PERFORMANCE ANALYSIS OF A STAR S-ALOHA PACKET RADIO NETWORK WITH DIFFERENT MODELS OF THE CENTRAL STATION

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In the paper a packet radio network with a Central Station/CS/ is presented and analysed. Four models of the CS with different buffer configurations and different service priorities are investigated. The throughput-delay performances are obtained by a steady-state analysis assuming the equilibrium conditions of a Markov chain. The performance parameters are particularly related to the local subnetwork in which a transmission strategy called the Station Priority Scheme is used. It is shown that the SPS that operates on a single slotted radio channel is an effective means of randomly accessing packet radio channels.

Introduction

The packet broadcasting technique offers an attractive solution for data exchange between different types of users. It brings together the advantages of both packet switching and broadcast communications. The decreasing cost of digital equipment has provided distant stations with many desirable distribution processing capabilities over LANs and MANs, such as communications with a central computer, and between fixed or mobile stations via a digital radio channel. When channel access is unpredictable, distributed and bursty random-access techniques are successfully applied. With these techniques users can share a common broadcast channel by transmitting packets without any coordination between them. In these schemes, collisions which destroy packets, may occur. Because collisions can occur, stations should be acknowledged whether or not their transmission attempts have been successful, so that appropriate action can be undertaken. In most studies it is assumed that acknowledgement packets are transmitted along a separate channel. In this paper we study the performance of a packet radio network which uses a single channel for bidirectional communications between local stations and the Central Station /CS/.

We consider a star packet radio network consisting of a large number of low-rate, bursty stations - possibly mobile - located or moving around the Central Station. The stations are assumed to be hidden one from another, so they can communicate only through the CS, which can hear the broadcast from any of the stations/or from other CSs over a separate channel/. Furthermore, the CS is assumed to be capable of storing in its buffers packets awaiting processing or retransmission. We investigate different models of the CS related to the different buffer configurations presented in Fig. 1.

![Fig. 1 Models of the Central Station.](image)

The CS retransmits the packets it receives and independently generates some traffic of its own. Depending on their later processing at the CS or retransmission to local stations, packets received from the local stations are stored in buffers B or B0. If they are relayed by the CS to local stations, the packets are inserted into buffer B. The CS maintains queues of packets to be relayed in FCFS order.

In the following we determine the maximum network throughput and the delay-throughput tradeoffs for different buffer configurations and different priorities given to the traffic components/local arrivals, traffic generated by the CS, arrivals from other CSs/.

Channel Access Scheme

In the paper we present and analyse the Station Priority Scheme [1], a transmission strategy based on the S-ALOHA protocol [2]. This transmission strategy is in some ways similar to the protocol presented in [3], where the single channel is shared by a large number of small users and one large user. The time axis is divided into slots of length T' = T + A, where T is the packet transmission time and A is the time needed to transmit ACK or to sense channel activity. For the sake of simplicity we neglect the packet processing time and the propagation delay. The amount of traffic generated by any pair of stations is assumed to be equal and the stations are identical, each with a single buffer for one data packet. Stations and the CS operate according to the Station Priority Scheme /SPS/ illustrated in Fig. 2.
This scheme states that:

- When a station has a data packet to transmit, it operates according to the S-ALOHA protocol and sends the packet at the beginning of a time slot [see Fig. 2].
- If a packet is not acknowledged at the CS, the station repeats its transmission after a randomly chosen delay.
- The CS transmits the ACK of a correctly received packet in the same time slot.
- If the CS has a packet to transmit it waits for a slot not used by stations.

Note that a station packet transmission is successful only if there is no collision and the CS has a buffer available for storage. The CS recognizes an empty slot by sensing the channel activity for a period $B$ at the beginning of a slot. The absence of the ACK in the time slot used by a station to transmit a packet signals the need for packet retransmission, thus minimizing the retransmission delay. Transmissions from the CS to local stations are not acknowledged because no collision can occur and the management of transmission errors is left to higher level protocols.

