

# A new high-speed general purpose I / O with real-time computing capability

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## INTRODUCTION

Real-time data acquisition and control systems incorporating a general purpose digital computer (GPC) are considered for convenience to be composed of three parts: transducers and transmission paths, an input-output mechanism (I/O), and the GPC. The transducers and transmission paths considered in particular are those resulting in the desired data being phase-modulated on carrier waveforms. This type of phase-encoded information may be received from a variety of sources, among which are shaft-angle resolvers with sine-cosine excitations, and Doppler navigation systems. The primary focus of attention in this paper is a new I/O which can receive the phase-modulated waveforms directly, perform a variety of processing functions on the raw data in its phase-modulated form, and present the processed data in a convenient binary format to the GPC. The new I/O can process data from a number of sources in parallel at relatively high speeds, thereby leaving the GPC time for monitoring, adaptive parameter adjustment, and other sophisticated decision and control functions. Similar techniques utilizing different phases of the computer clock signal permit the GPC to generate digital and/or analog commands to transducers via the I/O.

Some examples of transducers and transmission links are briefly presented as background material and motivation for a discussion of the I/O itself. The operation of the I/O is then discussed in detail with reference to a practical system for obtaining and processing whole-angle data from a multi-speed shaft-angle resolver and, in more generality, with reference to a wide variety of signal sources producing phase-modulated information.

The ease of performing analog/digital conversion

(and its inverse) with phase information is the key element in the I/O discussion. As a result, the real-time control problem and hybrid computer problem may be considered as essentially the same time shared computer problem, and the traditionally complex I/O problem of tying analog elements to the GPC may be solved in the same simple way.

Several significant improvements can be made in computer usage by the proper design of an I/O. In particular, the programming can be simplified, more useful computing can be carried out in a given time interval, and greater system flexibility can be achieved.

### *Transducers and transmissions links*

In preparation for the detailed discussion of the I/O, some examples of transducers and transmission links producing phase-encoded information are given. These examples serve to introduce some simple but useful mathematical notation and to provide a physical interpretation of the origin and meaning of the phase-encoded waveforms that are central to the discussion of the new I/O. The examples also serve to introduce the ideas that there may be several information sources (either independent or dependent) operating simultaneously and that the information can conveniently be multiplexed without altering the phase encoding. The new I/O has the natural capability of processing the phase-encoded information from several sources simultaneously.

A typical example of a transducer producing phase-modulated information is the shaft-angle resolver shown in Figure 1. These transducers are found in equipment ranging from inertial to machine-shop. The input signals  $e_1$  and  $e_2$  are any periodic waveforms with fundamental components  $E \sin 2 \pi f t$  and  $E \cos 2 \pi f t$ ,

respectively. The resolver produces an output

$$e_o = e_1 \cos \phi + e_2 \sin \phi \quad (1)$$

where  $\phi$  is the mechanical resolver angle. Hence the fundamental component of the output is

$$\text{fund}(e_o) = E \sin(2\pi f t + \phi) \quad (2)$$

which is linearly phase modulated by the resolver angle. Henceforth, we shall assume for convenience that the input waveforms are square waves with frequency  $f$  and phase angles  $0^\circ$  and  $90^\circ$ , these are denoted by  $f/0$  and  $f/90$ , respectively, as shown in Figure 1. Implicit in this notation is the assumption of a reference zero phase angle, which we shall assume is established by a reference clock.\* The output of the resolver we shall denote by  $f/\phi$  to indicate that the fundamental component of the waveform is of frequency  $f$  and phase angle  $\phi$ .

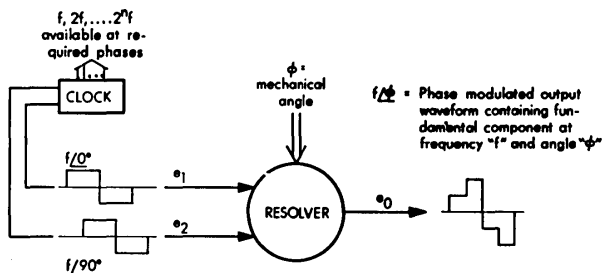


Figure 1—A shaft-angle resolver with phase-modulated output

Often an m-speed resolver\*\* yielding an output  $f/m\phi$  may be used with a one-speed resolver on a common shaft to assign roughly equal parts (in the sense of double precision) of the desired angle information to each of two modulated waveforms. Where there are several resolvers, their output may be time multiplexed over a common transmission path with, e.g., four samples of each waveform taken per cycle, without distorting the phase of the fundamental component.<sup>1</sup> Another possibility is to effect frequency multiplexing by choosing the excitation frequencies for different resolvers as binary multiples, e.g.,  $f$ ,  $2f$ ,  $4f$ , etc. Of course, both time and frequency multiplexing may be used simultaneously. In that case the outputs of the time multiplexer would be several zero-order-held waveforms, each resembling the resolver output waveform shown in Figure 1, but at different excitation frequencies. These phase-modulated outputs would be received by the new I/O. The frequency demultiplexing of the

\*A clock is a high-frequency oscillator followed by a (typically binary) countdown chain from which several signals with locked frequencies and phases may be obtained.

