in terms of possibility distributions. Definitions of temporal relations between two fuzzy temporal elements are proposed and a computational model for combining different uncertain bodies is then described in order to compute the fulfillment degree of a generalized propositional statement under uncertainty. Fuzzy temporal constraints propagation is discussed.

Introduction

In many areas of artificial intelligence, there is a pressing need for efficient ways of dealing with temporal knowledge. Among the work in this subject, The Allen's [1] and the McDermott's [10] are the most influential ones. Many applications based on these two temporal reasoning models have shown their capabilities for problem solving in time. In all the two models, dating of events have been assumed always precise and certain and temporal concepts in knowledge expressions are quite well defined. Unfortunately, a event happening in real world is often imprecisely dated and our everyday knowledge about time is also expressed in a no structured and fuzzy way. It seems quite natural and very important to search for an appropriate manner of dealing with temporal uncertainty.

Some theories for handling uncertainty of information and knowledge have been proposed [8]. Fuzzy logic(and therefore possibility theory), as a means of both capturing human expertise and dealing with uncertainty [11], was applied to various domains such as industrial control, medical diagnosis, management and decision-making systems and in many instances, better performances are achieved using fuzzy approach than applying conventional and precise system models.

Since a few years, some researchers have attempted to deal with temporal uncertainty [3][9]. But temporal constraints propagation under uncertainty has not been considered.

This paper presents a fuzzy logic based approach handling especially temporal uncertainty. First, some notions of fuzzy set and possibility theory are introduced. Both linguistic temporal uncertainty and event dating imprecision is represented in terms of possibility distributions. Temporal relations between two fuzzy time elements, and duration of events with its imprecisely dated endpoints are then defined. Finally, a computational model to find fulfillment degree (FD) of a form-generalized propositional statement under uncertainty is described and a method based on the fuzzy inference called generalized modus-ponens is proposed to perform fuzzy temporal constraints propagation.

1. Fuzzy Set and Possibility Theory

1.1 Fuzzy Set

Let U be a set of objects, a fuzzy set is a class of objects with a continuous grade of membership in U (whereas in probability theory, membership function is discrete grade) which associates a real number in [0,1] with each element x ∈ U. Possibility distributions are related directly to fuzzy membership function.

\[ \pi_H(x) = \mu_H(x). \]

where \( \mu_H(x) \) is the membership grade of x in H.

If both A and B are propositions, then A ∨ B denotes a proposition that is a composition of A and B. The two usually used composition modes are logical 'AND' and logical 'OR'.

1. \( \mu_A \land B(v_1, v_2) = \min [\mu_A(v_1), \mu_B(v_2)] \)
2. \( \mu_A \lor B(v_1, v_2) = \max [\mu_A(v_1), \mu_B(v_2)] \)
1.2 Possibility Theory and Logic

Possibility theory is based on fuzzy set theory. In possibility logic, each axiom $p_i$ is assigned a grade of possibility $\pi(p_i)$ and a grade of necessity $N(p_i) = 1 - \pi(\neg p_i)$ denoting the impossibility of contrary fact[6]. In fact, $N(p) = 1$ entails that $p$ is true, $\pi(p) = 0$ entails that $p$ is false and $N(p) = 0$, $\pi(p) = 1$ means total ignorance about the truth or falsity of $p$.

The support of a possibility distribution, $\text{supp}(\pi(t))$ is defined as

$$\text{supp}(\pi(t)) = \{t : \pi(t) > 0\}.$$ 

Furthermore, let us suppose now that one has two predicates $A$ and $B$ on the same universe. Given the true proposition "$X$ is $B$", what is the possibility that the proposition "$X$ is $A$" is true?

**Definition**

Let $A$ and $B$ be two predicates defined on a universe $U$. Knowing that " $X$ is $B$ " is true, the degree of possibility that the proposition " $X$ is $A$ " is true, $\Pi(A/B)$, is

$$\Pi(A/B) = \max_{x \in U} \min(\pi_A(x), \pi_B(x)).$$

where $\pi_A(x)$ is the possibility distribution defined by the predicate $A$ on the universe $U$ and $\pi_B(x)$ is the possibility distribution defined by the predicate $B$ on the universe $U$ and $\Pi(A/B)$ is called associated possibility of $A$ and $B$.

