Improved Methods of Simulating RLC Coupled and Uncoupled Transmission Lines Based on the Method of Characteristics*

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ABSTRACT

This paper describes new techniques for simulating lossy (RLC) transmission lines based on the method of characteristics. For uncoupled lossy transmission lines a method is presented which speeds up the simulation time by a factor of two compared with existing techniques. A method is also presented for the transient analysis of coupled lossy lines in an inhomogeneous medium. Previously, simulation techniques were limited to coupled lossy lines in a homogeneous medium.

I. Introduction

The accurate assessment of interconnection delay is key to estimating the performance of digital systems. The models for the interconnections between devices can generally be divided into two categories. In the first category the signal transition time is sufficiently longer than the delay of the signal propagation through the interconnection [1]. Interconnect models in the first category include the lumped capacitor and the distributed RC line. In the second category the transition time is less than twice the delay of the signal propagation, thus a transmission line model must be considered. Models in the second category include single (uncoupled) lossless and lossy (RLC) transmission lines and coupled lossless and lossy transmission lines.

Off-chip interconnections frequently fall into the second category and are modeled as lossless uncoupled transmission lines [2]. However, with device speed increasing and interconnect dimensions decreasing both on-chip [3,4] and off-chip [5,6], transmission line models must be used to model on-chip as well as off-chip interconnections. Since these models must include the effects of series interconnect resistance and, if applicable, the influence of the close spacing of adjacent lines, the modeling and the analysis of lossy uncoupled transmission lines and lossy coupled transmission lines has become increasingly important.

This paper will consider the modeling and the transient analysis of lossy (RLC) coupled and uncoupled transmission lines. We assume the transmission lines to be uniform along its length and that the line behavior can be completely described in terms of matrices of inductances, capacitances, and resistances (quasi-TEM analysis) [7].

The method of characteristics is a standard mathematical method for solving hyperbolic partial differential equations such as the wave equation. The equivalent circuit associated with the method has proved useful for the transient analysis of single lossless transmission lines [8,9], coupled lossless transmission lines [10,11], single lossy transmission lines [12], and coupled lossy transmission lines in a homogeneous medium (which is characterized by multiple modes of equal phase velocity) [12]. The analysis of lossy coupled lines has also been investigated using the integral equation method [13]. This type of analysis assumes no coupling between non-adjacent lines. As we will show, this assumption is not adequate for strongly coupled lines such as those found on an insulating substrate.

In this paper we present two enhancements to the transient analysis of lossy transmission lines based on the method of characteristics; one for coupled lines and one for uncoupled lines. For uncoupled lines we present a method which makes it possible to simulate single lossy lines in approximately half the time with no loss of accuracy at the ends of the transmission lines. We also present a technique which allows the transient analysis, based on the method of characteristics, of lossy coupled lines in an inhomogeneous medium, which is characterized by multiple propagation modes of unequal phase velocities. A hueristic to determine the lower bound on the number of sections a line must be partitioned into to accurately model an RLC line is also presented.

In section two, the method proposed by Gruodis [12] for the analysis of single lossy transmission lines is reviewed, an improved method is presented, and the performance of the two methods is compared based on computer simulation time. In section three, the theory of modal analysis is reviewed along with the transient analysis of lossless coupled transmission lines. The transient analysis of lossy coupled transmission lines is then presented. In section four, a hueristic is proposed to accurately model an *RLC* line. Examples are given in section five and the conclusion follows in section six.

II. Single Resistive Transmission Lines

Transient Analysis

The method of characteristics solution of the partial differential wave equations with the per-unit length resistance and conductance both negligible yields the following two solution equations in terms of terminal voltages and currents (see Fig. 1(a)) at times t and $t-\tau$

$$\begin{aligned} em(t) &= -Zo\ im(t) + ek(t-\tau) + Zo\ ik(t-\tau) \\ &= -Zo\ im(t) - Ek(t-\tau) \end{aligned} \tag{1}$$

$$\begin{aligned} ek(t) &= -Zo\ ik(t) + em(t-\tau) + Zo\ im(t-\tau) \\ &= -Zo\ ik(t) - Em(t-\tau) \end{aligned} \tag{2}$$

where τ is the time of flight of the lossless transmission line [9]. The updating equations

$$Ek(t) = -[2\ ek(t) + Em(t-\tau)] \tag{3}$$

$$Em(t) = -[2\ em(t) + Ek(t-\tau)] \tag{4}$$

provide a recursive method for solving equations (1) and (2).