**Performance Analysis**

We consider four models of the CS corresponding to different configurations of the CS buffers and various functions realized by the CS [see Fig. 1a,b], and different priorities given to the traffic components service:

**Model 1:** Configuration a of buffers, shown in Fig. 1a, and the order of priorities of the buffer service: B, B1, B2;

**Model 2:** Configuration a of buffers and buffer service order: B2, B, B1;

**Model 3:** Configuration b of buffers [see Fig. 1b] and the buffer service order as in model 1;

**Model 4:** Configuration b of buffers and the buffer service order as in model 2.

In models 1 and 3 the highest transmission priority is given to packets from buffer B, in models 2 and 4 to packets from buffer B2. Additionally, in models 1 and 2 we assume that packets from buffer B1 can be inserted in buffer B in all slots, except those in which local packets enter buffer B /local packets successfully transmitted with the probability $\lambda_1 = \alpha_1$; a $\epsilon (0,1]$ or buffer B is full.

Let us start our considerations with the analysis of a simple model of the CS with a single buffer B for local packets only [1].

To study the behaviour of the network whose CS is provided with a buffer able to store up to $K$ packets and only relays the packets it receives, the simple Markov chain shown in Fig. 3 can be used [4]. In this model the state of the network is described by the number of packets in the CS buffer. The network throughput $S_K$ normalized with respect to one slot, and the average number of packets stored in buffer B, $N_K$, are given by:

$$S_K = \sum_{k=1}^{K} \frac{K}{(1 - \rho) - \rho K}$$

$$N_K = \frac{P_0}{1 - \rho}$$

where

$$P_0 = \frac{1 - \rho}{1 - \rho K}$$

Let $\rho$ be the probability that the station transmits during a slot. Given that buffer B's queue is not full, a successful arrival at the CS occurs when exactly one of the $N$ stations transmits during a slot. The transition probability from state $k$ to state $k+1$: $k=0,1,2,...,K-1$ is therefore expressed as

$$\lambda = N \rho (1 - \rho)^N$$

Similarly, the transition probability $\mu$ from state $k$ to $k-1$: $k=1,2,...,K$ is the probability that no station has transmitted in a slot.

$$\mu = (1 - \rho)^N$$

If the number of stations tends to infinity, while $p$ tends to zero, so that $N=p$, the offered traffic becomes Poisson and the values of $\lambda$ and $\mu$ reduce to $\lambda = G e^{-G}$ and $\mu = e^{-G}$. Under such conditions from eq. (1), the network throughput $S_K$ is

$$S_K = G e^{-G} \frac{1 - \rho}{1 - \rho K}$$

The delay performances of this system are simply obtained by observing that the total packet transmission delay, $D_p$, is given by the sum of the delay in reaching the CS buffer, $D_B$, and the delay to be transmitted by the CS to the stations, $D_B'$. The average delay
\( D_t \) is the same as in the S-ALOHA system, and from the results given in [5], [6] we have
\[
D_{t, K} = \left( \frac{G}{S_K} - 1 \right) (1 + \frac{R - 1}{2}) + 1 \tag{8}
\]
where \( R \) is the length, measured in slots of the transmission interval during which the collided packet’s retransmission must be completed. The average delay \( D_t \) is obtained by Little’s result and is given by
\[
D_{t, K} = \frac{N_K}{S_K} \tag{9}
\]
Note that both \( D_t \) and \( D_B \) are also functions of \( K \). In fact, as we have already mentioned, the CS buffer capacity \( K \) affects the channel traffic because, when full, it may cause packets to be retransmitted.

In the following we determine the performance parameters of models 1-4, particularly related to the local subnetwork. We accept memoryless properties for both the arrival processes and the service-time distributions for all analysed buffers. In all cases we assume that the input streams are Poisson arrival processes, while the service times are geometrically distributed.

**Model 1**

This case is depicted in Fig. 1a. To simplify our considerations we assume that the input streams with rates of \( \lambda, \gamma \) and \( \beta \) are independent Poisson processes. Furthermore we also assume that the service of possible rejections of packets from input streams of buffers \( B_1 \) and \( B_2 \) is left to other protocols, not considered in this paper. Under these conditions analysis of buffers \( B, B_1 \) and \( B_2 \) can be carried out independently in the same way as that presented for the simple CS model. The total intensity of the input stream of buffer \( B \) can be represented as the sum of the buffer \( B_1 \) output rate \( S_{K1}(\gamma) \) and the arrival rate \( \lambda \) from the local stations.