2. Temporal Uncertainty

In real world, our everyday knowledge about time is often uncertain or imprecise. In which follows, two major sources of temporal uncertainty will be discussed: imprecise dating of events and fuzzy linguistic expressions of temporal knowledge.

2.1 Linguistic fuzziness about time

Temporal knowledge is often expressed in terms of linguistic predicates such as 'long time', or in terms of temporal quantities with their modifiers such as 'about 25 minutes', etc. They are represented in terms of possibility distributions. For instance, in the statement 'Mary arrives at about 15 h', the term 'about 15 h' may be represented by $\pi_{\text{about } 15 h}(t)$

$$\pi_{\text{about } 15 h}(t) = \begin{cases} 
(13-t)/2 & \text{if } 13 \leq t < 15, \\
(17-t)/2 & \text{if } 15 \leq t < 17, \\
0 & \text{otherwise.}
\end{cases}$$
exclusive candidates for the value of dating \( Dt \).

![Figure 3](https://via.placeholder.com/150)

**Figure 3** dating of events represented by possibility distributions

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2.3 Representation of Uncertain Temporal Elements

What we call a temporal element is either a time point or a time interval.

2.3.1 Time Points

A time point is a zero duration time and arises in situations where we need to reason about the beginnings and endings of events. Such points can not be viewed as having duration unless we are willing to accept truth gaps. But in the case of imprecise dating, they should be represented in terms of possibility distributions as described in 2.2.

2.3.2 Time Intervals

A time interval is defined as a period in which certain properties hold[1]. The representation of an interval by two points, the beginning point and the ending point, seems natural enough and simplifies our presentation so that it will be employed thereafter.

Since time points are imprecisely dated, a time interval described by two time points are also imprecise(Fig.3). A real imprecise time interval can then be represented by two possibility distributions, one describing its beginning time point and the other its ending time point.

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3. Temporal Relations Under Uncertainty

Three types of temporal relations are defined: relations between time points, relations between a time point and a time interval and, relations between two intervals. In fact, the first type of temporal relations, i.e. those between time points, is the basis one as an time interval is represented by two time points. The two other types of temporal relations can be represented in terms of relations between time points here. It is therefore worth writing all possible temporal relations between two time elements. The definitions of temporal relations between fuzzy time points will be given and based on them, all possible temporal relations between two temporal elements can be obtained[5].

3.1 Temporal Relations Between Time Points (Instants)

As described in the previous section, an ill-dated instant \( t_i \) is represented by a possibility distribution \( \pi_{t_i}(t) \) with all its possible values belonging to the support of this distribution.

With the definition of associated possibility (see 1.2), temporal relations between two fuzzy time points can be defined.

**DEFINITION**

The temporal relation \( \circledast \) between two fuzzy time points \( t_1 \) and \( t_2 \), \( \circledast (t_1, t_2) \) is defined as if there exist \( t \in A \) and \( t' \in B \) so that " \( t \circledast t' \) ", where \( A \) and \( B \) denote the supports of the possibility distributions of \( t_1 \) and \( t_2 \) respectively. And \( \circledast \) denotes the logical comparator corresponding to \( \circledast \).

The possibility of such \( t \in A \) and \( t' \in B \) exists can be viewed as the conditional possibility of " \( t \) exists knowing that \( t' \) exists", i.e.

\[
\max\{\min\{\pi_{t_1}(t), \pi_{t_2}(t')\}\}
\]

\( t \in A \)

\( t' \in B \)

and the possibility of \( \circledast (t_1, t_2) \) is then

\[
\Pi (\circledast (t_1, t_2)) = \max\{\min\{\pi_{t_1}(t), \pi_{t_2}(t')\}\}
\]

\( t \in A \)

\( t' \in B \)

