Validation of a Simple Relay-based PID Autotuner with Specified Modulus Margin

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***Abstract*— The aim of this paper is to present the validation of a simple relay-based PID autotuner with user-specified modulus margin. The testing is made on relevant benchmark systems proposed for evaluating PID controller design methods. Based on prior art results, the performance of the autotuner is then compared to other PID controllers from literature.**

Keywords- autotuner, relay feedback, modulus margin, Nyquist diagram, PID controller, robustness.

I. INTRODUCTION

Development of control theory goes far back in time as to the late eighteenth century, having played a vital role in the advance of science and engineering [1]. Since the beginnings of process control many sophisticated methods have been constantly developed in theory. Judging by automation literature one would think that advanced control strategies such as supervisory, adaptive, model predictive control should be the norm of industrial control. However a survey by Desborough and Miller (2001) has shown that 97% of regulatory controllers are of the proportional-integral-derivative (PID) type and the majority of them are very poorly tuned [3].

The proportional-integral-derivative (PID) control schemes in industrial applications were first introduced in 1939 [4]. The PID is a generally applicable technique that offers the possibility to control both transient and steady state behavior providing thus an efficient and generic solution to real-world control problems, deriving its success from simple and easy to understand operations [5].

Given the popularity of the PID regulators in industrial control, the problem of determining their parameters becomes thus of great importance in the professional control domain. To obtain appropriate controller parameters one can proceed with available analytical design methods or tuning rules. Mostly the analytical design methods assume known plant models [6],[1],[7], while the tuning rules assume known process responses [8],[9] and known plant models [10]. Every analytical design method and every tuning rule, however, have some particular conditions concerning the plant models, such as dead time, fast and slow poles, real and complex conjugated zeros and poles. These conditions make the design methods and tuning rules non-general. In 2000, Åström and Hägglund [11] proposed a collection of systems which are a departure point for testing PID controllers. These collected systems are considered benchmark problems for evaluating post-proposed PID controller design methods [12].

Process identification is usually a burden and remains a time consuming task. To simplify this task and to reduce the time necessary for it many controllers nowadays incorporate autotuning capabilities. This is the ability to compute automatically (sub-)optimal parameters for the controller when connected to the plant.

In what concerns autotuning capabilities there are numerous design methods. Some of these methods use the identification of one or more points on the process frequency response, while others use the knowledge of some characteristic parameters of the process open-loop step response. This information offers a rough model of the process which is then the basis of an algorithm for tuning the controller parameters. A *specific class* of autotuners use relay feedback in order to obtain information about the frequency response of the system [3],[13].

Based on prior results, the validation of a novel relay-based autotuner and a comparison with other methods made on Åström's benchmark systems [11] make the subject of this paper. The brief description of the algorithm is presented in the next section. To evaluate its performance the proposed method is applied to relevant plants, including several benchmark systems for PID control presented by Åström and Hägglund [11], in the third section. The evaluation of the new autotuner against other good-performance PIDs existing in current literature [5], [12], [14], [15] is made in the fourth section of this paper. A conclusion section summarizes the main outcome of this work.

II. A NEW AUTOTUNING APPROACH

Controller autotuning is based on the idea of exposing the process to an experiment, during the normal operation. This requires careful design of the excitation signal, to avoid de-stabilizing the process from its operating point. Traditional relay-based autotuning methods such as Åström - Hägglund (AH) autotuner and Phase-Margin (PM) autotuner, both described in [16],[17],[18],[19],- identify one point on the Nyquist curve of the process P: the intersection of the process beeline with the negative real axis, ref. Fig. 1.

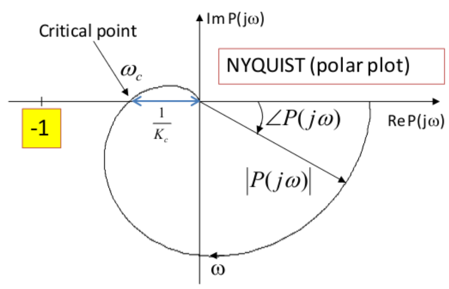

Figure 1. The process Nyquist plot and the intersection of the beeline with the negative real axis.