\*\*An m-speed resolver has windings with  $m$  pole pairs so that its output phase angle passes through 360 degrees when the mechanical shaft rotates through  $360/m$  mechanical degrees.

information is automatically achieved in the new I/O because an integral part of the new I/O is a set of phase-locked loops.

Other examples of transducers yielding phase-modulated information abound: Loran, Doppler radar, sonar, etc. In these systems phase shift data correspond to measurements of distance. Two or more carrier frequencies (often multiplexed on a third) are often used to measure the same distance variable. This technique is completely analogous to that of using one-speed and m-speed resolvers to measure a common shaft angle.

### The new I/O

A basic element in the new I/O is the phase-locked loop† shown in Figure 2a. It consists of a phase-sensitive detector (PSD), a low-pass filter (LPF) with transfer function  $F(s)$ , a voltage-controlled oscillator (VCO),‡ and a binary n-stage forward counter (count-down). The countdown output  $f/\theta$  is a square wave which in normal operation is locked in frequency and phase with the fundamental component of the input  $f/\phi$  to the phase-locked loop. Ideally,  $\theta$  is equal to  $\phi$  plus  $90^\circ$ ; careful loop design can insure that this relation is maintained reasonably well, in many cases to within a small fraction of a degree. The PSD can be a simple switching modulator that multiplies the input waveform by  $+1$  or  $-1$ , depending upon the state of the countdown output. The LPF extracts the average value of the PSD output and, in its most general form, performs several other filtering functions. The basic phase-locking operation of the loop depends upon the action of the PSD in producing an error signal to increase or decrease the frequency of VCO oscillation in order to drive the phase error to zero. This mechanism is illustrated by the block diagram in Figure 2b. Because of the presence of the n-stage countdown, the VCO frequency is  $2^n$  times the input frequency. Loops of this type have been operated satisfactorily with VCO frequencies as high as 5 mc and with as many as ten countdown stages. Standard heterodyning techniques can be used to accommodate input frequencies that are impractically high for the basic loop shown in Figure 2a.

The effect of the phase-locked loop is to create a set of square waves, each wave being the output of one stage of the countdown in Figure 2a, that track the phase  $\phi$  of the fundamental component of the input waveform. The phase angle  $\theta$  of this set with

†For good sources of information on the behavior, design and use of phase-locked loops the reader is referred to Tausworthe<sup>2</sup> and Gardner.<sup>3</sup>

‡Any simply controllable oscillator is usable, including digital oscillators made by DDA techniques.