1) before

In this case, \( \circledast = " < " \) and

\[
\pi_{\text{before}} (t_1, t_2) = \max\{\min\{\pi_{t_1}(t), \pi_{t_2}(t')\}\}
\]

\( t \in A \)

\( t' \in B \)

\( t < t' \)

2) equal

this is the case that \( \circledast = " = " \) and

\[
\pi_{\text{equal}} (t_1, t_2) = \max\{\min\{\pi_{t_1}(t), \pi_{t_2}(t)\}\}
\]

\( t \in A \cup B \)
3) after
It can be deducted directly from the definition of before relation.
\[ \pi_{after}(t_1,t_2) = \pi_{before}(t_2,t_1) = \max \{ \min [\pi_{t_2}(t), \pi_{t_1}(t)] \} \]
\[ t \in B \]
\[ t > t \]

The necessity degrees of temporal relations can be directly deduced form the definition of necessity:
\[ N(p) = 1 - \pi(\neg p) \]

In our approach, all possibility distributions are normalized, i.e.
\[ \max (\pi_1(t)) = 1, \]
\[ t \in A \]
where \( A \) denotes the support of \( \pi_1(t) \).

Intuitively, we have always that: "\( t_1 \) is wherever regarding to \( t_2 \)," or that "\( t_1 \) is before or at the same time as or after \( t_2 \)." Our definitions of temporal relationships are in agreement with this intuition since[5]
\[ \pi(t_1 \text{ before } t_2) \text{ or } (t_1 \text{ at the same time as } t_2) \text{ or } (t_1 \text{ after } t_2) = 1 \]

In the same way, "\( t_1 \) is not before \( t_2 \)" implies that "\( (t_1 \text{ after } t_2) \text{ or } (t_1 \text{ equal to } t_2) \)" with respect to exclusive and complete character of our definitions of basis relations between instants (before, equal and after). We have then:
\[ \pi(\text{after } (t_1,t_2)) = \pi((t_1 \text{ after } t_2) \text{ or } (t_1 \text{ equal to } t_2)) \]
\[ = \max (\pi_{equal}(t_1, t_2), \pi_{after}(t_1, t_2)) \]
and therefore:

\[ N_{before}(t_1,t_2) = 1 - \max (\pi_{after}(t_1,t_2), \pi_{equal}(t_1,t_2)) \]

Similarly, it can be obtained that:
\[ N_{equal}(t_1,t_2) = 1 - \max (\pi_{before}(t_1,t_2), \pi_{after}(t_1,t_2)) \]

\[ N_{after}(t_1,t_2) = 1 - \max (\pi_{before}(t_1,t_2), \pi_{equal}(t_1,t_2)) \]

3.2 Duration Computation of Events

There often appear statements such as 'The temperature remains very high during long time.' In this case, for being capable of treating this kind of information, we have to compute the duration of events (the duration of 'very high temperature' in this statement). As a event is described by a time interval during which it occurs, the computation of the duration of a event returns therefore to the computation of the length of its corresponding time interval.

Assume an time interval \( (t_1, t_2) \) with two possibility distributions \( \pi_{t_1}(t) \) and \( \pi_{t_2}(t) \), describing its beginning point and ending point respectively. The supports of \( \pi_{t_1}(t) \) and \( \pi_{t_2}(t) \) are \( E_1 \) and \( E_2 \). It has been assumed that all possibility distributions are unimodal so that \( E_1 \) and \( E_2 \) can be represented by intervals in the real line, i.e.
\[ E_1 = [S_{11}, S_{12}] \text{ and } E_2 = [S_{21}, S_{22}] \]

where \( S_{11} \) and \( S_{21} \) are strictly inferior or equal to \( S_{12} \) and \( S_{22} \) respectively.

