frequency of 20 radians so that no unusual stability problems resulted from the use of this amplifier.

A two-stage dc preamplifier was used ahead of the magnetic amplifier. A summing circuit compared the inputs with the outputs and produced an error voltage, which was then amplified.

The Summers Model 85A flight table was, as shown in Fig. 9, a two-axis, electromechanical system which simulated airframe motion on a one-to-one time scale. The equipment consisted of a control console and a two-axis flight table, which contained two independently controlled platforms, each driven by counter rotating electric motors.

The table could accommodate a 40-pound payload with a maximum height of 11 inches. It was capable of unlimited angular displacement in both yaw and roll and could attain an angular velocity of 5 radians per second in yaw and 6 radians per second in roll. The angular acceleration attainable for yaw and roll was 50 radians per second squared and 10 radians per second squared, respectively. It proved adequate in every respect, except for the fact that it was capable of but two degrees of freedom rather than three. Had it been a three-degree-of-freedom table, the study would undoubtedly have been extended to the combination of both lateral and longitudinal equations of motion.

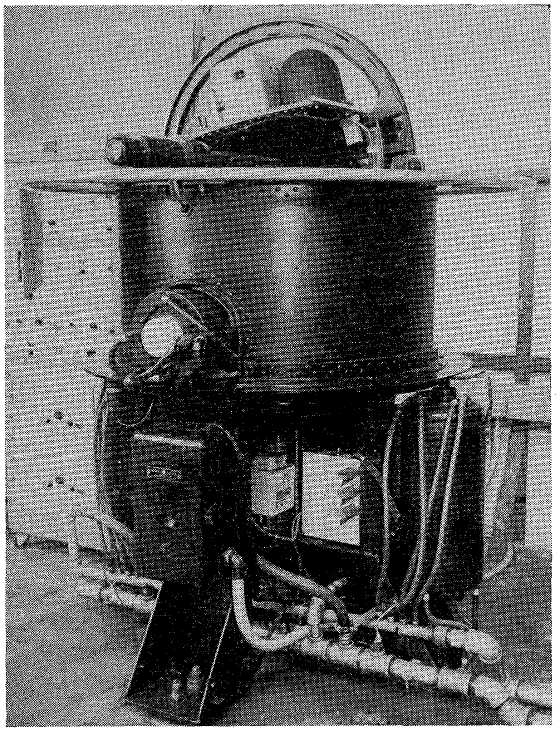

Fig. 9—Two-axis flight table.

Problems in Flight System Simulation

E. J. McGLINN[†]

INTRODUCTION

IN THE SUMMER of 1951, the Research Laboratories Division of Bendix Aviation Corporation initiated the development of a high-performance three-axis flight systems simulator for the Office of Naval Research, Department of the Navy. This effort was completed late in 1953 with the satisfactory construction of analog equipment designed primarily for the simulation of high-speed air-to-air missiles in real time. This equipment consists of a flight table and an associated simulator electronics unit. The three-axis flight table provides for the evaluation and testing of the actual control, stabilization, and guidance system equipment under the influence of angular motions of the simulated missile. The flight table contains three independently controlled gimbals as shown in Fig. 1 (opposite). The simulator electronics unit (computer) contains the significant elements required for representing high-speed dynamics on a one-to-one time scale. The computer (Fig. 2), when augmented by an analog facility, allows the flight table to be used in the complete trajectory simulation of a missile in three dimensions. (The Bendix three-dimensional flight systems simulator is described in more detail by Edwards and McGlinn.[1])

After completion of the simulator, the Bureau of Aeronautics established an operating program at the Bendix Research Laboratories in Detroit, Mich. For more than two years the simulation facility engaged in a wide variety of physical simulation and equipment evaluation studies. Early in 1957, the simulator was transferred to the Naval Air Missile Test Center at Point Mugu, Calif.

Although the flight table was especially designed for fast air-to-air missiles, the simulation program at Bendix also required that it be used in the real-time simulation of subsonic manned aircraft and missiles. A major difficulty in

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[†] Bendix Aviation Corp., Detroit 35, Mich.

[1] C. M. Edwards and E. McGlinn, "The use of the Bendix flight table," *Proc. Natl. Simulation Conf.*, pp. 6.1-6.7; 1956.