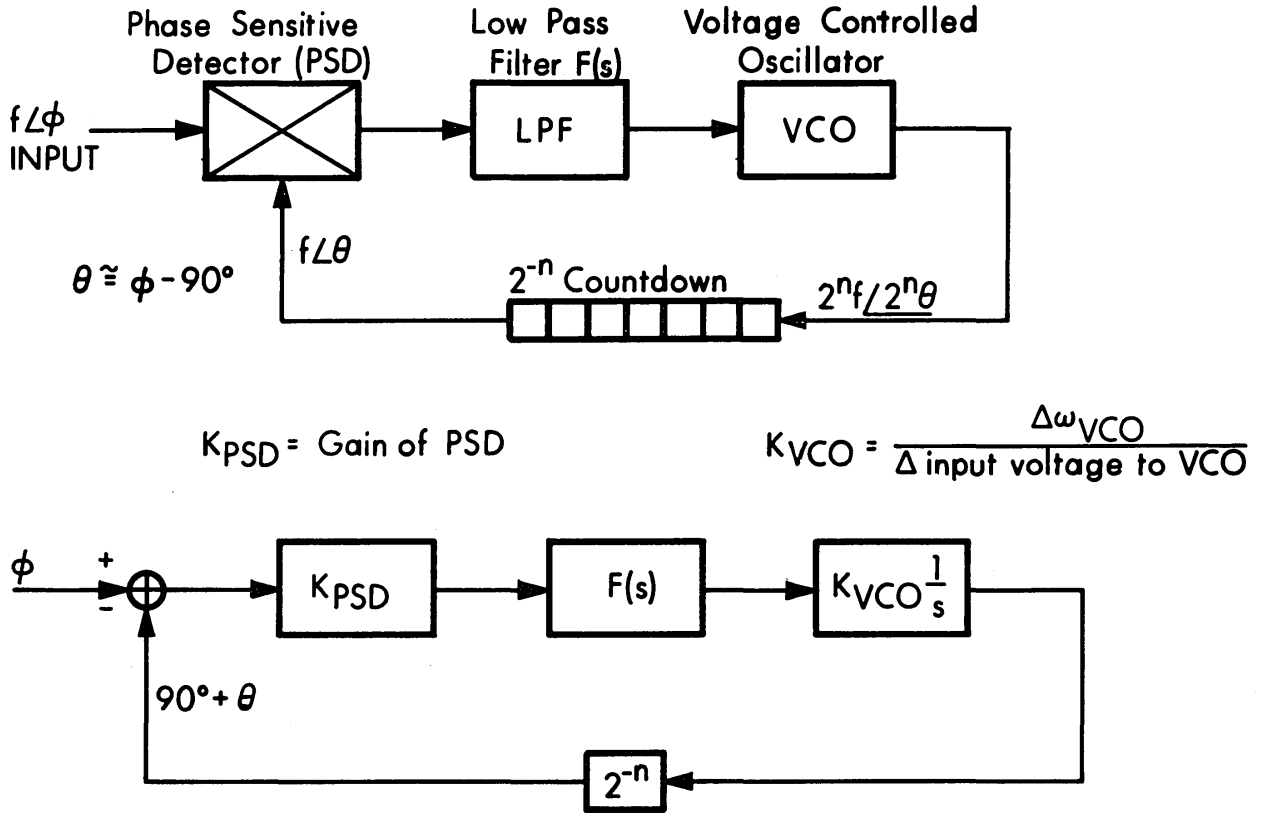


Figure 2—The phase-locked loop

respect to a reference clock waveform  $f/Q$  is easily determined by the process diagrammed in Figure 3. At the instants when the clock waveform  $f/Q$  changes states in the positive going direction a determination of the states of each of the countdown waveforms is made. Knowledge of whether  $f/\theta$  is up or down narrows the uncertainty in the angle to a  $180^\circ$  region; knowledge of whether  $2f/2\theta$  is up or down further narrows the uncertainty in the angle to a  $90^\circ$  region; etc. This process, referred to hereafter as "strobing

the countdown waveforms with the clock waveform,"\* results in a unique synchronous binary encoding of the angle and is the heart of the new I/O. A functional implementation of the strobing technique to measure a resolver angle is diagrammed in Figure 4.

The strobing process for obtaining binary encoding of phase angle can easily be extended to the case where there are two signals available corresponding to a common measurement variable, as from one-speed and m-speed resolvers on a common shaft. The one-speed signal  $f_1/\phi$  and the m-speed signal  $f_2/m\phi$  can, of course, be encoded separately, each with its own phase-locked loop strobed from a common clock. However, with this arrangement, as with any independent encoding scheme, unavoidable misalignment between the one-speed and m-speed data will cause,

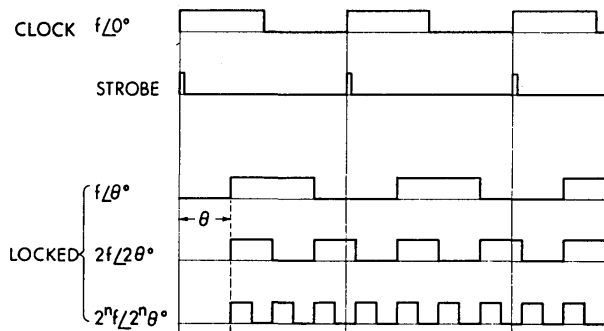


Figure 3—Strobing waveforms

\*Note that the process could be modified to obtain angle encoding by having the signal  $f/\theta$  from the phase-locked loop strobe the signals in the clock countdown, in which case the phase-locked loop would be acting as a filter and zero-crossing detector. However, with this arrangement, the precise times at which the data strobes occur would not be known *a priori*, and this uncertainty could be an important disadvantage when precise dynamic measurements are required.

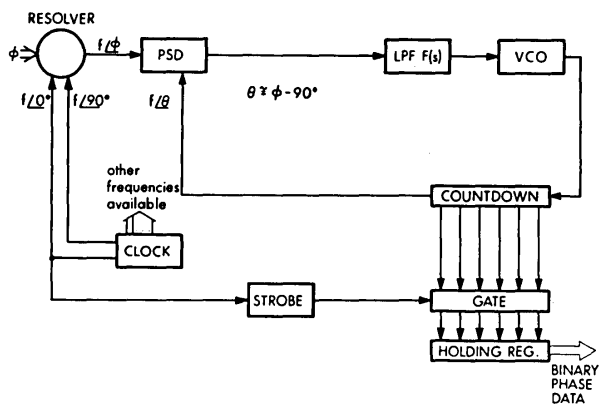


Figure 4—Strobing technique to obtain binary phase output

for certain angles, an inconsistency in the binary encoded data. An example of such data from 1-speed and 16-speed resolvers on a common shaft is shown in Figure 5. The inconsistency lies in the fact that the overlapping bits do not agree. In this case it is

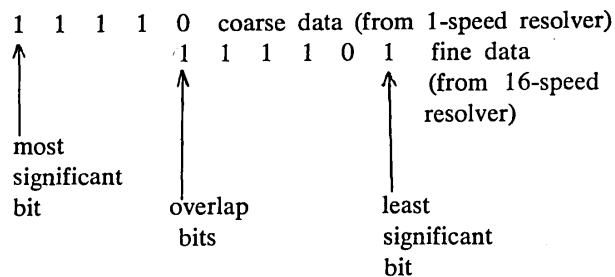


Figure 5—Data misalignment

necessary to establish the direction of phase misalignment and to add or subtract a unit from the coarse word accordingly. However, with the new I/O a bit can be added to the coarse word merely by delaying the coarse-word strobe. Hence, an efficient and easily implemented technique for data alignment is as follows:

