Analog, Digital, and Combined Analog-Digital Computers for Real-Time Simulation

C. G. BLANYER† AND H. MORI†

Introduction

A LARGE, important area of use for automatic computing machines is the study of the dynamic operation of physical systems, especially guidance and control systems. If the mathematical model under study describes in substantial detail an existing or contemplated piece of equipment and if stimuli are employed that correspond to those experienced by the equipment when in use, the process of study is referred to as simulation. The term simulation cannot be used to express a precisely defined area of work because widely varying interpretations are given to it. As used here, the term refers to an area of computation comprising analysis of large, complex systems that are represented in sufficient detail to include many of the distinguishing characteristics of practical equipment, even though few, if any, parts of the actual system are linked with the computational components.

Although rapid solution of a problem is advantageous in itself, the requirement for real-time speed in computation becomes necessary only when the simulation includes actual equipment. Additionally, the existence of identifiable signals related closely to physical variables in the real equipment provides both concrete and abstract advantages, particularly if the computer signals occur in the same time as in the actual equipment.

Simulation can be performed with either analog or digital computers. Furthermore, the many variations of the traditional form of these machines tend to blur the distinction between them. Most of the variations have resulted from attempts to combine the advantages of the two methods.

The word analog itself indicates that analog computers inherently involve a measure of simulation. Although simulation indeed has formed a large area of activity for these machines, the investigation of simple dynamic relationships including only a few variables and the education of engineers in the field of dynamics have been important uses of analog computers. Because the principle of operation of digital computers is the reduction of all relationships to a few simple forms that can be performed with extreme accuracy, a wide variety of operations, including bookkeeping and certain logical manipulations as well as simulation studies, can be accomplished with these machines. In general, they are most useful in fields where many arithmetic operations must be performed with great accuracy.

† Hydel, Inc., Cambridge, Mass. Formerly at the Dynamic Analysis and Control Lab., M.I.T., Cambridge, Mass.

Although both types of computers have been used extensively for simulation, most installations leave much to be desired from the point of view of convenience, accuracy, and efficiency. The disparate characteristics of the two types of machines indicate that either improvements in each should take widely different forms, or that features selected from each type should be combined to form the most satisfactory simulator. One major difficulty associated with improvement in simulation computers is the lack of a clear understanding of real requirements for a highly effective simulator, together with a realistic comprehension of what can be accomplished with various computing methods.

The purpose of this paper is to review the characteristics of analog and digital computers and of several variations of these basic machines, to point out the areas in which improvement is most needed, and to outline some of the methods being investigated for achieving such improvements. An attempt is made to describe the characteristics of an efficient real-time computer and to evaluate the demands that these characteristics would make on the computer designer.

Characteristics of Analog Computers

The distinguishing characteristic of analog computation is well known; physical quantities, such as voltages or shaft angles, are constrained to obey relationships analogous to variables existing in a real system. This process is feasible because many different physical systems obey mathematical laws of identical form. However, the ability of a system to perform certain elementary mathematical operations is not sufficient to make it useful for simulation. Generation of complex functions and communication with external equipment and personnel must be provided. Furthermore, combination of operations into large groups must be practicable. Certain abstractions that correctly belong in the field of human engineering determine, to a large extent, the utility of a computation scheme.

Before computer characteristics can be evaluated effectively, appropriate criteria must be established. Three useful criteria are the accuracy and the speed with which a computation is performed and the degree of difficulty associated with an operation. Error, or imperfect accuracy, includes any deviation of a response from the required result. Errors, which may exist statically or appear only in dynamic operation, almost invariably accumulate when functions are combined. Finite resolution results from unavoidable quantization of variables or from noise which limits the minimum discernible variation in a signal. Although speed can be measured quantitatively, the appropriate criterion in simulation is whether the dynamic error in an operation permits or prevents real-time operation. The term "difficulty" (as used above) applies particularly to operations that reduce significantly the convenience of computation, or that result in great effort or expense.