To incorporate the resistive component of a lossy transmission line into a method based on the method of characteristics, the lossy line is modeled as a number of ideal lines, separated by resistors. Consider a lossy line modeled by two ideal transmission lines (sections) separated by a resistor, R (see Fig. 1(b)). The updating equations for the network are

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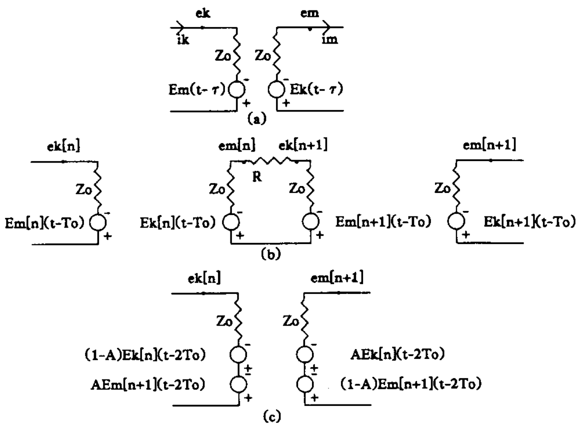

Fig. 1. (a) Equivalent impedance network of a lossless transmission line (b) Equivalent impedance network of a lossy transmission line (c) compacted version of (b).

$$Ek[n](t) = -[2\,ek[n](t) + Em[n](t-To)] \quad (5)$$

$$Em[n](t) = -[2\,em[n](t) + Ek[n](t-To)] \quad (6)$$

$$Ek[n+1](t) = -[2\,ek[n+1](t) + Em[n+1](t-To)] \quad (7)$$

$$Em[n+1](t) = -[2\,em[n+1](t) + Ek[n+1](t-To)] \quad (8)$$

where $To = x\sqrt{lc}$ is the time of flight of each ideal transmission line section, x is the length of the lossless section, l and c are the per unit length inductance and capacitance, respectively, and $(t-To)$ represents a time delay of To. Solving for $em[n]$ and $ek[n+1]$ in the closed loop of Fig. 1(b) yields

$$Em[n](t) = AEm[n+1](t-To) + (1-A)Ek[n](t-To) \quad (9)$$

$$Ek[n+1](t) = AEk[n](t-To) + (1-A)Em[n+1](t-To) \quad (10)$$

where A is the attenuation coefficient per section

$$A = 2Zo/(2Zo+R)$$

and $Zo = \sqrt{l/c}$ is the characteristic impedance of the lossless transmission line [12].

Assuming that To is also the time step or stepsize of the transient analysis, the transient analysis of the lossy transmission line in Fig. 1(b) is performed as follows. At each time step of the analysis, the voltages at the ends of the line, $ek[n]$ and $em[n+1]$, are solved for based on the voltage sources $Ek[n]$, $Ek[n+1]$, $Em[n]$, and $Em[n+1]$ (which have been computed in the previous time step) and the terminating networks at the ends of the line. Afterwards, the voltage sources are updated using equations (5), (8), (9), and (10). If the lossy line must be segmented into a larger number of lossless sections separated by resistors, equations of the form of (9) and (10) will be used to update the additional voltage sources of the equivalent impedance network. This method based on the above equations will be collectively known as method one.