- Introduce a phase misalignment in the waveform  $f_c/\theta$  from the countdown of the coarse-data loop such that the waveform corresponding to the least significant bit from that loop lags the waveform corresponding to the most significant bit from the fine-data loop by about  $90^\circ$ . Strobe the countdown of the coarse loop as usual (on the same clock signal as for the fine loop) if the overlap bits agree. Otherwise, delay the strobe of the countdown of the coarse loop until the overlap bits do agree. (Clearly, this is not the only possible method of data alignment. It bears similarity to a double-precision operation involving a carry bit.)

The introduction of the correction signal for misalignment angle is easily accomplished by adding a small fixed voltage to the signal at the output of the PSD of the coarse-data loop. Then the indicated strobing scheme results in whole-word data being obtained jointly from the one-speed and m-speed phase-modulated signals.

The elementary I/O as described has been constructed and operated satisfactorily to obtain 13 bits of whole-word data at a 1 KC synchronous strobing rate from a 1- and 16-speed shaft-angle resolver.

*Comparison with conventional angle-measurement techniques*

The elementary I/O is in essence a new and powerful technique for measuring and encoding phase angles of time waveforms. On this basis alone, without consideration of the additional processing and computing capabilities, the elementary I/O offers several advantages over conventional phase-measurement techniques.

Almost all phase measurement techniques require the conversion of the phase-modulated waveforms to rectangular or square waveforms which are in turn used directly for timing measurements. In the new I/O, as often in the field of radio telemetry, the conversion is made through the use of phase-locked loops. In conventional systems for measuring resolver angles the conversion to square waves is done through the use of zero-crossing detectors, perhaps preceded by band-pass filters. The phase-locked loop has a superior capability to discriminate against noise (thereby avoiding totally false readings due to multiple false triggering of level detectors) and, in addition, avoids errors that occur as a function of frequency shift in direct transmission through a band-pass filter. The new I/O broadens the area of application of phase-tracking techniques long known in the communication field.

Conventionally, the timing measurements on the square waves are performed by either of two basic methods: One of the methods is to use a linear analog phase detector to determine the phase of the zero-crossing detector output with respect to a clock waveform. The phase detector output must then be filtered and A/D converted to obtain a numerical representation of phase angle. This method relies strongly on the linearity of the phase detector, and results in significant dynamic errors due to the filter. The new I/O offers the advantage of utilizing a nulled phase detector, which need not be linear, and the advantage of greatly improved dynamic performance. It also has the advantage of relative ease and simplicity, (and therefore

economy) of A/D conversion. The other conventional method of processing the zero-crossing detector outputs is to use their rising and/or falling edges to start and stop a counter that is driven by the reference clock. The final counter outputs (at variable clock times) are the encoded phase angle data. This method is the open-loop counterpart to the strobing method of A/D conversion used in the new I/O. In the new I/O the counter is incorporated in the phase-locked feedback loop and numerical angle data is obtained *at known clock times* by the strobing process. Hence, the elementary I/O combines the signal-tracking capabilities of the phase-locked loop with the capability of synchronous angle encoding by the strobing process.

The new I/O has the additional advantage that it can easily be generalized to perform a wide range of simultaneous signal processing tasks that cannot be performed as effectively by conventional schemes.

*Advantages and computing capabilities of the new I/O*

**Sensor as part of system memory**

One important capability of the new I/O is the ability to provide synchronous whole-word data from a single or multi-speed source without the requirement of memory registers. After momentary interruptions in power sources, communication channels, etc., the current data word is completely restored. In a very real sense, the analog data source — e.g., the resolver — can function as part of the system memory and is an adjunct to the computer memory. Similarly, the I/O is akin to an addressing mechanism for the system.

Equally important is the capability to perform simply and rapidly a useful set of computing (data processing) functions, which will be enumerated presently. When a multiplicity of data sources are present these computations are performed simultaneously by the individual sections of the I/O, and the processed data can be addressed serially, randomly, or otherwise, by the GPC and/or by an off-line processor. In this manner the new I/O can take on a rather large real-time computing load which otherwise would have to be assigned to the GPC.

**Filtering**

One type of data processing function naturally performed by the I/O is filtering. Because the phase-locked loops are tracking filters with limited bandwidths, they serve not only to discriminate against electrical noise in the transmission path, but also to smooth the angle data itself. This smoothing function is particularly useful in the cases where the angle variations to be “smoothed out” are relatively rapidly vary-

ing. In those cases real-time smoothing by means of a GPC would require computations to be performed at a frequency more than twice the highest important frequency component in the power spectrum of the angle data. The new I/O performs the filtering operation automatically on the phase-modulated waveforms and thereby relieves the GPC of a rather large computing load that it is generally not designed to handle in the first place. The general purpose computer remains free to alter the filtering time constants according to either a programmed or adaptive control law by altering the parameters of the low-pass transfer function  $F(s)$ .

