Orthonormal sets of filters obtained by modulations and rotations of a prototype

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In the past decade the field of image processing has grown considerably, and although various successful techniques have been developed for tasks such as image compression, understanding and segmentation, one final piece is missing. Bearing in mind that ultimately, an image is evaluated by a human observer, it is obvious that the usual mean-square error is not appropriate and thus we still sorely lack subjective measures of image quality. Some physiological results point out that the early stage of the human visual system (HVS) works like a filter bank on the retinal image. The filters in such a filter bank can be seen as being obtained by rotation and modulation from an original prototype filter.

This work is just the first step and concentrates on the design of local bases obtained by unitary transformations of a single (or more than one) prototype filter. In order to be able to construct such a basis, we want to extend to a more general case the following well-known fact:

\[
(f(t), f(t - n)) = \delta(n) \leftrightarrow \sum_{n=-\infty}^{\infty} |F(\omega + 2\pi k)|^2 = 1, \quad (1)
\]

where \(f(t)\) is a continuous-time function and \(F(\omega)\) is its Fourier transform. Property (1) proves its usefulness both in testing the orthogonality of \(f(t)\) with respect to its integer translations as well as in producing functions enjoying such a property.

We extend (1) to a more general case, in which the group of integer translations is replaced by a more general group consisting of unitary transformations. Two cases are considered: when the group is abelian and when it is not. For each case, the equivalent form of (1) is found. Based on this, the design of the filters is discussed. Finally, the case that triggered this work, that where the group consists of modulations and rotations, is examined in detail. As an example, we design a two-dimensional filter, which together with its modulations and rotations forms an orthonormal set.

References