\[
\lambda' = \lambda + S_{K1}(\gamma) = \lambda + (1 - \lambda)(1 - P_{01}) \tag{10}
\]
where
\[
P_{01} = \frac{1 - \rho_1}{1 - \rho_1^2} \tag{11}
\]
and
\[
\rho_1 = \frac{\gamma}{1 - \lambda_1} \tag{12}
\]
Packets from buffer \( B_1 \) can enter buffer \( B \) only when there is no packet from a local station /received in a given slot/ directed to this buffer. The input rate \( \gamma \) and the buffer \( B_1 \) size will affect the performance of the traffic exchanged between the local stations. The total throughput as observed at buffer \( B \) output is as follows
\[
S_K = \mu (1 - P_0) = \mu \frac{1 - \rho}{1 - \rho K + 1} \tag{13}
\]
where
\[
\rho = \frac{\lambda_1 + S_K(\gamma)}{\mu} \tag{14}
\]
The throughput corresponding to the local traffic can be evaluated as
\[
S_{K1}(\gamma) = \frac{1}{\lambda + (1 - \lambda)} \tag{15}
\]
The output rate from buffer \( B_2, S_{K2}(\beta) \), is given by
\[
S_{K2}(\beta) = \mu_2 (1 - P_{02}) = \mu_2 (1 - P_{02}) \tag{16}
\]
where
\[
\beta_2 = \frac{\beta}{\mu_2} \tag{17}
\]
We assume that in the same slot it is possible to send a packet from buffer \( B_2 \) or \( B \) and insert a packet from buffer \( B_1 \) in \( B \). Thus the total network throughput is
\[
S_C = S_K + S_{K2}(\beta) = \mu_1 (1 - P_{01}, P_{02}) \tag{18}
\]
To complete the delay analysis, we need to evaluate:
1 - for local packets - the total delay, given by the sum of \( D_t \) and \( D_B \) /as in the simple model of the CS/
\[
D_{t, K} = \left( \frac{G}{S_K} - 1 \right) (1 + \frac{R - 1}{2}) + 1 \tag{19}
\]
where
\[
a = \frac{\lambda_1}{\lambda} \tag{20}
\]
and
\[
D_{B, K} = \frac{N_K}{S_K} /\text{see eq. (9)/}
\]
2 - for packets entering buffer \( B_1 \) - the total delay \( D(\gamma) \), expressed as the sum of the delay \( D_{B1} \) which packets have spent in buffer \( B_1 \) and \( D_B /\text{see eq. (9)/} \)
\[
D_{B1,K1} = \frac{N_{K1}}{S_{K1}} \tag{21}
\]
The average number of packets in buffer \( B_1, N_{K1}, \) is given by eq. (2) /for \( \rho \) substituted by \( \rho_1 /\).
3 - for packets entering buffer \( B_2 \) - the delay is equal to
\[
D(\beta) = D_{B2,K2} = \frac{N_{K2}}{S_{K2}} \tag{22}
\]
where \( N_{K2} \) can be expressed as in eq. (2) /for \( \rho = \rho_2 /\).

**Model 2**

In this case the highest transmission priority is given to traffic from other CSs. Applying to this model the same analysis as
presented for model 1 we get

a) the average throughput \( S_{K2}(\beta) \) /the intensity of packets leaving buffer B2/

\[
S_{K2}(\beta) = \mu (1 - P_{02}) = \mu \frac{\beta_2}{1 - \beta_2} \quad (23)
\]

where

\[
\beta_2 = \frac{\beta}{\mu} \quad (24)
\]

b) the output rate, \( S_{K1}(\gamma) \), from buffer B1

\[
S_{K1}(\gamma) = (1 - \lambda_1)(1 - P_{01}) \quad (25)
\]

where

\[
\lambda_1 = \frac{1}{\mu} \quad (26)
\]

c) the total throughput, \( S_K \), as observed at buffer B output

\[
S_K = \mu P_{01}(1 - P_0) = \mu P_0 \frac{\beta - \beta_2}{1 - \beta_2} \quad (27)
\]

where

\[
\beta = \frac{\lambda_1}{\mu} = \frac{\lambda_1 + S_{K1}(\gamma)}{\mu} \quad (28)
\]

Thus the total throughput, \( S_C \), in the output channel to local stations takes the form

\[
S_C = S_K + S_{K2}(\beta) = \mu (1 - P_{02}) \quad (29)
\]

Average packet delays related to different packet streams can be evaluated in exactly the same way as in model 1.