**Time-derivative data**

Often, particularly when the data is being used in a control loop, it is desirable to obtain the time derivative of the variable being measured. When the data represent mechanical angle the GPC is often assigned to compute the derivative, a task which it cannot efficiently perform at high speed. The new I/O can provide the time-derivative (angular velocity) data directly in the form of a shift in the VCO frequency from its nominal value  $2^n f$ . Because both the frequency shift and the nominal frequency of the transmitted phase-modulated signal are magnified in the phase-locked loop by the factor  $2^n$  (where  $n$  is the number of countdown stages), the determination of the shift can be made both accurately and rapidly. Many convenient and practical methods of measuring frequency shifts have been developed over the years for Doppler navigation, FM data transmission, etc. Any of these can be used to obtain the encoded time-derivative data. As an example, Figure 6 shows an analog frequency-difference detector followed by an A/D converter to provide encoded time-derivative data to the GPC. An alternate rate signal may be derived from the error signal to the VCO in the phase-locked loop and the choice is a matter of signal-level and hardware considerations.

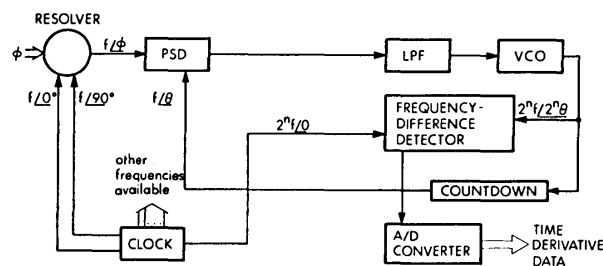


Figure 6—The generation of time-derivative data

**Compensation of periodic instrument errors**

Another type of computation that the new I/O can usefully perform is the removal of periodic instrument errors. Ideally the electrical phase angle of the transmitted data is linearly proportional to the quantity to be measured, e.g., shaft angle. Departures from linearity are objectionable and are often called "errors." However, the accuracy of the transducer may be considerably greater than its linearity, in which case it is desirable to remove the known non-linearity from the data numerically or otherwise. Although this is an extra and perhaps unwieldy task in real time for a GPC, the new I/O handles the task readily in the following manner: Suppose, for example, that the lowest-frequency signal  $f/\theta$  in the phase-locked loop in Figure 2a is multiplied (an "exclusive-or" operation on square waves) by the signal  $f/\theta$  from the clock. The average value of the product is a triangular function of the angle  $\theta$  and, hence, of the angle  $\Phi$ . If this product signal is added to the signal at the output of the PSD in the phase-locked loop, the output data angle  $\theta$  will be displaced from the input data angle  $\Phi$  by the triangular correction function (here assumed to be suitably small so that the effective gain of the PSD can be considered fixed). Similar correction functions can be generated with different periods and phase angles as a function of  $\theta$  by using different frequencies and angle references in time, as indicated in Figure 7. By using a set of correction voltages generated in this manner, essentially any nonlinearity can be compensated for if it is a known periodic function of  $\Phi$ . In this manner the binary angle (and angular rate) data is compensated for before delivery to the GPC. The latter remains free, for example, to oversee the correction process, perhaps to set adaptively the correction parameters, e.g.,  $K_c$  and  $\alpha$  in Figure 7, which can be stored temporarily or permanently in the I/O itself.

In Figure 7 the square wave  $mf < m\theta$  is shown as

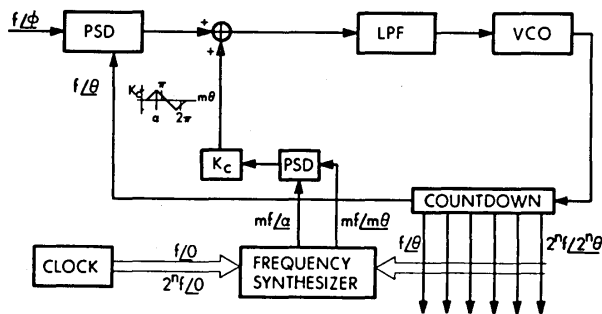


Figure 7—Correction for periodic instrument errors

an output from a frequency synthesizer. The function of the synthesizer in this case is to take the waveforms  $f/\theta, 2f/2\theta, 4f/4\theta, \dots, 2^n f/2^n \theta$  from the phase-locked loop counter and generate square waves at any desired missing integral multiples of  $f$ , such as  $3f/3\theta$ . This can be accomplished, with phase information preserved, by a novel technique, developed at MIT/IL, for frequency heterodyning.\* The function accomplished by the heterodyner is diagrammed in Figure 8.

