Unconstrained Vector Length in Fast Wavelet Transforms

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Abstract

In this paper, we propose a solution for obtaining fast discrete wavelet transforms for vectors of arbitrary even length. Signals encountered are represented as vectors of finite but arbitrairy length, not necessarily constrained in length to a power of two. Current solutions include padding or recomputing the wavelet transform coefficients for a specific length, a process that introduces unwanted complexity in the algorithms.

The padding techniques elongate the original vector by appending new points to the original vector. These new points are either zeros or some kind of extrapolation of the data that will introduce as few artifacts as possible. Padding techniques, depending on the method of padding, can lead to variants of the transforms that take advantage of the padded values. Such techniques are described in [Taswell94]. However, these solutions are computationaly more complex than the original fast transform algorithms and may be unsuitable for implementation in microcontroller-based devices and other lightweight applications. Rescaling techniques warp the vector onto a vector of the closest suitable length. This almost always leads to a loss of information when the vector is shrunk or extended and this makes reversible transforms virtually impossible to obtain. Padding and warping transform the original signal to a signal of amenable length for the current fast transform implementation. This means a power of two in the case of the Mallat and Haar transforms [Mallat89,Haar10] and a power of four in the case of the four tap Daubechies filter [Daubechies88].

In this paper, we propose modifications to the fast transform algorithms for the Mallat decomposition and the Haar Wavelet Transform to accept vectors of arbitrary even length. While the solution limits lengths to be even, it lifts the restriction that the length must be a power of the order of the transform. We give an interpretation of the rearranged coefficient pyramid. The modifications generalize to discrete transforms of any order.

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