Using an appropriate PID controller C, this point is then moved to a specific location in the complex Nyquist plane; e.g. for the AH autotuner, the beeline of C*P goes through the specific point -0.6-0.28j (distance to the point -1 is 0.5) and for the PM autotuner, the beeline of the C*P goes through the point with modulus '1' and phase '180°+specified PM'. The insights from Ionescu and De Keyser [20] show that the specification of only 1 point in the Nyquist plane might be sufficient for some type of processes, but might as well result in poor (low) modulus margin for other types of systems. It is generally valid that a robustness less than 0.5 is obtained for integrating processes when using the AH method, while using the PM method makes the system either unstable or cross the unit circle very close to the critical point -1 for the case of processes with large time delay [20].

Our novel autotuning algorithm has been described extensively in [2]. It is based on imposing a user-specified robustness. The robustness specification can be translated using Nyquist plots as a circle of specified radius (r) around the point -1 as in Fig. 2 (r=the Modulus Margin, 0<r<1).

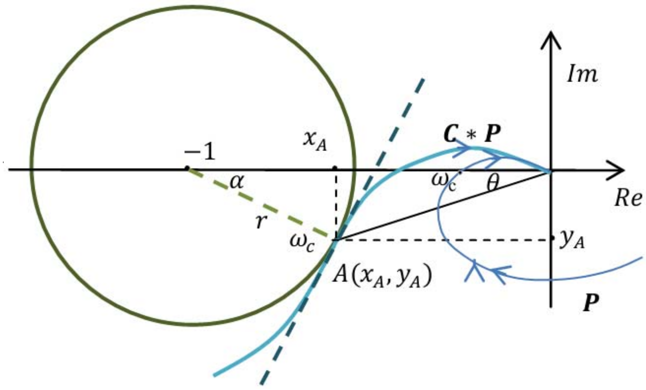

Figure 2. Novel principle for PID autotuning

In order to obtain a fluent curve for C*P, going smoothly around the circle with user-specified radius r, we impose that at the critical frequency $\omega_c$the C*P curve will be tangent to the robustness circle, resulting in the following equations at point A:

$$\begin{cases} M_A \cdot e^{j\cdot\varphi_A} = M_{(P\cdot C)}(j\omega_c)\cdot e^{j\cdot\varphi_{(P\cdot C)}(j\omega_c)} & (1) \\ \left.\frac{dM}{d\varphi}\right|_A = \left.\frac{dM_{(P\cdot C)}}{d\varphi_{(P\cdot C)}}\right|_{\omega=\omega_C} & (2) \end{cases}$$

That is: the modulus and the phase at point A should be equal to the modulus and the phase of the process plus controller at ω_c. Also the value of the derivative to the circle in A should equal the derivative of the C*P curve at ω_c.

As fully described in [2], to obtain the parameters of the PID controller a numerical algorithm is used, since an analytical solution for the system of equations (1)+(2) cannot be obtained.

By finding iteratively the angle $\propto *$ for which the error $\left|\left.\frac{dM}{d\varphi}\right|_A - \left.\frac{dM_{(P\cdot C)}}{d\varphi_{(P\cdot C)}}\right|_{\omega=\omega_C}\right|$ is minimum, the optimal parameters of the controller for a specified modulus margin r are found to be:

$$K_p = \frac{1 - rcos(\propto^*)}{M_P(j\omega_c)} \qquad (3)$$

$$T_d = \frac{M_A(\propto^*) + rsin(\propto^*)}{2\omega_c(1 - rcos(\propto^*))} \qquad (4)$$

$$T_i = 4T_d \qquad (5),$$

with the form of the controller being:

$$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right) = K\frac{(s+z)^2}{s} \qquad (6)$$

Necessary process information such as the critical frequency ω_c and the value of the modulus and the phase of the process at ω_c, $M_P(j\omega_c)$ and $\varphi_P(j\omega_c)$, are obtained via the usual relay test [3]. The same relay test, but with a small time delay, is used to find another set of data at a frequency very close to the critical one, in order to compute the value of the derivative of the process at ω_c.