Model 3

In this model, shown in Fig. 1b, we introduce a small modification in the buffer configuration as compared to models 1 and 2. Carrying out the same analysis as previously we get

a) the throughput, \( S_K = S_{K1}(\gamma) \), corresponding to the local traffic

\[
S_{K1}(\gamma) = \mu (1 - P_{01}) = \mu \frac{\beta_1 - \beta_1 K_{1+1}}{1 - \beta_1} \quad (30)
\]

where

\[
\beta_1 = \frac{\lambda_1}{\mu} \quad (31)
\]

b) the output rate, \( S_{K1}(\gamma) \), from buffer B1

\[
S_{K1}(\gamma) = \mu P_0(1 - P_{01}) = \mu P_0 \frac{\beta_1 - \beta_1 K_{1+1}}{1 - \beta_1} \quad (32)
\]

where

\[
\beta_1 = \frac{\gamma}{\mu} \quad (33)
\]

c) the throughput, \( S_{K2}(\beta) \), of packets leaving buffer B2

\[
S_{K2}(\beta) = \mu P_0 P_{01}(1 - P_{02}) = \mu P_0 \frac{1 - P_0}{1 - \beta_2 K_{2+1}} \quad (34)
\]

where \( P_0, P_{01} \) and \( P_{02} \) are the probabilities that buffers B, B1 or B2 respectively, are empty, while

\[
\beta_2 = \frac{\gamma}{\mu} \quad (35)
\]

The total throughput, \( S_C \), for model 3 is

\[
S_C = \mu (1 - P_{01} P_{02}) \quad (36)
\]

To complete the delay analysis we need to evaluate

1) for local packets: the total delay, given by the sum of components \( D_t \) and \( D_B \) as in model 1/

2) for packets entering buffers B1 and B2: the delays the packets have spent in these buffers /expressed by Little's result/.

Model 4

In this model, as in model 2, the highest service priority is given to buffer B2, with packets from other CSs. Applying the same analysis as described previously, we get the following results:

a) the throughput from buffer B2

\[
S_{K2}(\beta) = \mu (1 - P_{02}) \quad (37)
\]

/see also eqs. (16) and (23)/

b) the throughput corresponding to the local traffic

\[
S_{K1}(\gamma) = \mu P_{02}(1 - P_{0}) \quad (38)
\]

c) the throughput as observed at buffer B1 output

\[
S_{K1}(\gamma) = \mu P_{01}(1 - P_{01}) \quad (39)
\]

The delay performance of this model is expressed by the same formulae as in model 3.

Results

In this section the performances of the considered models of the CS are examined and compared. We analyse the parameters of a two-hop network in terms of maximum achievable throughput and the expected packet delay with respect to the local subnetwork.

In Fig. 4 we plot the total network throughput, \( S_C \), as observed in the CS-to-local stations channel, versus for total traffic \( G \) offered by local stations, for model 1, when \( \alpha = 0.5 \) and \( 0.75 \). In this and all the figures presented in this section the buffer capacities \( K, K_1 \) and \( K_2 \) are assumed to be identical and equal to 10.

The results show that the network throughput depends heavily on the values of \( \gamma \) and \( \beta \) as well as the parameter \( \alpha = \lambda_1/\lambda \). It is not surprising that higher values of \( S_C \) are achieved for higher input rates \( \gamma \) and \( \beta \).
The maximum values of $S_C$ correspond to rather low values of $G$. For instance, for $\gamma = 0.4$, $\beta = 0.2$ the network throughput reaches its maximum values $0.72$ and $0.68$, at $G = 0.3$, for $a = 0.75$ and $0.5$ respectively. We observe that the network throughput is a decreasing function of $\gamma$, $\beta$ and $a$. This is due to the fact that the buffer queues become longer and the probability that an empty slot, not used by the local station, will not be used by the CS decreases with the increase of these parameters. Note that for $\gamma = \beta = 0$ all the models considered reduce to the simple one shown in Fig. 3.