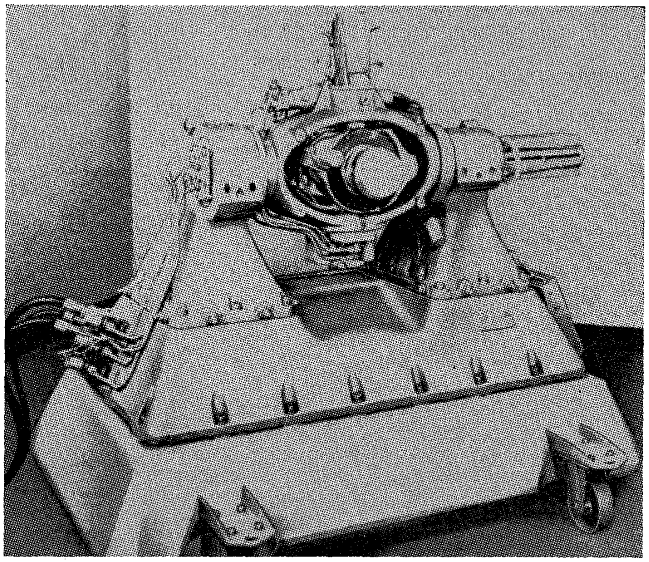

Fig. 1—Bendix three-axis flight table.

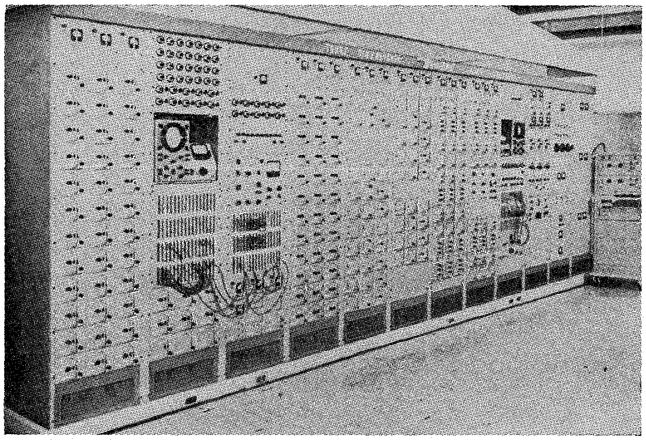

Fig. 2—Simulator electronics unit.

such a simulation was the low-speed performance required of the flight table. A technique of position control is described which extended the range of the flight table into the lower dynamic performance regions.

Another major problem connected with simulation of guided missiles in three dimensions was the target-missile geometry (*i.e.*, computation of the direction cosines of the line-of-sight vector). The large initial range and small miss distances required placed severe requirements on these computations. A technique involving continuous rescaling was chosen for the simulation work accomplished on the Bendix three-dimensional flight systems simulator.

Simulation of the Missile System

The simulated missile system block diagram is shown in Fig. 3. (Security regulations prevent a more detailed system description.)

As a reasonable approximation of the anticipated flight condition, the missile was assumed to fly at a constant Mach number. The aerodynamic coefficients used in the simulation were varied with altitude since, with a constant Mach number, the forward velocity varies with altitude. Subsystem transfer functions were included in this study and portions of the actual missile components were tested in their respected locations in the simulation loop to establish their compatibility in the over-all system. The simulation studies included the effects of heading errors, gyro reference errors, boresight errors, steady winds, wind shear, wind gusts, and target noise.

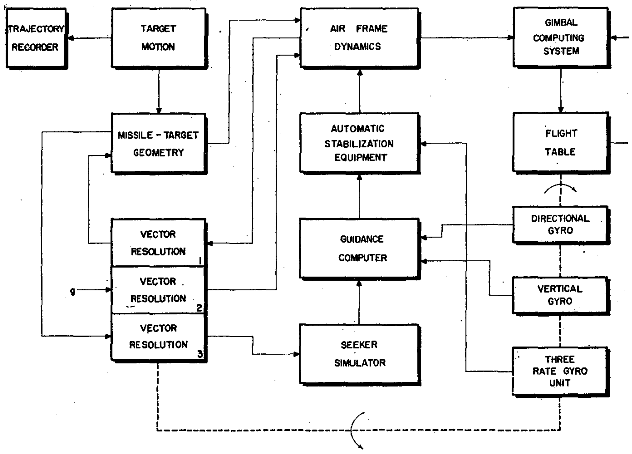

Fig. 3—Simulation block diagram.

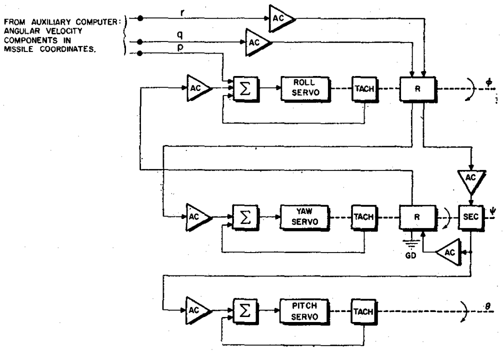

Fig. 4—Gimbal computing system—schematic diagram.

Two sections of this simulation were rather critical with respect to the flight system simulator. These were the flight table and gimbal computing system and the missile-target geometry. These portions of the simulation and the methods used to accomplish them provided the primary problem in this particular simulation effort.

