Application of Computers to Automobile Control and Stability Problems

ROBERT H. KOHR†

CURRENT advances in automobile stability and control studies can be credited largely to modern electronic computers. Solution of the lateral control problem, like problems from other areas of automotive technology, has been deterred by the extreme complexity of the automobile. Some understanding of static or steady-state stability was obtained in the late 1930's, but the solution of dynamic response to steering input was not obtained until the advent of modern high-speed computing equipment.

The Automobile Stability and Control Problem

Stability and control of the automobile, sometimes called the handling problem, is concerned with providing an automobile with the proper steering behavior. It is really two problems: 1) providing a vehicle with directional sense so that it will run straight of its own accord, and 2) providing sufficient steering control so that the automobile may be easily steered along some desired path.

In its simplest sense, the study of automobile stability and control is the study of the lateral motions induced in a car by the steering inputs of the driver. The entire system of automobile steering response, shown in Fig. 1, consists of a driver, a steering gear, and an automobile. The first block is the driver whose information input sets the system in operation. This may be the desire to go straight, to pass another car, or to turn a corner. In response to the information input, the driver decides to do something and as a result applies a torque to the steering wheel which turns the steering wheel to a given position. This action, working through the steering gear, turns the car's front wheels to some angle and finally the car begins to change its path down the road. The automobile's change in path is called its lateral response. Besides the flow of effects forward from the operator to the lateral response, there are also several feedback loops. For example, the lateral response is fed back to the steering gear as a torque, and to the operator in the form of visual inputs and lateral acceleration. There is also steering-wheel torque feedback from the steering gear to the operator.

Although the human operator steers his automobile by a combination of steering torques and steering displacements, it is possible to study the responses to these two types of steering inputs separately. The stability and control characteristics associated with a fixed steering wheel or the response produced by a steering-wheel displacement are called the "fixed control" characteristics. Conversely, the characteristics associated with a free steering wheel,

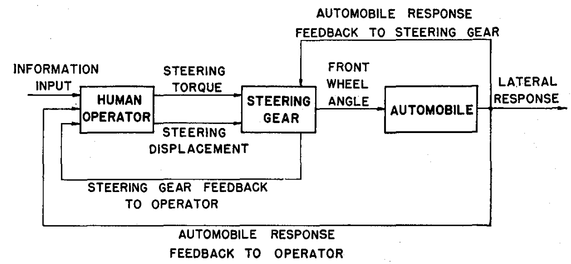

Fig. 1—Block diagram of car control system.

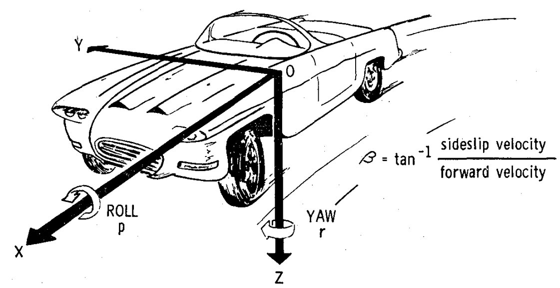

Fig. 2—Axis system for simplified automobile.

or the response produced by a steering torque, are called the "free control" characteristics. The remainder of this paper is concerned with the block labeled "automobile" and particularly its lateral fixed-control response to front-wheel steering inputs.

Development of Equations of Motion

The automobile motions, of concern in studying lateral response, are shown in Fig. 2. They are yawing, rolling, and sideslipping, and are defined as follows:

1) Yawing is the angular velocity about the vertical reference axis (OZ) and is denoted by the symbol r.
2) Rolling is the angular velocity about the fore-and-aft horizontal reference axis (OX) and is denoted by p.
3) Sideslipping pertains to the side velocity and is described by the sideslip angle β.

The axis system is effectively fixed in the unsprung mass (wheels and tires) so that Y and Z axes do not roll with the sprung mass (body, frame, and engine), but remain parallel and perpendicular to the road surface respectively. The car is considered to have a constant forward velocity along the X axis, and for the small angles of sideslip usually encountered, the sideslip angle β is defined as the side velocity along Y divided by the forward velocity along X.

† General Motors Corp., Detroit, Mich.

A first attempt was made in 1953 to describe the lateral motions of a car by use of several differential equations.[1] Later that same year, General Motors enlisted the aid of the Cornell Aeronautical Laboratories. Cornell's extensive experience with stability and control problems in the aircraft field, particularly their advanced instrumentation techniques, was brought to bear upon this problem.[2] The result of this joint effort was a verified set of equations which describe a car's lateral response to steering inputs. The set of three simultaneous linear differential equations which represent a car's handling motions is shown in Fig. 3.