We therefore next examine the variations of different components of the network throughput. Fig. 5 shows the particular throughputs as functions of the offered traffic $G$ for $a = 0.5$. It can be seen that the throughputs $S(\beta)$, $S(\gamma)$, and $S(\delta)$ behave in completely different ways when the order of service priorities changes.

In the case of model 1, for relatively small values of $G$ or $\gamma$, buffer $B$ is mainly occupied by packets that entered it from buffer $B_1$. For these values of $G$, the probability that any packet from buffer $B_1$ will be successfully transmitted is very close to 1 ($S(\gamma) = 1$). However, even for large values of $G$, the deterioration of $S(\gamma)$ as compared to $\gamma$ is relatively small. On the other hand, when the channel becomes heavily loaded, the output stream from buffer $B_2$ falls dramatically. Next, in model 2, we observe for large values of $\beta$ (e.g., $\beta = 0.4$) a significant degradation of the local throughput $S(\beta)$, especially for values of $G$ greater than 0.5. In this model the highest service priority is given to arrivals from other Central Stations (buffer $B_2$). The increase of intensity $\beta$ accelerates overloading of the local channel as the buffer $B$ fills up. Under these conditions the output stream from buffer $B_1$ falls quickly, reaching very small values at $G = 1$. In contrast to model 1, the throughput $S(\delta)$ is constant over a wide range of values $G$ and $\beta$. $S(\delta)$ behaved similarly in models 3 and 4. The most evident differences between models 1 and 3 or 2 and 4 are only reported in the case of $S(\gamma)$. For the buffer capacities $K = K_1 = K_2 = 10$ assumed in the figures, the differences between the local throughput values $S(\delta)$ can be neglected. These differences would be evident for smaller buffer sizes (e.g., $K = 1, 2$).

One of the principal concerns in this section is to investigate tradeoffs between the average packet delays and the local or total network throughput. Fig. 6 shows the station buffer delay $D_B$, the station transmission delay $D_{tr}$, and the total delay, $D(\delta)$, for local packets versus the local throughput $S(\delta)$. In this figure we assume $\alpha = 0.75$, $\gamma = \beta = 0.2$.

The best performance is obtained with model 3, the worst one with model 2.
The next figure /Fig. 7/ shows variations of packet delays, D(1), D(γ), D(8), in appropriate packet streams versus the total network throughput S. These delays depend very strongly on the CS model.

Once more the advantages of model 3 from the local stations point of view are demonstrated. In both cases /Figures 6 and 7/ the numerical values have been computed by setting R = 15, which is a value large enough to almost reach the maximum throughput /maximum input rate λ/ in the slotted ALOHA channel [5], [6].

Fig. 7. Variations of packet delays, D(1), D(γ), D(8) vs. total network throughput.

As expected, minimum transmission delays are observed in relation to packets from buffer B2 in models 2 and 4. By comparison with these models, packets from B2 buffer suffer the biggest delay in models 1 and 3. From the results presented in Fig. 7 it follows that the average delay D(γ) that packets generated by the CS spend in buffers B1 and B is less than D(1) for model 1. Note that the local packets are, in this model, privileged in accessing buffer B. On the other hand, packets from B1 can enter buffer B in all slots, except those in which successful local transmissions to buffer B have taken place. The delay D(γ) becomes much bigger when model 3 is used.

Conclusions

We have presented and analysed a packet radio network with a Central Station. Four models of the CS with different buffer configurations and different service priorities have been investigated. In all cases the memoryless priorities of both arrival processes and the service-time distributions are accepted. The throughput-delay performances are obtained by a steady-state analysis assuming the equilibrium conditions of a Markov Chain. The performances are particularly related to the local subnetwork in which a transmission strategy called the Station Priority Scheme is used. This channel access scheme, utilized by local stations and the CS, operates on a single slotted radio channel. It has been shown that the SPS is an efficient means of randomly accessing packet radio channels. The performance parameters of the SPS strategy presented in the paper have been normalized with respect to the slot T/4. However, it is evident that for A < T, the use of a single channel for bidirectional communications ensures a very high channel utilization, almost instantaneous acknowledgements, and simplifies the station equipment.

The models of the CS presented in the paper can be successfully applied in fixed and mobile station environments.

References