Examination of the above equations reveals that further compaction is possible. Substituting equations (9) into (5) and (10) into (8) yields

$$Ek[n](t) = -[2ek[n](t) + AEm[n+1](t-2To) + (1-A)Ek[n](t-2To)] \quad (11)$$

$$Em[n+1](t) = -[2em[n+1](t) + AEk[n](t-2To) + (1-A)Em[n+1](t-2To)]. \quad (12)$$

The compacted equivalent network is shown in Fig. 1(c). If two compacted sections are separated by a resistor R the updating equations for the network are

$$Ek[n](t) = -[2ek[n](t) + AEm[n](t-2To) + (1-A)Ek[n](t-2To)] \quad (13)$$

$$Em[n](t) = A(1-A)[Ek[n](t-2To) + Ek[n+1](t-2To)] + (1-A)^2En[n](t-2To) + A^2Em[n+1](t-2To) \quad (14)$$

$$Ek[n+1](t) = A(1-A)[Em[n](t-2To) + Em[n+1](t-2To)] + (1-A)^2Ek[n+1](t-2To) + A^2Ek[n](t-2To) \quad (15)$$

$$Em[n+1](t) = -[2em[n+1](t) + AEk[n+1](t-2To) + (1-A)Em[n+1](t-2To)]. \quad (16)$$

As before, if the lossy line must be segmented into a larger number of ideal lines separated by resistors, equations of the form of (14) and (15) will be used to update the additional voltage sources of the equivalent (compacted) impedance network. The methodology utilizing the above equations will be collectively known as method two.

Performance Analysis

An outline of a transient analysis algorithm for a single resistive transmission line with series resistance R_t follows:

```
determine number of sections
R = R_t/(sections-1)
for j=1 to simulation time/stepsize do
    calculate ek[in] and em[out]
    for i=1 to number of (compacted) sections do
        update Ek[i] and Em[i]
    end for
end for
```

We will discuss the determination of the number of sections in section IV. Assuming a single transmission line of length x, the stepsize is the time of flight of the line divided by the number of sections. Consider a lossy line partitioned into s sections with each ideal line segment having a time of flight of To. A time step of To may be used to simulate this line if method one is used. The same lossy line requires $s/2$ compacted sections and a time step of $2To$ if method two is used.

We implemented both methods in order to compare performance. In our application we are solving a tree of transmission lines where the input of the tree is a voltage source in series with a resistance. The output of a transmission line in the tree is connected to either the input(s) of another transmission line(s), a terminating resistor, or a terminating capacitor. In this application the update step represents the computational bulk of the algorithm.

We define the speedup, S, as the ratio of the simulation cost (in cpu seconds) * of method one to method two. The speedup for each of the configurations in Fig. 2 is shown in Table 1 for a varying number of sections, s. Recall, a lossy line partitioned into s sections is modeled with $s/2$ compacted sections if method two is used. For configurations (3) and (4), s is the number of sections in a 1/4 unit length conductor. As expected, as the number of sections in the transmission line model becomes large, the speedup increases. For example, consider configuration (4). For $s=4$ and $s=40$ the update step accounts for approximately 47% and 90% of the simulation time, respectively.

III. Coupled Resistive Transmission Lines

As discussed earlier, Gruodis [12] expanded the analysis of

* Programmed in C and executed on a Sun 3/50 with a floating point accelerator.

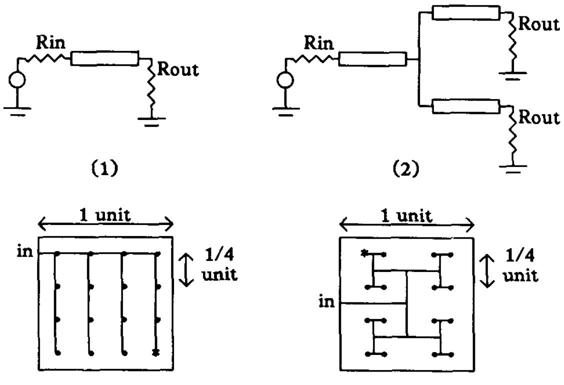

Fig. 2. Four lossy transmission line configurations. Configurations (3) and (4) connect sixteen equally spaced points on a one unit by one unit surface. For configurations (3) and (4), s is the number of sections in a 1/4 unit length conductor and R_t is the total series resistance of a 1 unit length conductor.