The Flight Table and Computing System

The gimbal computing system, as shown in Fig. 4, is a nonorthogonal transformation of missile body angular rates to the correct rates for driving the three flight table servos. This system is composed of analog equipment using 1000-cps suppressed carrier signals. The performance characteristics of the system were as follows:

Static accuracy	0.15 per cent of full scale
Bandwidth	50 cps
Drift	0.5 degree/minute (maximum).

The flight table, which contains the three servos, is an integral part of the gimbal computing system. (A complete discussion of the implications of this feature and the performance requirements for flight tables may be found in Blanton.[2])

The flight table is driven by three rate servos with the characteristics indicated in Table I.

TABLE I

	Roll	Yaw	Pitch
Maximum acceleration (rad/sec^2)	2500	500	500
Maximum velocity (rad/sec)	50	15	15
Attitude range (for noninterference of load—degrees)	Continuous on all gimbals		
Frequency for 90-degree phase shift (cps)	100	45	45
Transient time constant (sec)	0.002	0.003	0.003
Positional accuracy (degrees)	0.2	0.2	0.2
Input signal	1000-cps suppressed carrier voltage corresponding to a velocity command on all gimbals		
Steady state velocity error	Less than 0.015 per cent on all gimbals		
Maximum load	50 pounds (with the center of gravity not more than 2.5 inches from the center of rotation of the yaw gimbal; at reduced pressures, the center of gravity can be as much as ten inches from the yaw axis)		
Load dimensions for noninterference	8-inch diameter and 15-inch length		

These servos were, of course, designed for high-speed missile simulation. The use of this equipment in the simulation of a slower air vehicle might therefore be questioned, since accurate low-speed servo performance is limited by loop gain, static friction in the hydraulic motor and drive, and the low-speed capabilities of the tachometer.

In the design of the flight table servos, the principal factor that limited the low-speed performance was the tachometer. However, a careful choice of available tachometers provided excellent low-speed performance. A maximum load velocity variation of 0.5 per cent was achieved at a load speed of 0.3 radian per second. (A more complete discussion of the servo is contained in Bailey and Feder.[3])

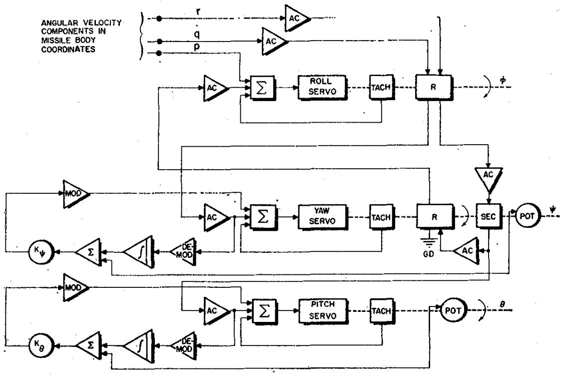

Fig. 5—Modified computing system.

Therefore, it is apparent that the low-speed properties of the flight table are really quite outstanding, and that the burden for accurate computation at any speed might be placed on the other computing elements in the system. Servo drift, however, must be considered. The flight table servos are electromechanical integrators that are exceptionally free of the type of drift normally attributed to the electronic integrator. Instead, the electromechanical integrator is subjected to a "counter" drift because of the tachometer. In effect, the tachometer feedback contains positional information which acts in the same manner as stiction in the servo. A more complete discussion of the effects of this type error is contained in Jones.[4] In this work, it was concluded that the errors of the two types of integrators, electronic and electromechanical, have a ratio on the order of

$$\frac{E_e}{E_m} = k \frac{T_s}{T}$$

where E_e is the error of the electronic integrator, E_m the error of the electromechanical integrator, k the ratio of the average drift of the electronic integrator to the aximum value of the tachometer positional error, T_s the total duration of the solution, and T the amount of time during which the input to the electromechanical integrator is very near the threshold. A good estimate for the value of k is 0.02. The value of T/T_s for the simulation of subsonic air vehicles involving lengthy flights is on the order of 2/3, and at the extreme approaches unity. Thus, for this type of simulation with the present configuration of the flight table, the more accurate integration would be accomplished with electronic integrators.

Therefore, to improve the integrating capability of the system at low speeds, and hence the flight table resolution, an electronic integration and a position feedback loop were added to the yaw and pitch gimbal servos of the gimbal computing system, as shown in Fig. 5. Although system bandwidth was decreased by a factor of approximately two with the position feedback circuit added, the response was more than adequate for the simulation study. In addition, this improvement was accomplished with a minimum of effort using standard flight simulator electronic components.

The improvement was immediately apparent in the simulation operation since the drift problem was all but eliminated. A trajectory with a duration on the order of three minutes was compared to a digital check solution and a maximum deviation of less than 2 per cent was obtained.