The irreducible minimum error in a simple analog computation is set by the accuracy that can be maintained in the basic circuit elements. Designers have concluded that the characteristics of materials limit this basic accuracy to roughly one part in 10^4, although the stability of available circuit elements and their freedom from parasitic effects have increased steadily during the past decade.[1] As a result, the relatively simple mathematical operations of summation, multiplication by a constant, integration, and the inverses of these operations, usually can be performed to an accuracy of three significant figures. Typical noise levels limit resolution to one part in 10^4 of full scale. Lags associated with these components are inconsequential, with certain exceptions noted later. Errors common to electronic analog integrators are of a type that may be almost negligible in the simulation of closed-loop systems. However, these errors can prevent open-loop operation of typical integrators in many circumstances, particularly if long computing times are involved. The unavoidable noise in computer circuits frequently prohibits the explicit use of differentiation. However, in many simulation problems, a lag can be associated with the differentiation without introducing unacceptable dynamic errors. Unless an indirect method of computation can be utilized, integration and differentiation with respect to variables other than time can be accomplished only with all-mechanical devices that are both slow and inconvenient.

The operations of multiplication and division of variables and function generation are difficult to perform in most analog installations. Static accuracy varies between 1/10 per cent for multiplication and 1 per cent for the generation of complex functions. Dynamic performance, even of the electronic methods, usually is not completely satisfactory. The use of servomechanisms to convert voltages into shaft positions makes possible various combinations of these operations in output elements; however, resolution as fine as 0.01 per cent in output potentiometers can be achieved only in linear elements. Furthermore, severe rate and acceleration limits impair servo performance. A typical unit requires a few tenths of a second to reach full scale at maximum speed and may require an equal time to accelerate from zero to full speed.

Programing and setup procedures for analog computers are readily intelligible to engineers. A computer diagram is very similar to the familiar block diagram. In many installations, the interconnections of individual circuit elements to form functional units must be planned; in others, only the functional units themselves need be connected. Although setup typically consists of making physical connections between units by means of patch cables and of setting controls, examples exist of installations employing remote, as opposed to manual, connections and settings.[2] Automatic or especially convenient manual methods of changing a few settings and a few parameters may be provided. The physical manipulation of connectors and controls included in setting up and operating the machine is time consuming, but forms a psychological aid for the operating personnel.

In summary, an analog computer is characterized by similarity to real systems, both in detail and in an over-all sense. Most simple operations of algebra and the calculus can be performed with approximately three-place accuracy and four-place resolution, although only a single independent variable is permissible in most integration and differentiation. Nonlinear operations, which are relatively difficult to perform, limit achievable resolution and tend to be at least an order of magnitude less accurate than linear operations. Speed is no problem in the simpler operations, however, the dynamic performance of servos and many nonlinear devices is marginal in large-scale real-time simulation. Over-all solution accuracy typically is 1 per cent in the simulation of a closed-loop system of medium complexity. This degree of accuracy suffices for many engineering applications.

Characteristics of Digital Computers

In a digital computer, bits of information are carried in groups by means of a spatial arrangement of signals or a time arrangement of pulsed signals. Because essentially all mathematical operations can be approximated by successive applications of addition and comparison, solutions to differential equations and other manipulations required for the simulation of systems can be obtained.

The basic operations must be combined to form functional relationships to a much greater extent than in analog computers; in most digital computers, these basic operations, which are separated in time by repeated communication with a memory, must be performed sequentially. Because the degree of approximation permitted in nonexact functions can be varied, the speed and accuracy of operation are not fixed quantities. Furthermore, the use of sequential operations causes speed and accuracy to be functions of problem complexity. If the details of the operational methods to be employed are left to the programer, the effectiveness of the speed-accuracy compromise depends, to a great extent, upon the skill of the programer in the fields of numerical analysis and logical manipulation, and upon the time allowed for program preparation.

Resolution, which usually can be measured in tens of binary digits, forms essentially the only limit to the exactness of basic operations such as addition and multiplica-

[1] G. A. Korn and T. M. Korn, "Electronic Analog Computers," McGraw-Hill Book Co., Inc., New York, N.Y.; 1956.

[2] H. E. Harris, "New techniques for analog computation," *Instr. and Automation,* vol. 30, pp. 895-899; May, 1957.

tion. If a problem requires a large number of multiplications, the time required for multiplication may determine, to a significant degree, the resolution that can be retained.

Operations of the calculus are performed by approximate arithmetic procedures, with the programer exercising some choice of the specific method to be used. Again, results are repeatable precisely and indefinitely, although the errors introduced by approximations and truncations may be serious if rapidity of operation consistent with real-time simulation is required. Functions may be generated by solution of a formula, consultation of a stored table of values, or interpolation between widely spaced stored values. The evaluation of integration presented here also is appropriate for function generation, except that the latter process may require a large part of the available fast-access storage space.