Table 1. Computer Simulation Time and Speedup in Simulation Time, S, for the Configurations (cases) in Fig. 2.

case	s	$t_{one}(sec)$	$t_{two}(sec)$	S
(1)	20	1.29	0.68	1.90
(1)	100	17.34	6.87	2.52
(2)	20	2.48	1.15	2.16
(2)	100	44.99	17.47	2.58
(3)	4	2.54	1.36	1.87
(3)	28	73.52	30.32	2.42
(4)	4	3.53	1.97	1.79
(4)	40	191.83	75.50	2.54

single lossy lines to coupled lossy lines in a homogeneous medium. To facilitate the generalization to inhomogeneous media, modal theory and the transient analysis of lossless coupled lines will be briefly reviewed. The transient analysis of lossy coupled lines will follow.

Modal Theory

Consider n coupled lossless transmission lines. If we limit ourselves to quasi-TEM waves the generalized telegraphists' equations may be used [14]:

$$\frac{\partial}{\partial z}\begin{bmatrix} v(z,t) \\ i(z,t) \end{bmatrix} = -\begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix} i(z,t) \\ v(z,t) \end{bmatrix} \tag{17}$$

where $v(z,t)$ and $i(z,t)$ are $n \times 1$ voltage and current vectors, respectively, and L and C are symmetric $n \times n$ inductance and capacitance matrices, respectively, which may be calculated using numerical techniques [15,16]. By applying the following linear transformations

$$v(z,t) = Me(z,t) \tag{18}$$

$$i(z,t) = (M^t)^{-1} j(z,t)$$

where M is the modal matrix of eigenvectors of the matrix LC and the superscripts t and -1 denote transpose and inverse, respectively, the generalized telegraphers equations can be decoupled into a set of $2n$ equations [17,10]

$$\frac{\partial}{\partial z}\begin{bmatrix} e(z,t) \\ j(z,t) \end{bmatrix} = -\begin{bmatrix} L' & 0 \\ 0 & C' \end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix} j(z,t) \\ e(z,t) \end{bmatrix} \tag{19}$$

where $L' = M^{-1}L(M^t)^{-1}$ and where $C' = M^tCM$. L' and C' are diagonal matrices. $j(z,t)$ and $e(z,t)$ represent the n eigenmodes of the system, each eigenmode being characterized by a characteristic impedance, $z_i = \sqrt{L'_i/C'_i}$, and a velocity of propagation, $v_i = 1/\sqrt{L'_iC'_i}$.

An equivalent representation of n coupled lines with a coupling distance of x is shown in Fig. 3 where the boxes labeled M represent transformation networks and where all voltages are referenced to ground.

Transient Analysis of Lossless, Coupled Transmission Lines

Consider the n lossless coupled lines of coupled length x shown in Fig. 3. To perform a transient analysis based on the method of characteristics, the network is modeled as shown in Fig. 4(b), where n_i is the number of sections each decoupled line must be divided into so that

$$x/v_in_i \approx x/v_jn_j \approx \Delta h \tag{20}$$

where Δh is the stepsize of the transient analysis. We are assuming only one past history of Ek and Em is stored, thus the

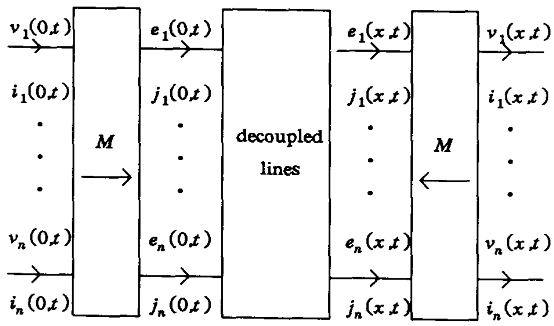

Fig. 3. Equivalent representation of n coupled lines of coupled length x. The boxes labeled M represent transformation networks.

need to subdivide the lines into sections of equal travel times. A more common approach is to store multiple past histories. The prior approach was chosen for illustrative reasons. Since the uncoupled lines are not lossy, the model of a *section* shown in Fig. 4(a) is used.