III. TESTCASE SYSTEMS

The algorithm has been validated on some relevant systems and for different robustness specifications in [2]. In the current paper, it is further validated on an extensive set of benchmark systems. Åström and Hägglund [11] have proposed 11 types of benchmark systems for evaluating PID controllers. The first 5 systems are standard that are well suited to parametric studies. The remaining part of benchmark systems are more specialized. They illustrate systems with various difficulties of control. PID control is not suited for all of them [14]. These systems (and similar ones) have served as reference for many previous papers including PID control design via genetic algorithms [12], robust PID controller tuning methods [15], multioptimal PID design with PIDeasy formulas [5] and the extensions to Ziegler-Nichols tuning rules proposed by Vesely [14]. Our autotuner will be tested against reference controllers from these papers focusing on the transfer functions that are similar or identical to the benchmark systems.

In this paper we will compare the above mentioned methods with the newly proposed autotuning algorithm, both for standard and more specialized testcases. An important remark to keep in mind is that the described method does not require the knowledge of the transfer function of the process!

A. Standard testcases

The first systems under discussion represent a selection of basic reference examples for PID design method evaluation and can be found in various recent studies. Systems (7) to (11), together with the evaluation of the new autotuner on Nyquist curves are depicted below. The controllers are designed for two different robustness specifications, a smaller modulus margin of 0.55 and a more exigent value of 0.7 (see figures 3 - 7).

$$G_{P_1} = \frac{1}{(1+s)(1+as)(1+a^2s)(1+a^3s)} \qquad a = 0.5 \quad (7)$$

$$G_{P_2} = \frac{1}{(s+1)^2} e^{-0.2s} \quad (8)$$

$$G_{P_3} = \frac{1}{(s+1)^2(s+4)} \quad (9)$$

$$G_{P_4} = \frac{-0.2s+1}{(s+1)^2} e^{-0.1s} \quad \text{(nonminimum phase system)} \, (10)$$

$$G_{P_5} = \frac{1}{(s+1)^8} \quad (11)$$

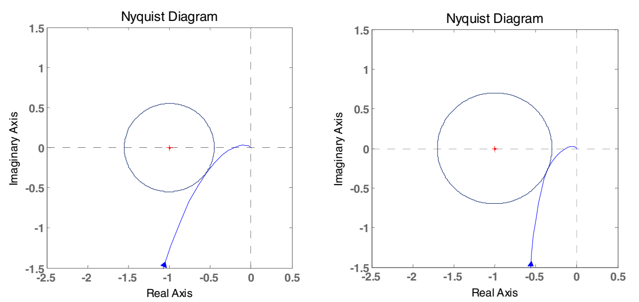

Figure 3. Process (7); MM=0.55 (left) and 0.70 (right)

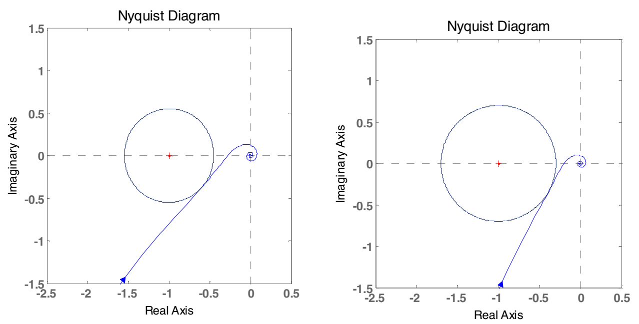

Figure 4. Process (8); MM=0.55 (left) and 0.70 (right)

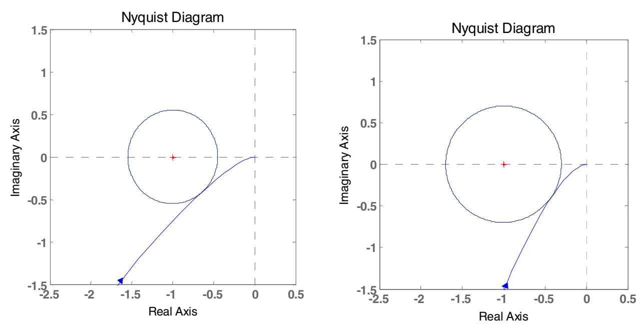

Figure 5. Process (9); MM=0.55 (left) and 0.70 (right)