The performance of general-purpose digital computers is difficult to assess, owing to the wide variety of computing methods that can be used and the close relationship between problem complexity and the speed-accuracy combination. However, the accuracy of typical installations might be estimated at a few per cent for applications such as real-time simulation of a missile in three dimensions.

Because of the method by which information is read in and stored, a large mass of detailed data, such as that involved in the description of an arbitrary function, can be inserted or changed quite rapidly. This advantage is offset partially by the fact that instructions for relatively simple operations also consist of a large mass of details. Programing essentially consists of an exercise in numerical computation with a limited choice of methods and language determined by specific design features of the computer.

Special routines such as the Differential-Equations Psuedo-Code Interpretor (DEPI)[3] and the Numerically Integrating Differential Analyzer (NIDA)[4] have been developed. These methods reduce the amount of work associated with programing and result in a setup procedure that is similar to that for an analog computer. These routines, of course, do not alter the capabilities of the machine, nor do they eliminate the necessity for knowledge of coding in specific machine language. However, they do reduce the problem of treating differential and integral relationships in terms of numerical analysis. The net result of using these routines, however, may be a decrease in speed of computation because the simplified code must be translated by the machine into digital-machine instructions and, moreover, these sets of instructions usually are not efficient.

The concepts embodied in special programs have been extended to special machines that are programed permanently to permit convenient solution of integro-differential equations. The most widely known of these is the Digital Differential Analyzer[5] or DDA. In such machines, special sections of the memory, designated integrators, are arranged to be processed in sequence by an arithmetic unit in accordance with a simple integration formula. These so-called integrators also can be used as servos, limiters, and generators of simple functions. Typical DDA machines offer no substantial advantage over general-purpose computers in speed or accuracy, and their performance may be poorer owing to the extremely simple methods of integration employed. However, most systems engineers undoubtedly would find these special machines easier to use.

Comparison of Computers

The task of comparing mechanisms that are basically as different (and diversified) as analog and digital computers perhaps can be accomplished best by concentrating on the results of operations instead of on the operations themselves and by commenting on the techniques necessary to utilize them. The following evaluation is based on the characteristics of existing computing equipment rather than on the inherent features of the two types of machines.

The accuracy of digital computers accounts for their superiority in algebraic operations, particularly in critical processes such as summation of large, nearly equal quantities. All-algebraic loops usually must be avoided in analog machines owing to stability problems. The range of problems that are possible with analog computers is restricted because most analog installations are capable of performing only a few multiplications of variables, with marginal accuracy. On the other hand, the time required for multiplication, either by constants or variables, may become significant in digital computers if many of these operations are necessary.

Digital integration on a real-time scale is quite difficult. Complex integration formulas require many operations and raise questions of convergence. Simple methods require a high density of points in time, furthermore, they introduce serious limitations on the rate of change of variables. For example, even an electromechanical analog servo or servo integrator may be capable of changing its output two or even three orders of magnitude faster than a digital-counter type of integrator with equal resolution.

The ease with which arbitrary functions can be set up and changed in a digital computer is an advantage that is particularly significant in systems studies involving empirically derived relationships. This advantage extends to a variety of analytic functions as well. However, a speed problem may exist in real-time simulation if many functions are to be represented.

Analog computers have superior characteristics for programing and communication. The advantages lie not

[3] F. H. Lesh and F. G. Curl, "DEPI: An Interpretive Digital Computer Routine Simulating Differential-Analyzer Operations," Jet Propulsion Lab., Pasadena, Calif., Memo. No. 20-141; March 22, 1957.

[4] H. H. Anderson and J. R. Johnson, "Numerical Integration of Differential Equations on the 704 EDPM," Eng. Lab., IBM Corp., Poughkeepsie, N.Y., Tech. Rep., Code: 011.070.592; January 11, 1956.

[5] M. L. Klein, F. F. Williams, H. C. Morgan, and S. Ochi, "Digital differential analyzers," *Instr. and Automation,* vol. 30, pp. 1105-1109; June, 1957.

only in the close parallelism between the system and machine diagrams—an advantage shared in part by DDA machines—but also in the parallelism between control systems and analog-computer elements. The physical aspects of communication, such as connection of real components and read-out of data, cause complexity or speed difficulties in digital computers.