The support of the length of this interval, named \( L_e \) can then be obtained from \( E_1 \) and \( E_2 \)
\[ L_e = [L_{e1}, L_{e2}] \]

where \( L_{e1} = \max ((S_{12} - S_{11}), 0) \) and \( L_{e2} = S_{22} - S_{21} \)
and the possibility distribution corresponding describing the during of event can be obtained [5].

\[ \pi_{le}(l) = \left\{ \begin{array}{ll}
\max (\min (\pi_{l_1}(t), \pi_{l_2}(t))) & \text{if } l \in L_e \\
0 & \text{otherwise}
\end{array} \right. \]

\( t \in E_1 \)
\( t \cdot t' = l \)
4. Temporal Reasoning Under Uncertainty

Generally, it is desired to handle linguistic vagueness under imprecise measurements and, to propagate temporal constraints with uncertain knowledge. In what follows, these two types of problems will be studied separately.

4.1 Combination of Linguistic Vagueness and Imprecise Measurements

There are several types of linguistic vagueness [4]. In this paper, only fuzzy temporal predicates (e.g., long time) and time quantifiers with their modifiers (e.g., about 15 h) will be considered.

Given a fuzzy temporal linguistic predicate represented by its possibility distribution \( \pi_{tc}(t) \), and its corresponding measurement described by \( \pi_{tm}(t) \) and its support \( L_{tm} \). The intersection of these two possibility distributions, named \( \pi_{tmc}(t) \), represents the possibility of variable described by its measurement \( \pi_{tm}(t) \) firing the fuzzy linguistic predicate.

\[ \pi_{tmc}(t) = \min \left( \pi_{tc}(t), \pi_{tm}(t) \right) \] if \( t \in L_{tm} \)
\[ = 0 \] otherwise. (2)

Generally, a propositional statement is of the form [4]

\[ p-1 \]
\[ \text{OR} \left[ \quad \text{AND} \left( T_{mij} \right) \right. \text{AND} \left( T_{cij} \right) \left. \right] \] (3)
\[ i=0 \quad j=0 \]

where \( \text{OR} \left( \text{BODY}_i \right) = \text{BODY}_0 \text{OR} \ldots \text{OR} \left( \text{BODY}_{p-1} \right) \left( \text{BODY}_i \right) \)
\[ i=0 \]

Each \( \text{BODY}_j \), connected by \( \text{AND} \), is called an atomic body and that connected by \( \text{OR} \) a secondary body.

The number of atomic body contained in each secondary body connected may be different and this fact is represented by \( q(i) \) which differ along with \( i \).

Given \( \pi_{Tcij}(t) \) possibility distribution of \( T_{cij} \) and \( \pi_{Tmij}(t) \) as measurement of \( T_{mij} \), it can be obtained from (2)

\[ \pi_{Tmij}(t) = \begin{cases} \min \left( \pi_{Tcij}(t), \pi_{Tmij}(t) \right) & \text{if } t_{ij} \in L_{Tmij} \\ 0 & \text{otherwise.} \end{cases} \] (4)

To find the fulfillment degree of the statement under these imprecise dating, there are three different ways. All the three methods are based on fuzzy logic combination operations, i.e. \( \min \) operator for conjunction 'AND' and \( \max \) for disjunction 'OR'.

1) Maximum value method

In this method, the maximum value of \( \pi_{Tmij}(t) \) in \( L_{Tmij} \), named \( \pi_{Tmij}^{max} \), is taken to be the possibility measure (it can also be called the fulfillment degree) of an atomic body of statement \( T_{mij} \) is \( T_{cij} \).

Applying fuzzy logic combination operations, we have finally

\[ p-1 \]
\[ \text{FD} = \max \left\{ \min \left( \min \left( \pi_{Tmij}(t) \right) \right) \right\} \]
\[ i=0 \quad j=0 \]

2) Local average value method

It is similar to the maximum value method escape that here the average value for an atomic body of the statement is taken instead of the maximum one.

3) Global average value method

A computational approach has been proposed to find the FD of a statement under uncertainty. It seems tedious to repeat all its details. Only its principle idea will be reviewed here and the interested readers can find it in the paper referred to earlier [4].