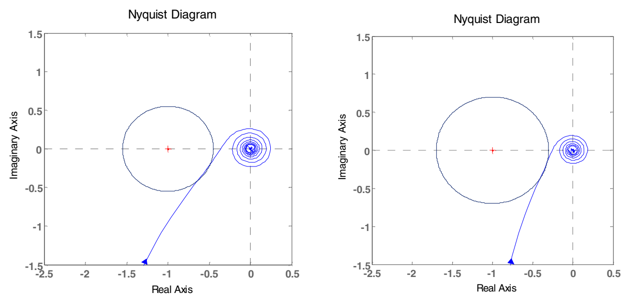

Figure 6. Process (10); MM=0.55 (left) and 0.70 (right)

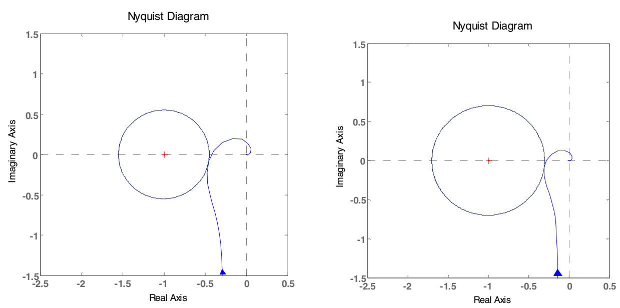

Figure 7. Process (11); MM=0.55 (left) and 0.70 (right)

B. Specialized testcases

The new tuning method has also been tested on systems that do not make the subject of simple PID design methods. Similarly to the previous subsection, three examples of such benchmark systems (transfer functions (12) – (14)), along with the Nyquist validation for the two different robustness specifications are shown in figures 8 - 10.

$$G_{P_6} = \frac{(s+6)^2}{s(s+1)^2(s+36)} \quad (12)$$

The above system has a pure integrator in the process, which results in a double integrator in the loop. These types of systems are usually representative for positioning control.

$$G_{P_7} = \frac{{\omega_0}^2}{(s+1)(s^2+2\xi\omega_0 s+{\omega_0}^2)} \quad (13)$$

$\xi = 0.1; \; \omega_0 = 1,2,5,10$

The oscillatory system from (13) has a small damping ξ. This system is easy to control for large ω_0 [11]. In this paper we will consider the case ω_0=1.

$$G_{P_8} = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \qquad (14)$$

The dynamics of system (14) has a fast mode with time constant 1 and a moderate gain (of value 1) and a slow pole with time constant 20 and a large gain (of value 20). Simple tuning rules do not normally give good results for systems of this type because it is difficult to get a good estimate of the gain and the time constant[11].

Another problematic system proposed by Åström and Hägglund [11] is an unstable second order system. It should be mentioned that since the novel autotuner computes the tuning parameters using the critical frequency of the process, it is not a proper method for controlling second order systems which do not have a finite value for ω_c.

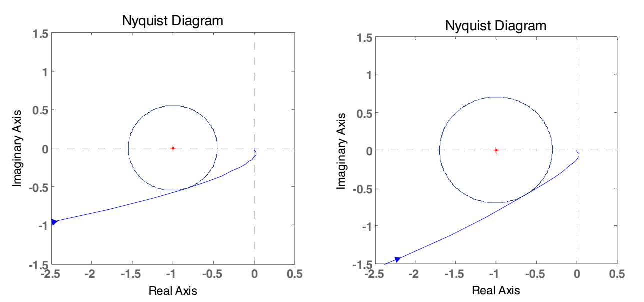

Figure 8. Process (12); MM=0.55 (left) and 0.70 (right)

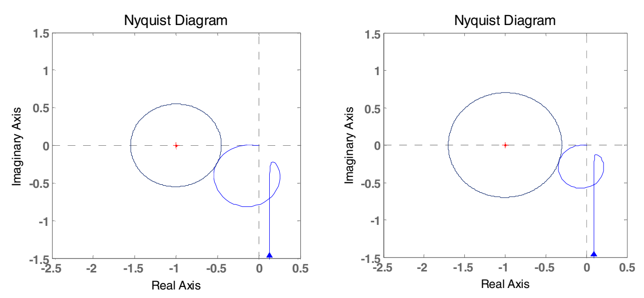

Figure 9. Process (13); MM=0.55 (left) and 0.70 (right)