Although special programs and machines have increased the usefulness of digital computation in simulation, numerous factors tend to limit the speed of digital machines, and therefore, analog computers are superior in many applications requiring high operating speed. The margin of difference is difficult to assess quantitatively. However, some conception of the problems involved can be gained from consideration of the operations performed in a large-scale simulation. A recent three-dimensional guidance study performed on the large-scale analog computer at M.I.T. involved the following individual mathematical operations (exclusive of scale-factor changes):

Additions of two quantities:	65
Multiplications by a constant:	102
Multiplications and divisions by variables:	51
Integrations:	23
Generation of analytic functions:	25
Generation of arbitrary or piece-wise linear functions:	11.

This simulation was conducted in real time (a solution time of approximately 20 seconds). In a similar situation, the programing and operation of a digital computer to obtain a check solution for this analog machine became so tedious that the check solution had to be simplified to an extent that rendered it almost useless.

Certain abstractions assist the analog user. First, the tangible features of these machines aid in orienting the engineer and often help him to feel at ease with the mechanical aid. Second, the computer may be one of the few contacts with the realities of the physical world for the systems analyst who does not work closely with components. An example in system synthesis occurred recently at M.I.T.[6] A design of a time-variable control system, that was ingenious theoretically but unfeasible practically, was reconsidered after troubles in the simulation procedure illustrated the difficulties involved.

Trends in Computational-System Improvement

Activities in the development of computer components, control systems, and programing methods have included the refinement of commonly used techniques and the trading of methods between digital and analog computers. Also, a number of schemes have been devised for making more efficient use of existing facilities.

Much of the development effort in analog systems has been devoted to providing faster means for setting up and checking computers. Arrangements exist whereby parameter setting and function-generator calibration instructions are reduced to taped commands that can be fed to the machine automatically. Automatic digital print-out of data is being included in many installations.

In the digital field, the effort to increase the rate at which operations are performed probably will go on indefinitely. Because memory storage has been a bottleneck in attempts to increase both capacity and speed, much work has been concentrated in this area. Most installations now provide several means of storage characterized by a variety of capacities and access time. An example of an advanced system is a photographic technique for high-scanning-rate storage that is under development at M.I.T.[7] This scheme utilizes a projection system and a rotating mirror to sweep digital information stored on a photographic medium past a row of stationary photoelectric transducers. Preliminary studies indicate that a reading rate greater than 10^7 bits per second is obtainable.

Better methods for taking advantage of the capabilities of existing facilities are being devised. For example, highly accurate solutions can be obtained with relatively low accuracy equipment by the solution of variational equations describing deviations from a known solution. This technique is particularly valuable with analog computers. Basic equations can be solved with very small error by digital or other means; standard conditions are employed in this solution for parameters and relationships that are to be varied. Equations describing the dependent variables of interest then can be linearized about the standard solution. If this technique is applied to a systems study, such as an investigation of ballistic missile trajectories, the variational equations can be generated as linear equations in time with coefficients that vary as integral powers of time. Because the equations describe relatively small deviations, the effects of errors are reduced by one or two orders of magnitude. The equations, in general, include a large number of multiplications and similar functions that may prove difficult for many computers. However, the situation is one in which an increase in the quantity of available equipment increases the accuracy by reducing truncation errors in the expansions. In a recent simulation study using these variational techniques at M.I.T., an equivalent over-all accuracy of four significant figures was obtained.

Techniques for more efficient utilization of digital computers usually include methods for reducing the detail required in programing, and frequently involve procedures based on the block-diagram concept. Examples of trends in this direction are the DDA machines and the DEPI and NIDA programs referred to previously.

Advanced digital computational schemes, as well as

[6] P. Goldberg, R. V. Morris, and L. H. Walker, "Multicondition Terminal Control and Its Application to Aircraft Landing," Dynamic Analysis and Control Lab., M.I.T., Cambridge, Mass., Rep. No. 109; September 30, 1957.

[7] D. M. Baumann, "A High-Scanning Rate Storage Device for Computer Applications," presented at Twelfth Natl. Meeting, Assn. for Computing Machinery, Houston, Texas; June 19-21, 1957.

convenient programing techniques, are being devised. Improved integration formulas and other computational algorithms have been incorporated into an all-digital system for an operational flight trainer. This work,[8] which was done at the University of Pennsylvania, was supported by studies of the rate at which discrete computer solutions are required to ensure stability. A study of faster integration schemes for DDA's has been conducted at Harvard.[9] The results indicate that the use of complex integration schemes instead of the simple methods presently used in DDA's will improve the speed of computation considerably. An extension of DDA techniques has resulted in the Incremental Computer,[10] which solves a single basic equation that can be rearranged to perform many different mathematical operations.