The recursive updating equations for coupled lossless lines are of two forms. Let all updating equations which must consider the transformation network M in them be termed terminal updating equations. The terminal updating equations of Fig. 4(b) are

$$\begin{aligned}[Ek[0](t)] &= -2(e(0,t) + [Em[0](t-\Delta h)]) \\ &= -2(M^{-1}v(0,t) + [Em[0](t-\Delta h)])\end{aligned} \tag{21}$$

$$[Em[n_i-1](t)] = -2(M^{-1}v(l,t) + [Ek[n_i-1](t-\Delta h)]). \tag{22}$$

$[Em[n_i-1](t)]$ is an $n \times 1$ vector whose entries are the final Em values to be updated for each uncoupled line. For $n_i \geqslant 2$, the remainder of the updating equations are not vectorized and thus, will be of the form

$$Em[j](t) = Em[j+1](t-\Delta h) \tag{23}$$

$$Ek[j](t) = Ek[j-1](t-\Delta h) \tag{24}$$

for each uncoupled line.

Transient Analysis of Lossy, Coupled Transmission Lines

To model a set of lossy coupled transmission lines, a series resistance is inserted between lossless coupled lines as shown in Fig. 4(c). A third form of updating equation is needed for the terminals of the internal coupled section:

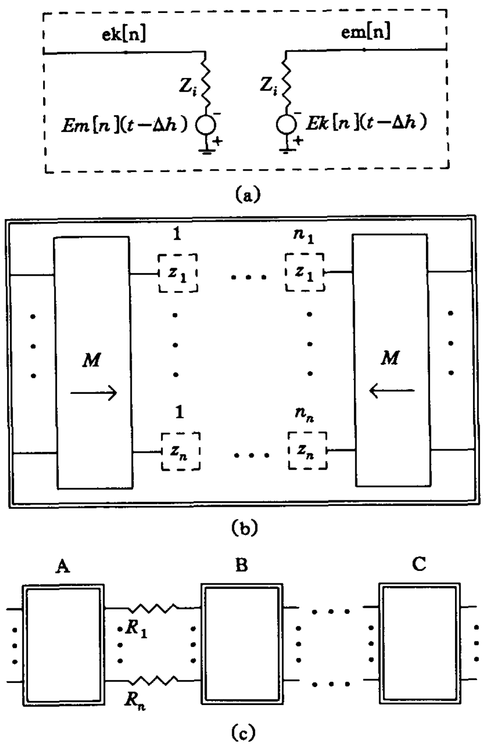

Fig. 4. (a) Equivalent impedance network of a section of one of the lossless decoupled lines. (b) Equivalent network of n lossless coupled lines using the model of (a). (c) Equivalent network of n lossy coupled lines using the model of (b).

$$[Ek_B[0](t)] = M^{-1}(I - A)M[Em_B[0](t-\Delta h')] + M^{-1}AM[Ek_A[n_i-1](t-\Delta h')] \tag{25}$$

$$[Em_B[n_i-1](t)] = M^{-1}(I - A)M[Em_B[n_i-1](t-\Delta h')] + M^{-1}AM[Ek_C[0](t-\Delta h')] \tag{26}$$

where I is the identity matrix, $\Delta h'$ is the updated stepsize, and

$$A = 2Zo\ (2Zo + Rdiag)^{-1}$$

where $Zo = MZdiagM^t$ and $Rdiag$ and $Zdiag$ are diagonal matrices of the series resistance and modal impedances, respectively.