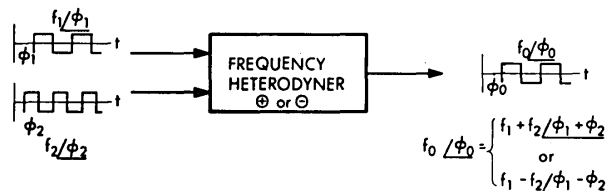


Figure 8—Fundamental element of frequency synthesizer

**Correction of phase delay as a function of frequency**

In some resolver applications, energy-storage mechanisms associated with the transmission path can introduce a phase shift into the carrier waveform as an approximately linear function of frequency. The new I/O can easily compensate for this phase shift, which can be considered as a dynamic phase error, by adding a small voltage from the frequency-difference detector to the voltage from the PSD as shown in Figure 9. This type of correction, not easily handled by the GPC, would be particularly valuable in applications where instantaneous angle data is desired from a resolver rotating at high angular velocities.

**Redundancy reduction**

In many applications, the behavior of the phase

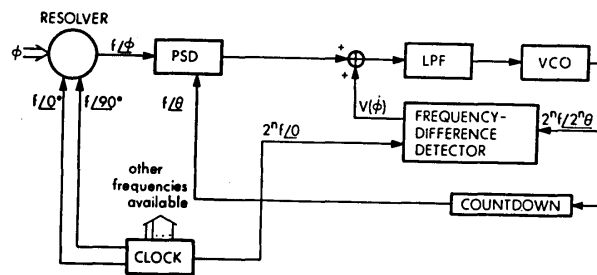


Figure 9—Phase correction as a linear function of frequency

\*The technique, which is akin to single-sideband operations and results in a flexible frequency synthesizer,<sup>4</sup> was developed by Edmund Foster and Kenneth Fertig.

and frequency of the incoming waveform to the I/O is approximately known *a priori*, and the real data of interest is the departure of the actual data from the nominal data. Examples of this situation arise in a number of Doppler-type navigation systems when the approximate course of the vehicle is known from other sources of navigation information. In such cases it is useful to be able to reduce the computing load of the GPC by subtracting the expected data in real time from the actual data. This is easily accomplished in the new I/O.

Suppose that the expected Doppler shift is  $K_1 f$  cps, where  $K_1$  is some rational number less than unity. This can be removed from the data by using a strobe waveform

$$f/0 \oplus K_1 f/0 = (1 + K_1) f/0 \quad (3)$$

where  $\oplus$  indicates the heterodyning operation in Figure 8. If this strobe signal is used, and if the actual data is equal to the expected data, the strobed binary angle is a fixed number. Small deviations in the actual data from the expected data result in slowly varying binary angle data. The GPC in turn has to process only the slowly varying data, which contain the significant information, and can ignore the rapidly-varying unprocessed data containing redundant information on the already-known nominal path. By using

the strobe waveform in place of the clock waveform as an input to the frequency-difference detector, the redundancy reduction of the time-derivative data is also effected.

The I/O can perform frequency correction also as a function of the frequency of the incoming signal. This is accomplished by letting the strobe waveform be  $f/0 \oplus K_1 f/0 \oplus K_2 f/K_2 \theta$  (4) as shown in Figure 10.

Figure 11 shows a functional diagram of the new I/O with provision included for the computing options mentioned thus far. By placing the phase-pre-