First, each \( \pi_{Tmij}(t) \) represents a function in a two dimension space \( R^2 \) with its support \( L_{Tmij} \). Secondly, each of the \( q(i)-1 \) atomic bodies in the \( i \)-th secondary
body is extended to a space of \( q(i) \) dimensions, denoted by \( R^q_{i} \), and they then form a new function \( \pi_{Tmc}^{(i)}(t_0', t_1', \ldots, t_{1,q(i)-1}) \) with its support

\[
L_{Tmi} = L_{Tmi0} \times L_{Tmi1} \times \ldots \times L_{Tmiq(i)}
\]

It is achieved by taking the minimum value of all these \( q(i) \) function in each point included in the \( L_{Tmi} \). After this, \( p \) functions can be obtained as there are \( p \) secondary bodies. These \( p \) functions are also extended into a \((n+1)\) degree space \( R^{n+1} \) and they are then combined to form a final function \( \pi_{Tmc} \) with its support \( L_{Tmc} \) formed by \( L_{Tmi} = L_{Tmi0} \times L_{Tmi1} \times \ldots \times L_{Tmiq(i)} \) at \( t_{1,q(i)-1} \).

Therefore, the final FD of the statement (3) can be found by simply averaging \( \pi_{Tmc} \) within its support \( L_{Tmc} \).

4.2 Fuzzy Temporal Constraints Propagation

The temporal constraint propagation is regarded as inferring new temporal relationships between events that share a common reference event directly or indirectly.

Given \( a, b, c \), three events and \( R_1(a, b) \) and \( R_2(b, c) \) denoting temporal relations between \( a \) and \( b \) and that between \( b \) and \( c \) respectively. What the temporal constraint propagation does is to reason about all possible relationships existing between the two event \( a \) and \( c \), \( R_3(a, c) \), or simply to verify existence of some relationships between \( a \) and \( c \) based on the known relations \( R_1(a, b) \) and \( R_2(b, c) \). This can be described as

\[
\text{event}(a), \text{event}(b), \text{event}(c), R_1(a, b), R_2(b, c) \rightarrow R_3(a, c)
\]

The precise temporal constraint propagation has been studied since a decade. In what follows, the possibility to perform temporal constraint propagation under uncertainty based on fuzzy logic and possibility theory, called fuzzy temporal constraint propagation, will now be discussed.

4.2.1 Propagation of Temporal Relations Between Fuzzy Time Points

An approach of dealing with uncertain facts and default rules has been proposed by Farreny[7]. Only the \( \min \) and \( \max \) operators are used for computing the possibility and necessity degrees corresponding to different alternatives.

A fact \( p \) is represented by \( \pi(p) \) and \( \pi(-p) \) and a rule 'if \( p \) then \( q \) ' is represented in form of matrix

\[
\begin{bmatrix}
\pi(q/p) & \pi(q/-p) \\
\pi(-q/p) & \pi(-q/-p)
\end{bmatrix}
\]

where \( \pi(q/p) \) is read as 'possibility distribution of \( q \) if \( p \) is true'.

The inference, called Generalized Modus Ponens (GMP), is then modeled in a matrix form:

\[
\begin{bmatrix}
\pi(q) \\
\pi(-q)
\end{bmatrix} =
\begin{bmatrix}
\pi(q/p) & \pi(q/-p) \\
\pi(-q/p) & \pi(-q/-p)
\end{bmatrix}
\begin{bmatrix}
\pi(p) \\
\pi(-p)
\end{bmatrix}
\]

where the matrix product is defined by analogy with the usual one by changing the sum into \( \max \) operation and the product into \( \min \) operation.

Indeed, this matrix is a representation of two equations

\[
\begin{align*}
\pi(q) &= \max\{\min[\pi(q/p), \pi(p)], \min[\pi(q/-p), \pi(-p)]\} \\
\pi(-q) &= \max\{\min[\pi(-q/p), \pi(p)], \min[\pi(-q/-p), \pi(-p)]\}
\end{align*}
\]

The analogue of (5) in probability theory is

\[
\text{Prob}(q)=\text{Prob}(q/p)\times\text{Prob}(p)+\text{Prob}(q/-p)\times\text{Prob}(-p))
\]

Given now three fuzzy instants \( t_1, t_2 \) and \( t_3 \) and \( R_1(t_1, t_2) \) and \( R_2(t_2, t_3) \), temporal relations between \( t_1 \) and \( t_2 \) and those between \( t_2 \) and \( t_3 \) respectively.