The increased use of analog-style programing for general-purpose digital computers can be expected to continue because the versatile general-purpose machines are becoming common pieces of equipment to satisfy demands other than those of simulation studies, and because block-diagram organization possesses undeniable advantages for engineers. The extension of the combination of digital circuitry and analog form is resulting in the appearance of two types of hybrid systems that differ mostly in the degree to which the principle of combination is carried.

One hybrid device operates internally on a digital basis, but accepts analog inputs and delivers analog outputs. An example is the DIANA system[11] that was under development at M.I.T. This system performs function generation and multiplication by digital techniques. Functions of a single variable are stored on a magnetic drum and read out at command from a coincidence detector that compares an analog input with the stored values of the independent variable. The function is delivered to the output through an analog amplifier in which the gain is determined by the digital signal. In a more extensive project, a complete combined system is being designed at M.I.T. for use as an operational flight trainer.[12] This real-time analog-digital simulator employs conventional analog integrators, but performs all other computation by floating-point digital techniques. Programing requires no familiarity with the logical structure of the computer. Another method under study at M.I.T. involves the unique combination of digital equipment and a time-shared electronic analog multiplier.

A combination of individual analog and digital computers incorporating the best features of both systems has proved useful. In such a combined analog-digital simulator, each subsystem should be handled by the type of computer to which it is most readily adaptable.[13] This procedure has been implemented at the National Bureau of Standards in the simulation of a closed-loop sampled-data system involving a human operator and control equipment.[14] The problems of accuracy and drift in a long-time kinematic problem suggested the use of a digital computer, and the need for a large number of solutions to gather statistical data (owing to the nonrepeatability associated with a human operator) indicated the desirability of analog computation. The simulation of this system by either analog or digital computers separately could have been done, but with difficulty. In the combined system finally selected, the analog machine is used for dynamic-equation solution in real time and for activation of display devices. The use of digital computation is restricted to operations requiring precise and accurate computation, logical decisions (including control), and the statistical analysis of resultant data.

A combined analog-digital arrangement is being studied at the Ramo-Wooldridge Corporation to reduce the amount of time required to perform a three-dimensional missile simulation.[15] Simulation of the missile is being accomplished on the analog machine, and the kinematics has been assigned to a digital computer. One technique being employed is the combination of analog and digital integration, in order to produce a fast, accurate system, by integrating the high-frequency signal components by analog techniques, and the low-frequency components by digital methods. The combined simulation has been estimated to require twice as much equipment, but only one fifth the time of the equivalent operation performed completely by digital methods. An all-analog computation was ruled out in this instance by the accuracy requirements. Although real-time simulation was not a necessity here, high computation speed was desirable because a considerable amount of statistical work was contemplated.

Another combination of an analog computer and a digital computer is being attempted at Convair.[13] A rather complex weapons system is being simulated in real time because physical system components are to be included. An all-analog simulation is impractical in this situation owing to the accuracy requirements. An all-digital simulation would prevent real-time operation.

Considerable activity is occurring in this area; the general trend is to use digital equipment with a fixed program for the portions of a simulation that demand extreme accuracy and to reserve an analog computer for high-

[8] W. H. Dunn, C. Eldert, and P. V. Levonian, "A digital computer for use in an operational flight trainer," IRE Trans. on Electronic Computers, vol. EC-4, pp. 55-63; June, 1955.

[9] R. L. Alonso, "A Special Purpose Digital Calculator for the Numerical Solution of Ordinary Differential Equations," Computation Lab., Harvard Univ., Cambridge, Mass., Prog. Rep. No. AF47; 1957.

[10] W. J. Moe, "A Digital Computer System for Airborne Applications," presented at Natl. Conf. on Aeronautical Electronics, Dayton, Ohio; May 14-16, 1956.

[11] P. A. Hurney, Jr., "Combined analogue and digital computing techniques for the solution of differential equations," *Proc. Western Joint Comp. Conf.*, pp. 64-68; 1956.

[12] E. W. Pughe, Jr., "Logical design of a real-time analog-digital simulator," M.S. thesis, Elec. Eng. Dept., M.I.T., Cambridge, Mass.; 1957.

[13] J. H. McLeod and R. M. Leger, "Combined analog and digital systems—why, when, and how," *Instr. and Automation*, vol. 30, pp. 1126-1130; June, 1957.