In each of these equations, the mass, inertia, and acceleration terms are given on the left, while the forces and moments that cause the acceleration are given on the right. The forcing terms on the right each consist of the product of some motion variable and a "stability derivative." For example, in the side-force equation, δ is the front-wheel steer angle put in by the driver, and Y_δ is the stability derivative with the dimensions of force per unit angle of front-wheel steer. The stability derivatives appear in each equation and are composed of various car parameters, like tire lateral stiffness, weight distribution, suspension characteristics, and various other terms. In all, there are 20 car parameters included in the equations of motion. In deriving these equations, it was assumed that the tires, springs, and shock absorbers all behave linearly and that the car is operating on a flat road with no wind blowing so that the only force input to the system is caused by the driver's turning of the front wheels of the car.

Measurement of Vehicle Parameters

The initial step in the experimental program consisted of determining the actual values of the mass, chassis, and tire characteristics of a particular automobile. The yawing moment of inertia, Iz, was measured by hanging the car on four cables and swinging it as a multifilar pendulum. By measuring the frequency of the yaw oscillation, it was possible to compute the yawing moment of inertia. The rolling moment of inertia, Ix, and the product of inertia linking the yawing and rolling motions, Ixz, were both determined by oscillating the car on knife edges about a horizontal axis. Ix was found from the rolling oscillation frequency and Ixz from the yawing moment produced by the rolling oscillation. The total weight and the longitudinal center of gravity locations were obtained by a simple weighing process.

Chassis characteristics, such as roll-spring rates, rear-axle roll steer, and front-wheel camber due to body roll, were determined by standard General Motors Proving Ground tests. The damping characteristics of the shock absorbers were determined with a stroking machine, and the damping produced by the shock absorbers in the suspension system became a simple geometrical calculation.

[1] R. Schilling, "Directional control of automobiles," *J. Indus. Math. Soc.*, vol. 4, pp. 64-77; 1953.
[2] W. F. Milliken, Jr., "Dynamic Stability and Control Research," Cornell Aeronautical Lab., Report No. CAL-39; 1951.

Side force equation

$$MV(\mathring{\beta} + r) + M_s h\mathring{p} = Y_\beta \beta + Y_r r + Y_\delta \delta + Y_\phi \phi$$

Yawing moment equation

$$I_z\mathring{r} - I_{xz}\mathring{p} = N_\beta \beta + N_r r + N_\delta \delta + N_\phi \phi$$

Rolling moment equation

$$I_x\mathring{p} + M_s hV(\mathring{\beta} + r) - I_{xz}\mathring{r} = L_p p + L_\phi \phi$$

Fig. 3—Lateral equations of automobile motion.

Variable of Motion	Sensing Instrument
1) Left front-wheel position, δ_L	Angular potentiometer
2) Right front-wheel position, δ_R	Angular potentiometer
3) Steering-wheel position, δ_{SW}	Angular potentiometer
4) Lateral acceleration, η_Y	Statham lateral accelerometer
5) Roll attitude, ϕ	Minneapolis-Honeywell attitude gyro
6) Pitch attitude, θ	Minneapolis-Honeywell attitude gyro
7) Angular yaw velocity, r	Doelcam rate gyro
8) Angular roll velocity, p	Doelcam rate gyro
9) Forward velocity, V	Fifth-wheel-generator set

Fig. 4—Measured variables and associated transducers.

Tire side force and moment characteristics were supplied by the tire manufacturer who obtained the data by running the tire on a moving drum. These tire characteristics were determined for wide variation in tire pressure and in the load carried by the tire.

In addition to providing numerical data to insert in the equations of motion, these tests demonstrated that the assumption of linearity for the various car parameters was valid for lateral motions of a reasonable magnitude.

Verification of the Equations

The equations of motion were verified by response tests made with an instrumented 1953 Buick. The instrumentation that was required to measure the car's lateral response is shown in Fig. 4. Both left and right front-wheel positions were measured and averaged to yield the effective front-wheel angle. Since it was not convenient to measure the sideslip angle, the total lateral acceleration along the Y axis was measured with a lateral accelerometer. The pitch attitude is not a lateral degree of freedom, but its measurement was made to determine whether any coupling occurred between the vertical and lateral motions. The outputs of the various motion-sensing instruments were recorded on an oscillograph.

In the early stages of the experimental work, it was assumed that information would be recorded in the frequency range of 0 to 10 cycles per second. After the first shakedown runs, it was discovered that both engine vibrations, and wheel vibrations at the natural frequency of the wheel on the tire, were being picked up by the various transducers. These unwanted vibrations were removed with low-pass filters utilizing a tuned galvanometer so that only frequencies from 0 to 3 cps were recorded.