What can we say about \( R_3(t_1, t_3) \) the possible relationships between \( t_1 \) and \( t_3 \) ?

In this case, the equation of GMP is be obtained by replacing \( p \) and \( q \) with \( R_1R_2R_3 \) in (6) respectively.

The coefficients matrix (the first matrix in the right hand) will be determined now. Due to the fact that a dating has only one value (mutually exclusive), these coefficients may be the same as in non-fuzzy cases. These coefficients represent transitivities between certain and precise temporal relationships which can be found in [1]. In the case of time points, these transitivity properties are summarized in table_1.

<table>
<thead>
<tr>
<th>t1,t2</th>
<th>t2,t3</th>
<th>before or at same time</th>
<th>after</th>
</tr>
</thead>
<tbody>
<tr>
<td>before</td>
<td>before</td>
<td>before</td>
<td>?</td>
</tr>
<tr>
<td>at same time</td>
<td>before</td>
<td>at same time</td>
<td>after</td>
</tr>
<tr>
<td>after</td>
<td>?</td>
<td>after</td>
<td>after</td>
</tr>
</tbody>
</table>
As for example, given $R1 = \text{'before'}$ and $R2 = \text{'before'}$, i.e.

$t_1 \text{ is before } t_2$, and $t_2 \text{ is before } t_3$.

What are the possible relationship between $t_1$ and $t_3$?

All the three possible relations between $t_1$ and $t_3$ should be considered.

1) 'before'

In this case, the coefficients matrix has values as

$$
\begin{bmatrix}
1 & \lambda_1 \\
0 & \lambda_2
\end{bmatrix}
$$

where $\lambda_1 \in [0,1]$ and $\lambda_2 \in [0,1]$.

We have then

$$
N[\text{before}(t_1, t_3)] \geq \min \{N[\text{before}(t_1, t_2)], N[\text{before}(t_2, t_3)] \}
$$

since

$$
\pi[\text{before}(t_1, t_3)] = 1 - N[\text{before}(t_1, t_3)]
$$

and

$$
\pi[-\text{before}(t_1, t_3)] = \max \{\pi[-\text{before}(t_1, t_2)], \pi[-\text{before}(t_2, t_3)] \}
$$

$$
= \max \{1 - N[\text{before}(t_1, t_2)], 1 - N[\text{before}(t_2, t_3)] \}= 1 - \min \{N[\text{before}(t_1, t_2)], N[\text{before}(t_2, t_3)] \}
$$

It can be also deduced that

$$
N[\text{after or at the same time}(t_1, t_3)] \
\geq \min \{N[\text{after or at the same time}(t_1, t_2)], N[\text{after or at the same time}(t_2, t_3)] \}
$$

$$
\pi[\text{after or at the same time}(t_1, t_3)] = \max \{\pi[\text{before}(t_1, t_2)], \pi[\text{before}(t_2, t_3)] \}
$$

2) 'after' or 'equal'

In this case, nothing can be deduced because the coefficients matrix has only as its elements 0 or unknown.

4.2.2 Propagation of Temporal Relations Between Fuzzy Time Intervals

In the case of fuzzy time intervals, the propagation may be more difficult than that between fuzzy time points.

It has been proposed that fuzzy time intervals are represented by its two fuzzy ending points and relations between two such intervals are defined in terms of relations between their corresponding ending points. For this reason, propagation of relationships between fuzzy time intervals will be also performed based on that between fuzzy time points.