[14] H. K. Skramstad, "Combined Analog-Digital Simulation of Sampled Data Systems," presented at the AIEE Summer General Meeting, Montreal, Can.; June 24-28, 1957.

[15] J. H. McLeod, Jr., "The simulation council newsletter," *Instr. and Automation*, vol. 30, p. 695; April, 1957.

frequency dynamic simulation involving extensive parameter variation.

The Future of Simulation Computers

The temptation to describe a Utopian computation scheme is difficult to resist when the advantages and disadvantages of existing simulation equipment are outlined. However, specifications of this nature are best modified by economic and practical factors which may play a large part in determining future trends in machine computation. Such factors are considered briefly here.

A computer for simulation purposes should not be designed to unnecessarily stringent specifications. Errors of approximately 1 per cent in the final solution can be tolerated for many simulation problems, although a machine capable of repeating solutions to approximately 0.1 per cent is desirable. Certain operations probably should be performed to four-place accuracy to achieve this goal. Variable accuracy would be desirable in a computer for simulation if accuracy could be traded for simplicity. Although over-all accuracy to four places may be highly desirable in a few applications, the computer should be designed for ease of preparation for less accurate computation. Complexity in preparation and equipment for the high-accuracy computation probably could be tolerated.

As indicated earlier, the speed requirement can be stated simply in that real-time operation is required. In any case, speed should be sufficient to permit the operator to compare successive solutions mentally. Provision for a variable time scale is useful for checking and exploratory purposes.

Preparation, setup, and control of the computer should be characterized by convenience and flexibility. Furthermore, rapid, convenient programing should be possible without the need for special skills. Setup for a new problem should require a short time in comparison with the computing period; little is gained in reducing to fifteen minutes the setup time for a two-week program if multiple-shift computer operation is not used. Adjustment of parameters in a system as well as changes in subsections of a system should be convenient operations. Means for incorporating real equipment and human operators in a systems study on a computer should be provided.

A few additional comments on operations and methods may be worthwhile. The machine must be capable of performing multiplications, divisions, and common types of nonlinearities easily and in quantity. Purely arithmetic computations must be possible without special techniques. Machine capabilities should include dynamic operations such as integration and differentiation on more than one independent variable. Automatic scale factoring should be provided to reduce the drudgery of preparation and checkout; components should be checked automatically, and means for repeating a preset static check should be available. Both digital and analog read-out displays should be provided to preserve the advantages of multiplace accuracy as well as the graphical form of data presentation. If substantial sections of the machine are unused during studies, the installation should be arranged for simultaneous use on more than one problem.

Machine setup and physical arrangement should be based as thoroughly as possible on concepts that are familiar to the engineer. The block-diagram or flow-chart representation of relationships has proved its value repeatedly, not only in engineering, but also in unrelated fields in which organization is important. The widening range of subjects with which an engineer must become acquainted suggests that additional burdens of learning should not be placed on technical men to enable them to use computational aids. Not only should specialized machine languages be avoided, but optimum machine exploitation should not require detailed knowledge of electronic circuitry.

Comparison of these specifications with the capabilities of analog and digital computers indicates that a computing facility composed of both types of equipment suitably interconnected would be of great immediate utility. Functions could be assigned to the type of mechanism best suited to the specific operation; in addition, individual operations might be performed by combined types of equipment separated on a time basis or some other basis. For example, slowly varying components of signals might be calculated by digital means while the analog equipment is being utilized essentially for time-wise interpolation. Of course, any such combination schemes place a heavy burden on analog-to-digital conversion mechanisms. Because many of the troubles associated with digital simulation could be alleviated by improved conversion equipment, the development of fast, convenient converters probably should be emphasized in simulation-computer development programs.

To establish firm goals toward which computer designers should be working, accuracy and speed requirements should be determined carefully. As indicated previously in this section, accuracy to four and five or more significant figures is not vital to much successful simulation. Experience in a large analog computing facility has shown that over-all agreement between analog and digital check solutions can be maintained to approximately ½ per cent for complex systems by careful and laborious trimming of the analog. On the other hand, the effort required to achieve these small errors is not applied during large parts of the simulation because the desired information on system behavior is obtainable without it.