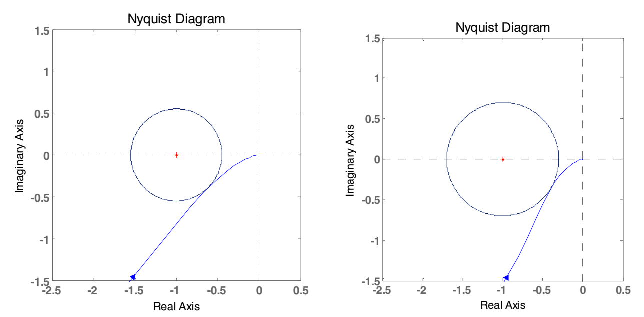

Figure 10. Process (14); MM=0.55 (left) and 0.70 (right)

IV. COMPARISON TO OTHER METHODS

The proposed controller will further be compared against reference controllers from recently developed PID tuning literature [5],[12],[14],[15]. The comparison will be made for setpoint trajectory and for input disturbance rejection.

The fourth order system from (7) is the first transfer function used to perform the tests. The system has four poles with their location being determined by a. In this work a is set to 0.5.

A multioptimal PID designed with PIDeasy as described in [5] and a controller developed with a genetic algorithm (GA) method as in [12] were compared with two different specifications for the autotuner. The responses for the setpoint step of value +1 and for an input disturbance step of value -1 are depicted in Fig. 11.

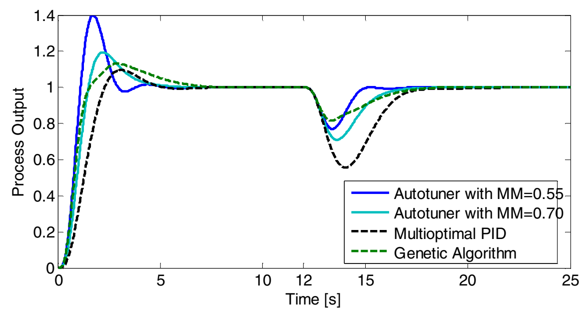

Figure 11. Process (7); unit step response and disturbance rejection

As it can be seen the user can choose to compromise overshoot over faster disturbance rejection by specifying a smaller modulus margin. The overshoot at a setpoint step can also be reduced by either setpoint-filtering or by using the derivative action in the feedback path. In this sense the proposed method is found to be more flexible and easily adaptable than the other methods, yielding similar or better results.

A generic second order system plus time delay from (8) is compared in a similar manner with a ‘robust’ PID controller developed by Garcia et al. [15] and a PID controller using Kappa Tau tuning rules [21], an empirical method that also uses the modulus margin as a design parameter. The autotuner outperforms the PIDs giving similar results as in the previous example (see Fig. 12).

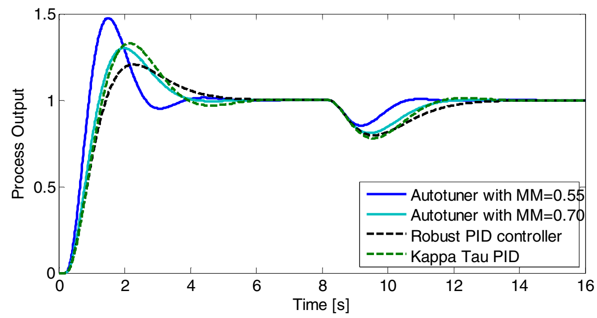

Figure 12. Process (8); unit step response and disturbance rejection

The same good behaviour can be observed for the systems from (9) and (11), as depicted in Fig. 13 - 15.