Given three time intervals $t_1 = [t_{11}, t_{12}]$, $t_2 = [t_{21}, t_{22}]$, $t_3 = [t_{31}, t_{32}]$, and

$\pi[\text{identical}(t_1, t_2)]$ and $\pi[\text{identical}(t_1, t_3)]$, the possibility and necessity measures of $t_1$ is identical to $t_2$. The coefficients matrix becomes now an unit matrix (refer to table I), i.e.

$$
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$

Therefore

$$
\pi[\text{equal}(t_{11}, t_{21})] = \min \{\pi[\text{equal}(t_{11}, t_{21})], \pi[\text{equal}(t_{12}, t_{22})] \}
$$

and

$$
N[\text{equal}(t_{11}, t_{21})] = \min \{N[\text{equal}(t_{11}, t_{21})], N[\text{equal}(t_{12}, t_{22})] \}
$$

which can be described in terms of given relations

$$
\pi[\text{meet}(t_{11}, t_{31})] = \min \{\pi[\text{identical}(t_{11}, t_{21})], \pi[\text{meet}(t_{12}, t_{32})] \}
$$

and

$$
N[\text{meet}(t_{11}, t_{31})] = \min \{N[\text{identical}(t_{11}, t_{21})], N[\text{meet}(t_{12}, t_{32})] \}
$$

One question may be asked 'Does $t_1$ meets $t_3$?'

For answering this question, it is sufficient to test if $t_{11}$ is equal to $t_{32}$.

From what are given, it is obvious that

$$
\pi[\text{equal}(t_{11}, t_{32})] \geq \pi[\text{identical}(t_{11}, t_{21})]
$$

$$
N[\text{equal}(t_{11}, t_{32})] \leq N[\text{identical}(t_{11}, t_{21})]
$$

$$
\pi[\text{equal}(t_{11}, t_{32})] = \pi[\text{meet}(t_{11}, t_{21})]
$$

$$
N[\text{equal}(t_{11}, t_{32})] = N[\text{meet}(t_{11}, t_{21})]
$$

since

$$
\pi[\text{identical}(t_{11}, t_{32})] = \min \{\pi[\text{equal}(t_{11}, t_{21})], \pi[\text{equal}(t_{12}, t_{32})] \}
$$

and

$$
N[\text{equal}(t_{11}, t_{32})] = N[\text{equal}(t_{12}, t_{32})]
$$

$$
\pi[\text{meet}(t_{11}, t_{32})] = \pi[\text{equal}(t_{12}, t_{32})]
$$

$$
N[\text{meet}(t_{11}, t_{32})] = N[\text{equal}(t_{12}, t_{32})]
$$

The coefficients matrix becomes now an unit matrix (refer to table I), i.e.

$$
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$

Therefore

$$
\pi[\text{equal}(t_{11}, t_{32})] = \pi[\text{equal}(t_{12}, t_{32})]
$$

and

$$
N[\text{equal}(t_{11}, t_{32})] = N[\text{equal}(t_{12}, t_{32})]
$$

which can be described in terms of given relations

$$
\pi[\text{meet}(t_{11}, t_{32})] = \min \{\pi[\text{identical}(t_{11}, t_{21})], \pi[\text{meet}(t_{12}, t_{32})] \}
$$

and

$$
N[\text{meet}(t_{11}, t_{32})] = \min \{N[\text{identical}(t_{11}, t_{21})], N[\text{meet}(t_{12}, t_{32})] \}
$$

where $\pi[\text{equal}(t_{11}, t_{21})]$ and $N[\text{equal}(t_{11}, t_{21})]$ are assumed to take their minimum value of $\pi[\text{identical}(t_{11}, t_{21})]$ and $N[\text{identical}(t_{11}, t_{21})]$ respectively.
CONCLUSION

In designing intelligent systems, it is desirable to perform on one hand temporal reasoning and on the other hand the reasoning under uncertainty. It seems quite natural and necessary to study the possibility of integrating these two kinds of reasoning. This paper presented a fuzzy logic-based approach dealing with temporal uncertainty. Both linguistic fuzziness about temporal knowledge and dating imprecision of events have been taken into consideration. Based on fuzzy logic and possibility theory, temporal constraints propagation can be achieved.

Our further research will be the temporal evaluation of system behavior under uncertainty.

Reference: