Abstract

One of the central questions in topology is determining whether a given curve is knotted or unknotted. An algorithm to decide this question was given by Haken in 1961, using the technique of normal surfaces. These surfaces are rigid, discretized surfaces, well suited for algorithmic analysis.

Any oriented surface without boundary can be obtained from a sphere by adding “handles”. The number of handles is called the genus of the surface, and the smallest genus of a spanning surface for a curve is called the genus of the curve. A curve has genus zero if and only if it is unknotted. Schubert extended Haken’s work, giving an algorithm to determine the genus of a curve in any 3-manifold.

We examine the problem of deciding whether a polygonal knot in a closed triangulated three-dimensional manifold bounds a surface of genus at most $g$. 3-MANIFOLD KNOT GENUS. Previous work of Hass, Lagarias and Pippenger had shown that this problem is in PSPACE. No lower bounds on the running time were previously known. We show that this problem is NP-complete.