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[2] H. E. Blanton, "Performance Requirements for Flight Tables," Wright-Patterson Air Force Base, Ohio, WADC Tech. Rep. No. 54-250, pt. 10; September, 1954.

[3] K. V. Bailey and M. S. Feder, "Design of a high performance hydraulic control system," *Proc. Natl. Simulation Conf.*, pp. 7.1-7.6; 1956.

[4] T. Jones, Jr., "The propagation of errors in analog computers," Master's thesis, Mass. Inst. Tech., Cambridge, Mass.; May, 1952.

MISSILE TARGET GEOMETRY

Probably the most critical portion of any three-dimensional missile simulation, with regard to the electronic computer, is the missile-target geometry, or the kinematics of the problem. The mathematical structure of the simulated system was

$$l = \frac{x}{R}$$

$$m = \frac{y}{R}$$

$$n = \frac{z}{R}$$

where

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$x = \int u dt + x_0$$

$$y = \int v dt + y_0$$

$$z = \int w dt + z_0.$$

In this formulation, R represents the missile-to-target range; x, y, and z, the relative components of the line-of-sight expressed in inertial coordinates; u, v, and w, the translational velocities; and l, m, and n, the direction cosines of the line-of-sight in inertial coordinates.

An extensive preliminary study was conducted to obtain a geometry system which would be suitable for this simulation and compatible with the high resolution capacity of the flight table. Square root and division loops, with an accuracy of better than 5 per cent over a range of 100:1, were considered necessary because the required miss distance was relatively small with respect to the initial range.

A number of methods were evaluated and found unsuitable. The system adopted was one involving a continuous rescaling process during the computation. The equations simulated were:

$$10kR = \left[(10kx)^2 + (10ky)^2 + (10kz)^2\right]^{1/2}$$

$$l = \frac{10kx}{10kR}$$

$$m = \frac{10ky}{10kR}$$

$$n = \frac{10kz}{10kR}$$

$$k = \int_0^T \left[a(10kR) + b\right] dt + \alpha \qquad \text{for } 0 \leq t \leq T$$

where

$$\left[a(10kR) + b\right]_{t=0} = 0$$

$$k = 1 \qquad \text{for } t \geq T.$$

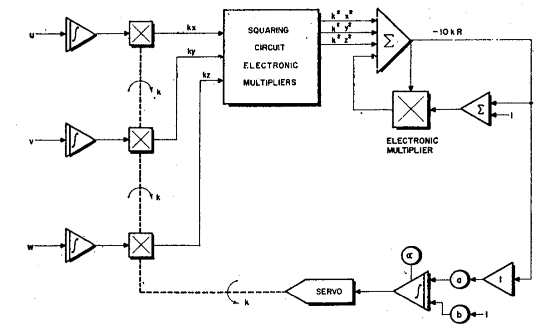

Fig. 6—Missile-target geometry simulation diagram. *Note:* a, b, α are adjusted for best results. $a = b \cong 0.2$, $\alpha \approx 0.10$.

The solution of these equations is shown in Fig. 6. Electronic multipliers were used throughout since rapid response was necessary when target noise was introduced. A rescaling servo was driven by the function k as indicated. With the computer in the initial condition state, k approximately equaled 0.1 (where unity indicated one machine unit, or full scale). When computing, k increased as R decreased according to the defining equation and when R equaled 0.1, k was approximately 1. The unscaled circuit provided good accuracy to 0.1 machine unit, while the rescaled circuit was accurate to approximately 0.01 machine unit.

Resolution of the direction cosine information into missile axes was accomplished by the resolver computing section of the flight systems simulator. For the over-all system, accuracy was approximately 1 per cent for $0.1 < R < 1$, and better than 5 per cent for $0.01 < R < 1$.

CONCLUSIONS

It has been demonstrated that a real-time simulator with a high dynamic range can be used for applications which require performance on the lower portions of the dynamic range. Such a simulation, using the Bendix three-dimensional flight systems simulator, has been described. Problems encountered in the flight table and gimbal computing system were solved by careful selection of critical components (*e.g.*, the feedback tachometer) and the addition of an electronic integrator and a position feedback loop to the yaw and pitch gimbal servos of the system. Problems presented by the missile-target geometry were minimized by use of electronic multipliers and a continuous rescaling process during the computation.

ACKNOWLEDGMENT

The successful culmination of the simulation studies performed with the aid of the Bendix flight systems simulator has been the result of the combined effort of many people under the guidance of C. M. Edwards. The author especially wishes to thank F. B. Lux, who suggested the rescaling circuit for the kinematics section, W. H. Baur for his effort in synthesizing the computer setup, and J. Kaiser for his valuable assistance in the simulation program. Other Bendix Research Laboratories Division personnel also contributed support to this program.