The accuracy of the transient analysis depends on the the number of coupled sections as well as the number of internal sections in each coupled section. If N_C is the number of coupled sections, the stepsize for a set of lossy coupled lines is

$$x/v_i n_i N_C \approx x/v_j n_j N_C \approx \Delta h/N_C \approx \Delta h'. \tag{27}$$

The determination of a stepsize for lossless transmission lines is considered in [11, 18]. The number of coupled sections needed to accurately model the lossy transmission lines imposes an additional constraint on this process.

The analysis of lossy coupled lines in a homogeneous medium requires a subset of the updating equations of lossy coupled lines in an inhomogeneous medium. If the medium is inhomogeneous, three types of updating equations are needed: two types are of the terminal form (those of the form of (21) and (22) and those of the form of (25) and (26)) and the third type is of the form of (23) and (24). If the medium is homogeneous, all modal delays would be equal and thus, assuming the step size chosen is equal to the delay of one of the decoupled lines, $n_i = n_i' = 1$. In this case equations (23) and (24) would not apply.

IV. Determination of the Number of Sections

The incorporation of the resistive component of lossy lines into a method based on the method of characteristics allows one to represent a resistive line to any desired accuracy. For large nets it may not be practical to have the user determine the correct number of sections for each line. We present a hueristic method to find a lower bound on the number of sections a single line must be partitioned into, given the impedance and the total series resistance of a lossy transmission line, as well as, a user specified tolerance. The hueristic is generalized to coupled lossy transmission lines.

Given a general transmission line of length x, where l, c, and r are per unit length inductance, capacitance, and resistance, respectively, the input at any point z on an infinitely long line driven by a zero impedance step of amplitude one is [19]

$$v(z,t) = \left[e^{-rz/2Zo} + (rz/2Zo) \int_{t=z\sqrt{lc}}^{t} \frac{e^{-(r/2l)t}}{\sqrt{t^2-(z\sqrt{lc})^2}} \times I_1\left[(r/2l)\sqrt{t^2-(z\sqrt{lc})^2}\right] dt\right] \mu(t-x\sqrt{LC}) \tag{28}$$

where $I_1(y)$ is a modified Bessel function of the first kind

$$I_1(y) = -jJ_1(jy). \tag{29}$$

At $t = x\sqrt{lc}$, the time the waveform reaches the far end of the line, the voltage at the far end of the line is

$$v(x,t) = e(out) = e^{-R_t/2Zo} \tag{30}$$

where $R_t = rx$ is the total resistance of the line. If the method of characteristics is used to determine the voltage at the far end of the line at $t = x\sqrt{lc}$, we find

$$v(x,t) = e(out) = A^{s-1} \tag{31}$$

where $R = R_t/(s-1)$.

Having established a comparison point between theory and simulation methods based on the method of characteristics at time $t = x\sqrt{lc}$, it is possible to specify a tolerance, tol, on the accuracy of the waveform down the line. The tolerance is basically the voltage difference between expected (theory) and actual (simulation) for a normalized step voltage. Given a tolerance, the number of sections, s, can be increased until

$$tol \geqslant \left| e^{-R_t/2Zo} - \left[\frac{2Zo}{2Zo + R_t/(s-1)}\right]^{s-1} \right|. \tag{32}$$

For n lossy coupled transmission lines each line has on it the superposition of the n eigenmodes. Thus in the most general case we can use equation (32) with

$$R_t = \max\ Rdiag[i],\ 0 \leqslant i \leqslant n \tag{33}$$

and

$$Zo = \min\ Z'[i],\ 0 \leqslant i \leqslant n. \tag{34}$$

V. Transient Analysis Examples

Some experimental evidence of the validity of the application of the method of characteristics to lossy lines has been reported [12]. More experimentation is needed to verify that the quasi-TEM approximation is indeed valid for VLSI interconnections [20]. We will demonstrate the theory and techniques presented in this paper based on the quasi-TEM approximation.