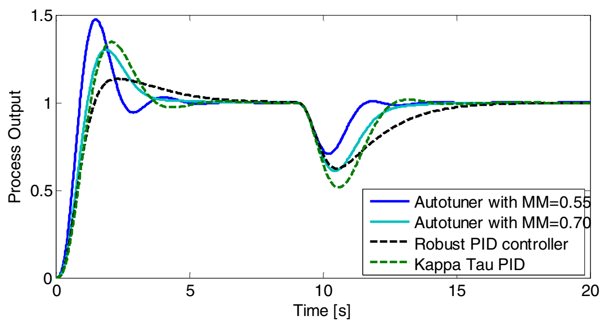

Figure 13. Process (9); unit step response and disturbance rejection

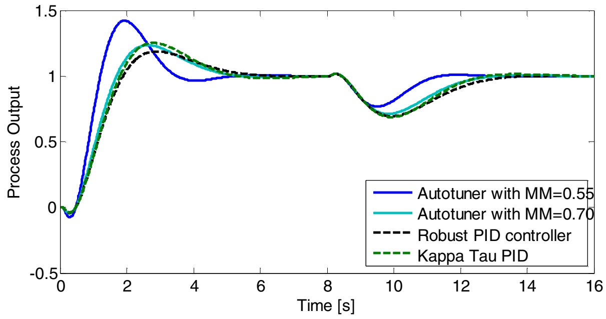

Figure 14. Process (10); unit step response and disturbance rejection

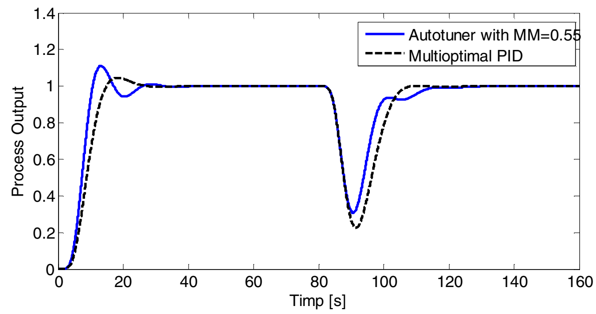

Figure 15. Process (11); unit step response and disturbance rejection

In Fig. 16 an autotuner with modulus specification of 0.70 designed for the marginally stable system from (12) is compared against a reference controller based on Ziegler-Nichols tuning rules, as proposed by Vesely [14]. The better performance of the proposed method can be seen both in settling time and disturbance rejection.

The following testcase belongs to the class of poorly-damped oscillatory systems with the transfer function from (13). For comparison purposes we use a genetic algorithm PID fully described in Garcia et al. [15].

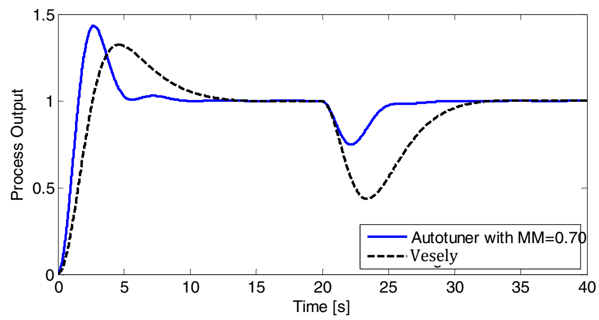

Figure 16. Process (12); unit step response and disturbance rejection

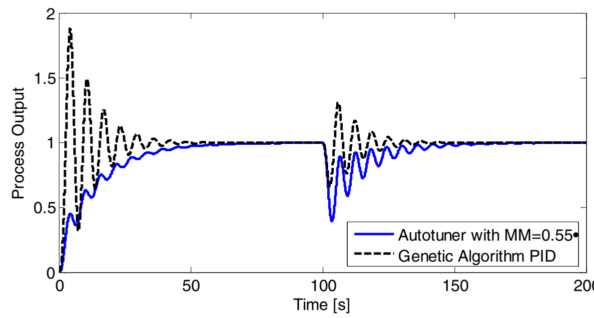

Figure 17. Process (13); unit step response and disturbance rejection

A reasonable performance of the proposed method can be observed, having no overshoot and similar disturbance rejection settling time (see Fig. 17). However, it is well known that a PID is not the most appropriate approach for controlling such poorly damped systems.