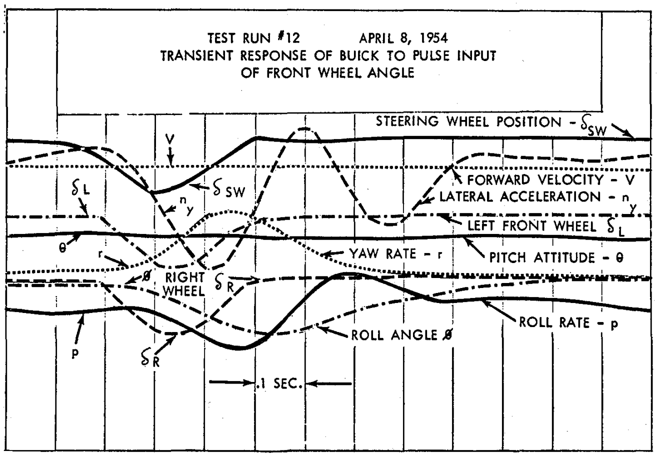

Fig. 5—Typical test response.

Response tests were conducted by stabilizing the car's forward speed, then performing a steering maneuver and recording the resulting car responses. The record of a typical test is shown in Fig. 5. This record shows the transient responses induced by a pulse steering input. All the motion variables are shown here. It is of interest to note that there was no appreciable change in the car's pitch attitude during this test. Numerous tests were made using both step and pulse steering inputs. The "step" and the "pulse" steering inputs used in this test work are only approximations to the mathematically pure step and pulse inputs used by servo engineers. The use of these two inputs with different harmonic content made certain that all of the frequencies of interest were introduced as inputs at some time during the experimental program. In addition to varying the inputs, response data were obtained in which the stability derivatives were varied from normal. With these response data in hand, a comparison was made between the actual measured responses and the responses predicted by the differential equations.

Comparison of Theory and Experiment

This comparison was made on the basis of frequency response, that is, the steady-state response of the automobile to a sinusoidal input of front-wheel steer angle. The theoretic frequency response was obtained by applying the Laplace transformation to the system of equations, and then replacing the Laplace operators by $j\omega$ where ω is the frequency of front-wheel oscillation and j is $\sqrt{-1}$.

The determination of yawing, rolling, and sideslipping frequency responses was then a matter of algebraic computation which was quickly accomplished by use of a digital computer.

The experimental frequency responses were determined from transient responses like the one shown in Fig. 5. This procedure is based on the use of the Fourier integral which, under certain conditions, enables a time function of a system $f(t)$ to be transformed into a complex frequency function $F(j\omega)$. The Fourier integral may be expressed as

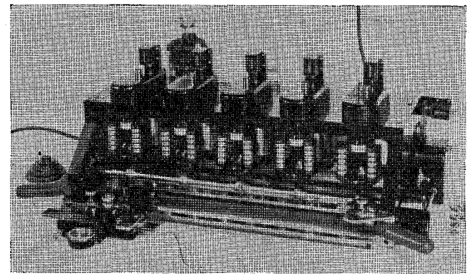

Fig. 6—Rolling sphere harmonic analyzer.

$$F(j\omega) = \int_0^\infty f(t)e^{-j\omega t}dt.$$

Since this integral must be evaluated along the interval from zero to infinity, it is necessary to know the behavior of $f(t)$ for an infinite time. This is most easily arranged by applying a disturbance to the system such that $f(t)$ reaches a steady value in some finite time, T. Under these conditions, the frequency function $F(j\omega)$ may be broken into its real and imaginary parts as:

$$F(j\omega) = R + jI$$

where

$$R = \int_0^T f(t) \cos \omega t\, dt - \frac{f_T}{\omega} \sin \omega T$$

$$I = -\int_0^T f(t) \sin \omega t\, dt - \frac{f_T}{\omega} \cos \omega T.$$

These integrals may be evaluated in a number of ways.[3] One convenient method utilizes the rolling-sphere harmonic analyzer shown in Fig. 6.

This method simply involves following the curve to be analyzed with a cross hair eyepiece that is attached to the analyzer. As the eyepiece is moved along the curve from the starting point (initial conditions zero) to the point where steady state is reached, it actuates a number of rolling ball integrators. Each integrator is equipped with a recording dial, and after the eyepiece has completed the traverse of the curve, the individual dials produce readings which are proportional to the real and imaginary components of the Fourier integral. The machine used in this work produces five harmonics for one traverse of the transient curve.

In order to obtain experimental response data for comparison, it was necessary to analyze the input to the system (steering angle), and the various system responses. Once this was done, a comparison was made between theoretic and experimental responses.

Fig. 7 shows the excellent agreement between the predicted yawing velocity response and that actually obtained on the road. Good agreement was also obtained for the rolling and sideslipping motion. It should be pointed out that the use of the frequency-response technique has two

[3] J. M. Eggleston and C. W. Mathews, "Applications of Several Methods for Determining Transfer Functions and Frequency Response of Aircraft from Flight Data," NACA Report 1204; 1954.