We will first consider four uncoupled transmission line configurations. The output voltage response to a step input is shown in Fig. 5 for each of the four uncoupled transmission line configurations of Fig. 2 for two values of total resistance, $R_t = 25\Omega$ and $R_t = 100\Omega$. Recall, for configurations (3) and (4), R_t is the total series resistance of a 1 unit length conductor. Consider Fig. 5.1(a). The magnitude of the initial waveform at the end of the line is

$$2\,Zo\,e^{-R_t/2Zo}\,/\,(Zin+Zo).$$

Thus, the magnitude of the far end waveform is 1.04 and .49 for $R_t = 25\Omega$ and $R_t = 100\Omega$, respectively. To demonstrate the effect of varying the tolerance, *tol*, expanded views of selected waveforms are shown in Fig. 6.

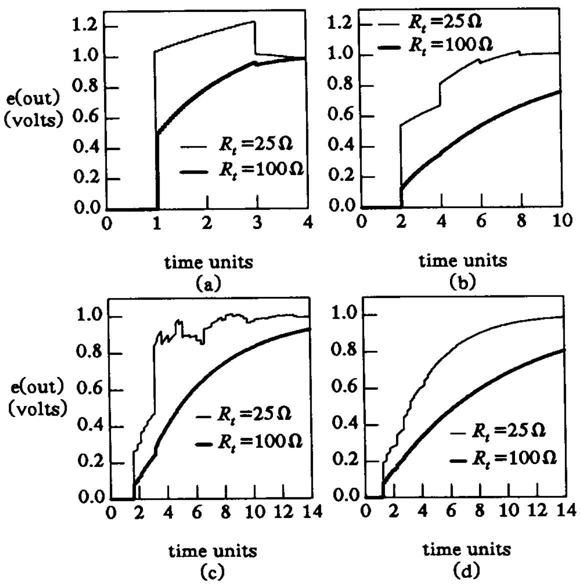

Fig. 5. Output response to a step input of Fig. 2 configurations (a) (1) (b) (2) (c) (3) (d) (4). The outputs shown for configurations (3) and (4) are marked by a '*' in Fig. 2. All transmission lines have 50Ω impedances. Each transmission line configuration is driven through a 25Ω impedance and the outputs are terminated in 1*M*Ω. For configurations (1) and (2), each transmission line has one unit delay. For configurations (3) and (4), one unit length of transmission line has one unit delay and all transmission lines are scaled accordingly. For example, the distance to the outputs in configurations (3) and (4) are 13/8 and 19/16, respectively.

The study of coupled lines is important not only in determining an accurate delay estimate, but also in ascertaining the impact of coupled noise on a design [21]. To demonstrate the significance of coupled noise we will consider the eight parallel coupled lines of coupled length .5cm shown in Fig. 7(a). The conductor configuration and dimensions are shown in Fig. 7(b). We will also consider two substrate types: gallium arsenide and silicon. We assumed the slow wave mode of propagation on silicon [22], that is, the substrate behaves as a perfect conductor for the electric field and as a poor conductor for the magnetic field [23]. The capacitance and inductance matrices were calculated using [16]. Fig. 8 shows the voltages at the endings of three of the eight parallel lines with the lines considered lossless ($R_t = 0\Omega$) or lossy ($R_t = 500\Omega$). As expected the resistive

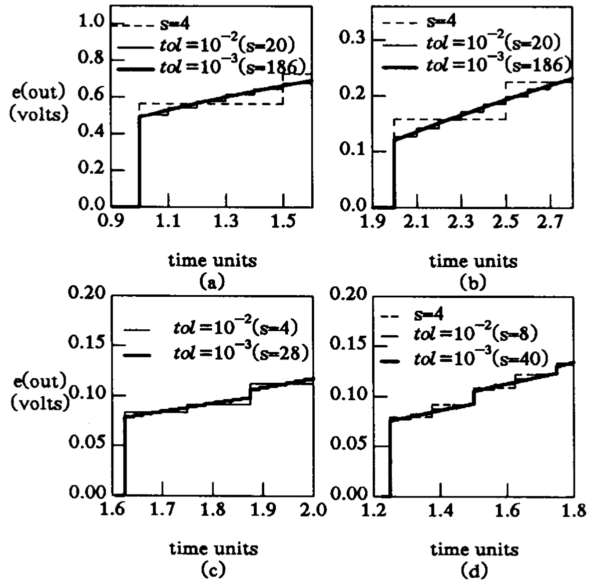

Fig. 6. Expanded views of the output waveforms shown in Fig. 5 (a) (1) (b) (2) (c) (3) (d) (4). R_t = 100Ω.

