A Theoretical Analysis of Data Reduction using the Weber Quantizer

Julius Kammerl, Peter Hinterseer, Subhasis Chaudhuri and Eckehard Steinbach

I. SUMMARY

We present a theoretical analysis of a perceptual coding approach, the so called Weber quantizer. Extensive studies performed by experimental psychologists and physiologists have unveiled one major conclusion: Human perception often follows Weber’s Law. Ernst Weber was an experimental physiologist who in 1834 first discovered the following implication \( \Delta I = kI \), where \( \Delta I \) is the so called Difference Threshold or the Just Noticeable Difference (JND). It describes the smallest amount of change of an (arbitrary) stimulus \( I \) which can be detected just as often as it cannot be detected and defines the Weber bound at \( [(1 - k)I, (1 + k)I] \).

The Weber quantizer analyzed in this work, in comparison to well known approaches like scalar- and vector-quantization, focuses on the reduction of signal update rates. In our earlier work we have shown that this type of data reduction scheme is particularly suitable and efficient for haptic data communication in telepresence and teleaction systems. In our analysis, we consider a normally distributed input sequence \( x_i \in N(0, 1) \). At the beginning of the sequence, the first signal sample \( x_1 \) has to be transmitted. From now on we propose to build a binary tree structure based on the events of either lying within the Weber bound of \( q_{i-1} \) or not, stated by the superscript \((0,1)\). Due to strong conditional probabilities in the sequences, we have to iteratively determine the PDF of the leafs \( q_t^i \) in the tree. The PDF itself, here denoted as \( D \), is calculated as follows:

\[
D_{q_1^i}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \text{ initial normal distribution}
\]

\[
D_{q_t^i}(x) = D_{q_{t-1}^i}(x) \int_{x(1-k)}^{x(1+k)} D_{x_t}(y) dy \text{ if bound is not violated}
\]

\[
D_{q_t^i}(x) = D_{x_t}(x) \left( \int_{-\infty}^{\frac{x}{(1+k)}} D_{q_{t-1}^i}(y) dy + \int_{\frac{x}{(1-k)}}^{\infty} D_{q_{t-1}^i}(y) dy \right) \text{ if bound is violated}
\]

For a particular threshold parameter \( k \), simulation of the Weber quantizer system with the given signal model showed that the probability of violating the Weber bound at a certain step in the input signal quickly converges to a fixed value \( P_S \) for all new input samples. Therefore we can estimate \( P_S \) by simply adding up all probabilities for a Weber bound violation at a lower level in the binary tree. We showed that simulation and numerical solution of the aforementioned formulae are in very good agreement. The presented approach can be easily applied to any i.i.d. input signal and provides the necessary theoretical basis for signal models, where dependencies within the signal further complicate the analysis. Therefore models like an autoregressive model or a Wiener process model can be analyzed in the near future.

J. Kammerl, P. Hinterseer and E. Steinbach are with the Institute of Communication Networks (LKN), Media Technology Group, at the Technische Universität München, Arcisstrasse 21, 80290 München, Germany, +49 (89) 289 25810, {kammerl,ph.eckehard.steinbach}@tum.de. J. Kammerl has been supported by a grant from the Universität Bayern e.V.

S. Chaudhuri is with the Department of Electrical Engineering at the Indian Institute of Technology in Bombay, Powai, Mumbai 400 076, India, (91-22) 2576 7437, sc@ee.iitb.ac.in