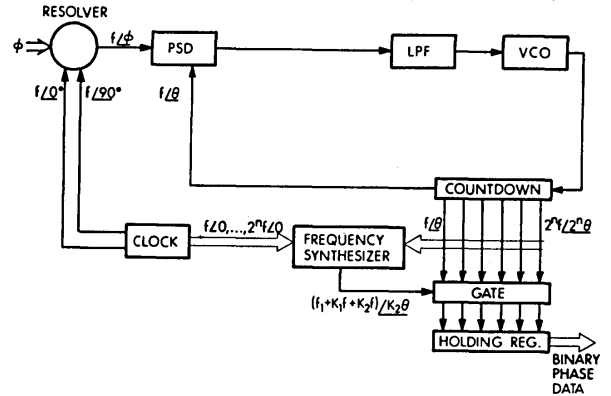


Figure 10—Method of correcting frequency as a function of frequency

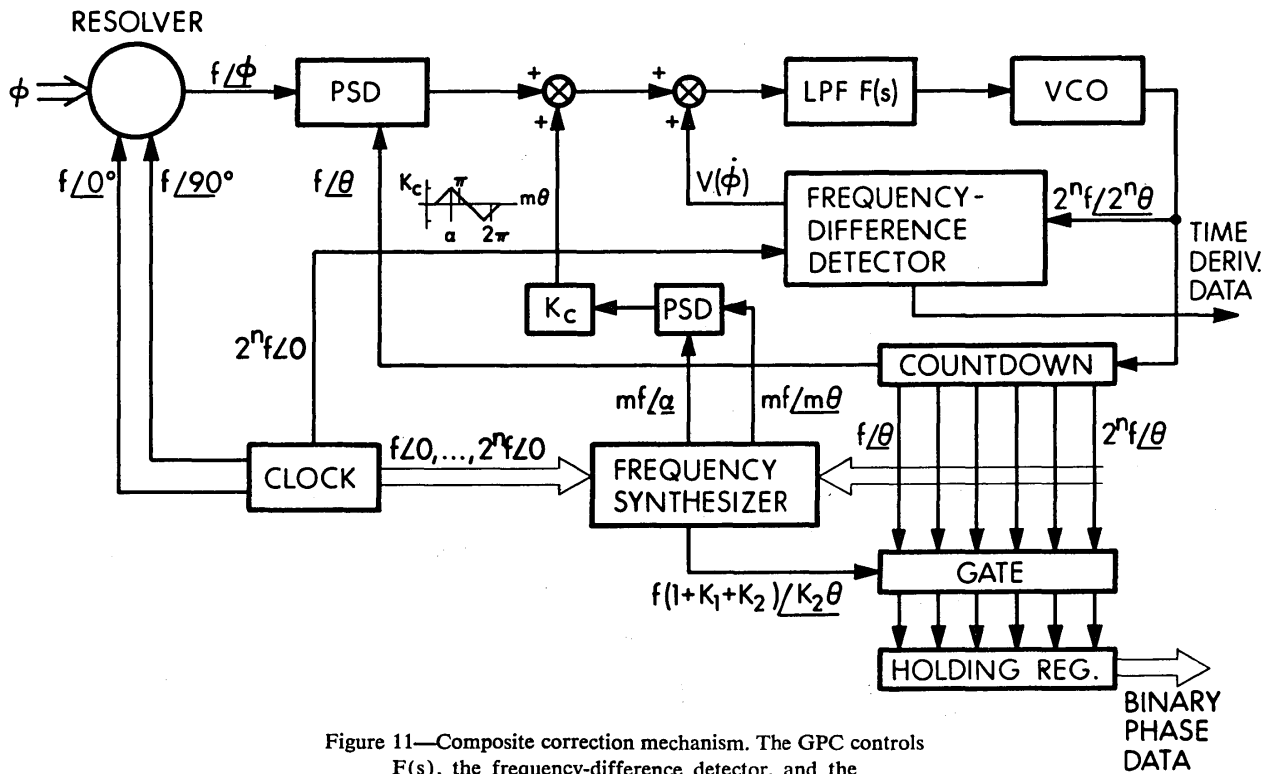


Figure 11—Composite correction mechanism. The GPC controls  $F(s)$ , the frequency-difference detector, and the frequency synthesizer

serving frequency synthesizer under GPC control any schedule of expected data may, in principle, be used in the redundancy reduction process. Moreover, the GPC may be used to perform a monitoring function to determine the extent of redundancy and error reduction and to modify the synthesizer accordingly.

**Other computing capabilities**

The computing capabilities discussed in the foregoing are substantial but rather obvious once the basic operation of the new I/O is understood. Many other types of computations can be performed, and it is likely that those mentioned are only the beginning of a long list. Some of the other possibilities presently under investigation are briefly summarized below:

- (1) Ladder networks can be used in conjunction with the holding register to form linear and transcendental functions of the strobed angle data.
- (2) A countdown in one phase-locked loop can be strobed with a signal from another phase-locked

- loop to obtain relative-angle data.
- (3) Digital-differential-analyzer ideas can be incorporated to allow products of data words to be obtained.
- (4) Generalized hybrid computation may be considered where the ladder networks of (1) are excited by voltages related to real signals. Depending upon where the feedback loop is closed with respect to input and ladder output, analog multiplication, division, etc., can be performed as shown in Figure 12. If the ladder is on the countdown, a continuous presentation of data on a modified carrier may be obtained. By using the multiplication and data-storage capabilities, correlation of data can be performed.
- (5) By incorporating logic circuitry in the clock to allow generation of waveforms at independently controllable phase angles, GPC outputs may be phase encoded for subsequent processing by the new I/O. The I/O could present the results in digital, analog, or phase-encoded form.