The performance of system (14) will be compared against a 'best-performance' controller designed using FRTool, a Computer-Aided-Design tool, based on the full knowledge of the system transfer function, as described in [22]. It is remarkable that the novel PID gives similar results as the CAD design, considering that the autotuning algorithm computes the parameters of the controller without knowing the transfer function of the process.

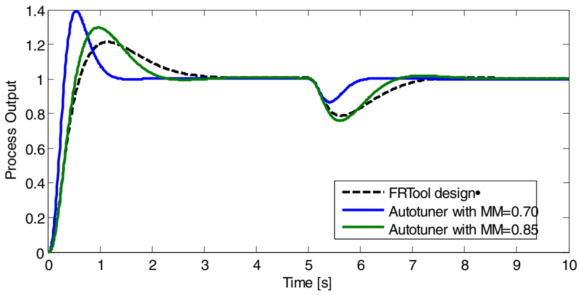

Figure 18. Process (14); unit step response and disturbance rejection

For the presented testcases input disturbance steps of value -1 have been used, with the exceptions of systems (10) and (13), where values of -10 and -0.5 were used respectively.

The parameters of the controllers presented in this paper are given in Table I. The forms considered for the transfer function of the PID controllers are as in (6).

TABLE I. PID controller parameters

Process	PID Controller Parameters					
	Tuning Method	K_p	T_I	T_D	K	z
$G_{P_1}(s)$	Autotuner MM=0.55	3.71	1.22	0.30	1.13	-1.64
	Autotuner MM=0.70	2.41	2.78	0.69	0.87	-1.39
	Multioptimal PID	1.19	1.36	0.17	0.20	-1.47
	Genetic Algorithm	3.78	1.75	0.61	2.31	-1.14
$G_{P_2}(s)$	Autotuner MM=0.55	6.33	1.16	0.29	1.84	-1.72
	Autotuner MM=0.70	4.09	1.41	0.35	1.44	-1.42
	Robust PID	3.57	1.64	0.41	1.46	-1.22
	Kappa-Tau	3.39	1.31	0.32	1.08	-1.53
$G_{P_3}(s)$	Autotuner MM=0.55	30.87	1.23	0.31	9.74	-1.63
	Autotuner MM=0.70	20.99	1.53	0.38	8.01	-1.31
	Robust PID	19.9	2.10	0.53	10.55	-0.95
	Kappa-Tau	16	1.40	0.34	5.44	-1.43
$G_{P_4}(s)$	Autotuner MM=0.55	3.78	1.34	0.33	1.26	-1.49
	Autotuner MM=0.70	2.42	1.56	0.39	0.94	-1.28
	Robust PID	2.15	1.64	0.41	0.88	-1.22
	Kappa-Tau	2.11	1.45	0.37	0.77	-1.38
$G_{P_5}(s)$	Autotuner MM=0.55	0.87	6.20	1.55	1.35	-0.32
	Multioptimal PID	0.5	4.31	1.01	0.51	-0.46
$G_{P_6}(s)$	Autotuner MM=0.70	3.77	2.74	0.68	2.58	-0.73
	Ziegler-Nichols	1.51	4.35	1.09	1.65	-0.46
$G_{P_7}(s)$	Autotuner MM=0.55	0.22	2.86	0.71	0.16	-0.70
	Genetic Algorithm	0.24	0.20	5.08	0.05	-10.0
$G_{P_8}(s)$	Autotuner MM=0.70	6.365	0.43	0.11	0.69	-4.64
	Autotuner MM=0.85	2.85	0.56	0.14	0.40	-3.57
	FRTool	3.20	0.75	0.19	0.6	-2.67

V. CONCLUSIONS

The validation of a novel relay-based PID autotuner has been presented in this paper. The main characteristic of the autotuner is perhaps its simplicity: the user has to specify only one design parameter - the desired modulus margin - which has a clear interpretation as a trade-off between performance and robustness. The PID autotuner has been successfully tested on several examples which have been published as benchmark systems in the control literature. Based on the results from a series of simulations the proposed PID tuning method is found to yield satisfactory performance compared to other methods which have appeared in the control literature.

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