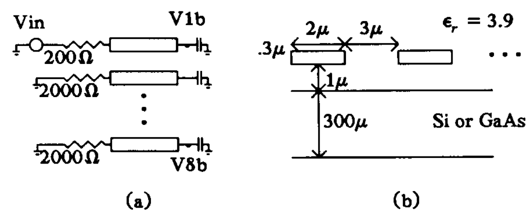

Fig. 7. (a) Eight parallel transmission lines of coupled length .5cm terminated in 20fF. (b) Cross sectional view of the conductors.

losses in the lines do have an effect on coupled noise. Also, as mentioned earlier, the technique which requires only adjacent conductor coupling would not be adequate to accurately model the coupled lines of Fig. 7(a), especially on an insulating substrate such as gallium arsenide. This is evident since the model would assume the voltage on the third line of Fig. 7(a) would be zero, which is not the case in Fig. 8(a).

VI. Conclusions

An improved method for the transient analysis of single *RLC* transmission lines based on the method of characteristics has been presented. The improved method has approximately twice the performance of prior methods. We have also presented a technique for the transient analysis of lossy coupled lines in inhomogeneous media which is characterized by multiple propagation modes of unequal phase velocities. A hueristic to predict the number of ideal transmission line sections a line must be partitioned into, and therefore the number of resistors which must be inserted into the line to model the resistive nature of the line, has also been developed. This hueristic may be easily incorporated into a transient analysis simulation program.

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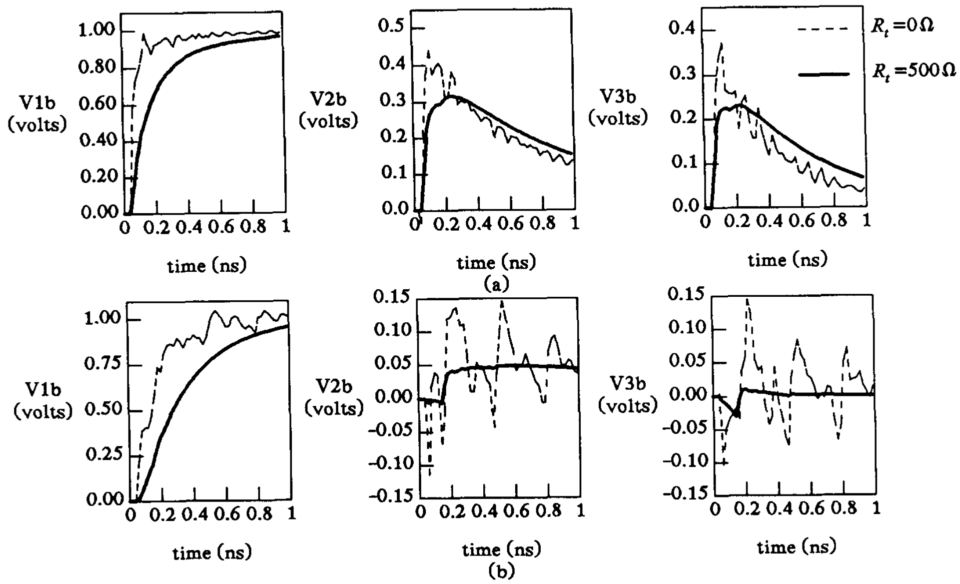

Fig. 8. Voltages at the endings of three of the eight lines shown in Fig. 7 with and without the series resistance included. (a) gallium arsenide (b) silicon. The input rise time is 10 psec.

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