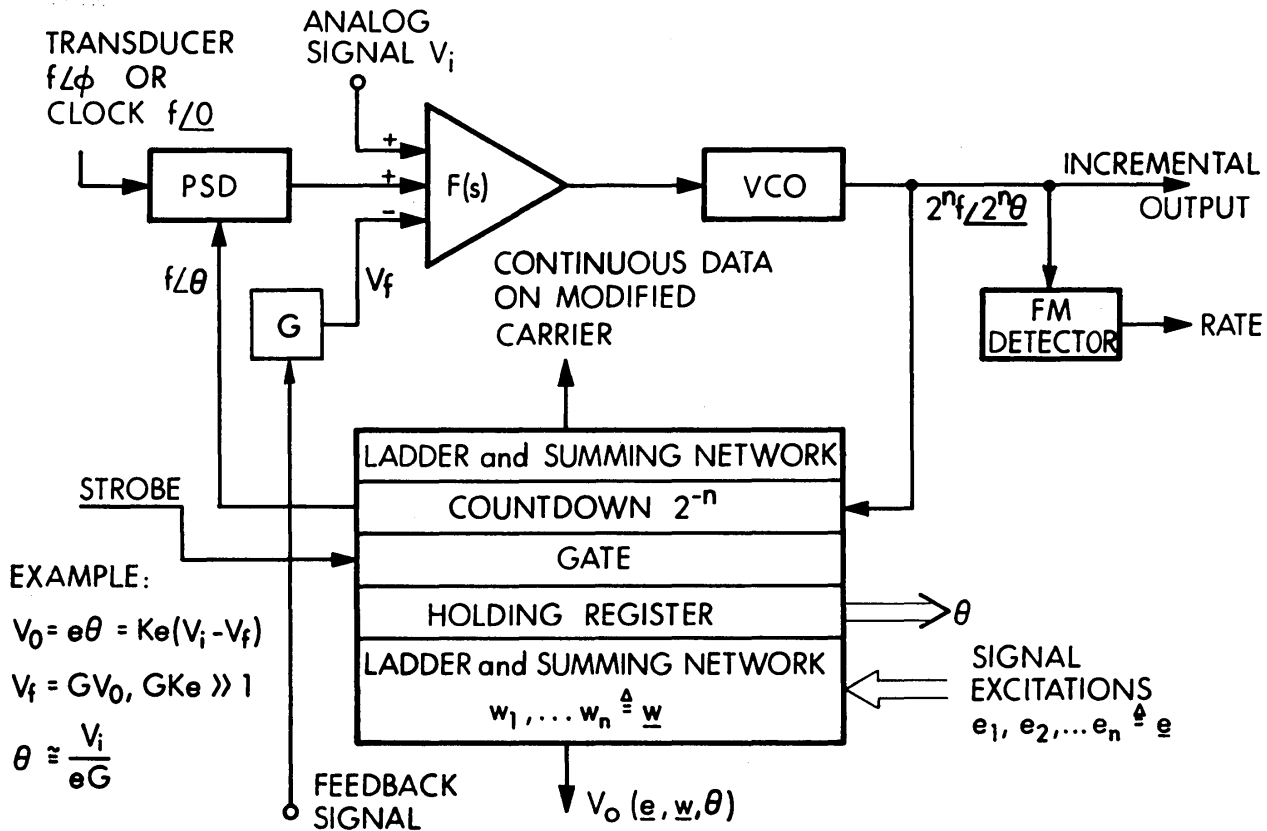


Figure 12—Hybrid computer element



### Generalization: Instrument servo and new I/O

The properties of the new I/O result from the action of the VCO in the phase-locked loop. The properties of the instrument servo of prehistoric analog computer days were largely dependent on the motor which acted as an integrator in the sense that a voltage applied to the motor control winding resulted in an angular velocity, the integral of which is angle. Similarly, the voltage input to a VCO controls frequency whose integral is phase. In the old servo, the motor shaft could be loaded by tachometers, resolvers, synchros, potentiometers, etc. In our phase-locked loop, the VCO and counter may be loaded by FM detectors (note that the loop error signal is the Doppler shift), ladders, etc., as shown in Figure 12. Clearly, the similarity between the phase-locked loop and the instrument servo is being stressed. From this similarity, all of the computing capability obtained previously with the instrument servo may be obtained from the electronic servo — the phase-locked loop.

As a result of the preceding discussion, a further generalization may be made which is incidental to the discussion of I/O. The phase-locked loop is really an accurate operational amplifier and the technique may be used to make a family of analog computing elements, servos, etc.

### Hardware implications

The circuit elements which are used to make a phase-locked loop lend themselves to microminiaturization. The developments in integrated circuits and large scale integrated arrays make hardware I/O improvements attractive.

Further, examination of existing I/O's for several real-time computer/control systems shows that the system designers, in order to alleviate the computer data-processing load, have placed in the I/O adders, forward-backward dual-rank counters, A/D converters, etc. The phase-locked loop, utilizing a forward-only counter, etc., appears to lend itself to a building block concept which represents no increase in hardware but rather a regrouping and a considerable increase in flexibility.

### CONCLUSION

A simple high-speed I/O with real-time computing capability has been described. The I/O is particularly well suited for performing high-speed calibration and redundancy reduction on data received as phase modu-

lation on carrier waveforms, and for presenting the processed data in a convenient whole-word binary format for further processing by a general purpose computer. The I/O is non-incremental in nature and, hence, is completely self-restoring after momentary equipment interruptions and can be adaptively manipulated under general purpose computer control.

The new I/O is a step in the direction of freeing a general purpose computer in a real-time measurement and/or control application from high-speed but routine data-reduction tasks (for which it is not well suited) to perform computation, sophisticated monitoring and adaptive adjustment tasks (for which it is particularly well suited). Although the examples have been about inputs to the general purpose computer from continuous sources, the techniques described would apply to the generation of outputs from the general purpose computer which commands or controls some element of a system.

The concepts of I/O have been generalized to include the general problem of analog computation and simulation and to lead naturally to hybrid computation techniques.

### ACKNOWLEDGMENT

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