Undeniably, real-time speed is mandatory for the class of simulation including environmental component testing, human-response investigation, and similar activities. However, in many cases, inclusion of real-system equipment in a simulation is a mistake even if the possibility exists and is attractive. An analogy exists between this situation and the process of obtaining an answer to a problem without understanding the method of solution. In a variable-time-scale computer, operators frequently adjust the length of a problem to suit personal preference (limited by machine capabilities, of course) without much regard for the realism of the time scale employed. This suggests that factors other than physical tie-in, or the desire to obtain data very rapidly, are operative in the selection of an appropriate solution time. In wide varieties of dynamic systems, the real-time scale may be so short or so long that real-time simulation is either absurd or highly undesirable. Recent work at M.I.T., for example, has included simulation of systems in which transients are completed within microseconds at one extreme and last for many years at the other extreme. Study of systems with such diversified characteristics should not be excluded from consideration in planning simulation computers. Thus, the goal in speed for such computational systems probably should be based on the preference of operators; rapidity of computation that is fitted to an engineer's direct memory span and other mental processes appears to be consistent with the needs of most simulation where real-time scales are desirable.

Although analog equipment approaches fairly closely the accuracy required for most simulation purposes, substantial improvement in this respect seems doubtful. On the other hand, the tenfold or hundredfold increase in speed necessary for satisfactory utilization of general-purpose digital computers for simulation probably cannot be accomplished efficiently by the brute-force speedup of individual operations. At least one combination of characteristics of the two basically different machines would be highly satisfactory; that is, a system arranged and handled in analog form, in which operations are performed simultaneously by digital techniques. Parallel operation is probably the most efficient mechanism for obtaining the speeds required. By means of this technique, a simulator can be developed in which sections, if not individual operations, are independent and can be isolated; an increase in complexity or accuracy would involve the utilization of more equipment and would increase the cost, but would not entail a loss in speed.

Any scheme that results in specialization of a computer leads to economic repercussions, some of which can be severe. The tremendous number of digital computers in use may be attributed to the extremely wide range of activities in which these machines are useful, if not vital. Any computer that is limited to simulation work by inherent design features is restricted to a smaller market, a smaller volume of use, and substantially less support both in operation and in crucial developmental activities. Although advanced programing procedures may simplify the use of digital computers in the field of simulation, they are not likely to remove the obstacles inherent in the application of ordinary general-purpose digital machines to real-time simulation. If this assumption is correct, the future of simulation computers depends upon two things: the ingenuity and activity of the users of such machines in extending the field of systems analysis to the immense areas in which these methods are applicable, and the ability of designers to devise machines that retain the versatility of general-purpose digital computers while fulfilling the unique requirements of simulation computers.

ACKNOWLEDGMENT

The authors wish to thank Dr. William W. Seifert of M.I.T. for furnishing information and ideas for the preparation of this paper. The consultation and advice of colleagues at M.I.T. are also acknowledged.

Discussion

A. S. Baron (Westinghouse Electric Corp., Baltimore, Md.): Can you furnish the names and the locations of the hybrid installations to which reference was made?

Mr. Mori: We have a number of copies of this paper available; it has a complete bibliography and the references are in there.

H. McKenna (General Motors Research): What analog computer are you talking about when you quote 1 per cent accuracy for a simulated system? How many amplifiers in this system and is the system linear or nonlinear?

Mr. Mori: The computer to which I referred mostly was the home-built analog computer at the Dynamic Analysis and Control Lab of M.I.T. This was originally built in conjunction with the three-axis flight table. A part of it consists of ac carrier equipment. There are 15 electromechanical rate servos or integrators, each of which can have five output elements; in conjunction with this we have about 130 dc amplifier operational amplifier positions. The entire system has been estimated as the equivalent of 400 to 600 amplifier positions. It is very definitely nonlinear. Multiplications and so on are done in the servo part of the system.

H. F. Meissinger (Hughes Aircraft Co., Culver City, Calif.): In reference to the variational approach mentioned, does this mean the behavior of the system is predicted on the basis of linear (or higher order) extrapolation from a solution obtained on the computer?

Mr. Mori: You obtain a solution first on digital computer or by hand computation methods or in some cases you can get an analystic solution if you leave out some of the complications. Then you must develop some variational equations from this. You can expand terms in Taylor series, or similar operation. These variational equations, which describe deviations from basic equations, then are placed on the analog computer and simulated.

E. L. Harder (Westinghouse Electric Corp.): How long did the digital check solution take?

Mr. Mori: On a simulation of a transonic aircraft that we studied, it took one programmer four months.