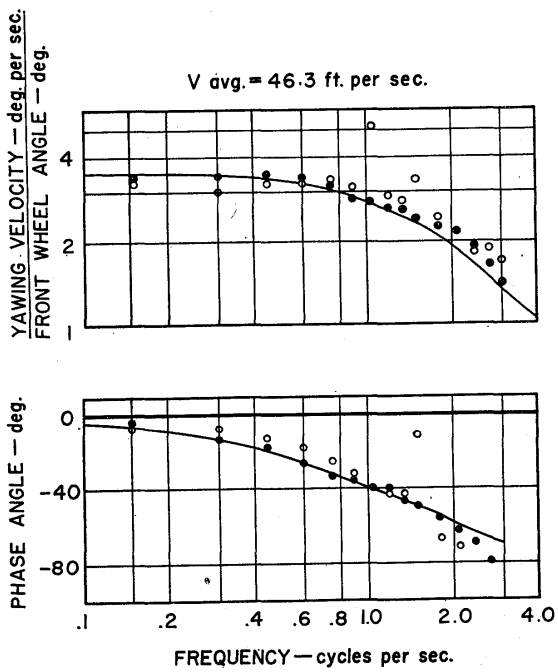

Fig. 7—Theoretic and experimental frequency-response, yawing velocity.

important advantages: 1) it provides for easy removal of dynamic effects produced by the filters in the recording channels, and 2) it provides a more general solution than does a transient response. It may be noted here that the first experimental frequency responses that were obtained did not match the equations exceptionally well, and, through use of the frequency-response plot, it was possible to determine some additional terms which were added to the equations of motion.

Computer Simulation of Vehicle Lateral Responses

When the equations of motion had been verified, they were then used to study the effects of the various car parameters on its lateral response. Both digital and analog computers have been used in this work. The analog computer has been used only to determine the transient response, while the digital computer has been used to determine the transient response, the frequency response, and the roots of the characteristic equation.

Analog Computer Studies

The mechanization of the differential equations on the analog computer follows the standard procedure of summing the various quantities which determine the various accelerations in the system, and then integrating acceleration to velocity and velocity to displacement.

The block diagram in Fig. 8 shows the general procedure that was used. Any of the motion variables, for example, the yaw acceleration *r*, is found by summing, horizontally, the products of the term in each box and the corresponding vertical input. The yaw acceleration is thus found as

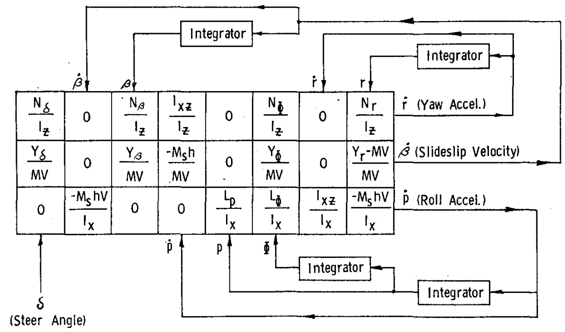

Fig. 8—Block diagram of automobile lateral-motion simulation.

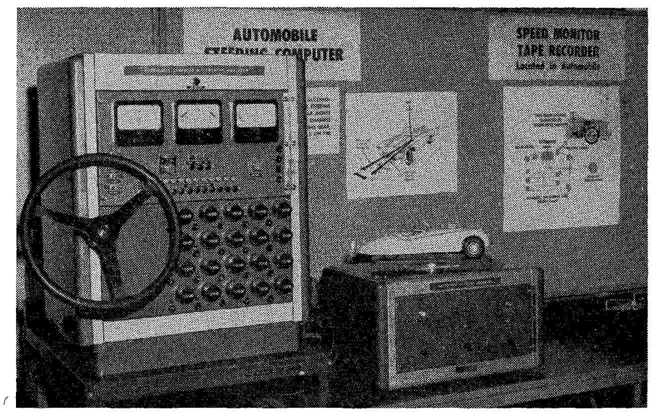

Fig. 9—Real-time automobile-handling simulator.

$$\dot{r} = \left(\frac{N_\delta}{I_z}\right)\delta + (o)\dot{\beta} + \left(\frac{N_\beta}{I_z}\right)\beta + \left(\frac{I_{xz}}{I_z}\right)\dot{p} + (o)p + \left(\frac{N_\phi}{I_z}\right)\phi + (o)\dot{r} + \left(\frac{N_r}{I_z}\right)r, \qquad (1)$$

and is exactly the yawing moment equation that is given in Fig. 3. The rate of change of sideslip and the rolling acceleration are calculated in a similar manner. Each row then represents a summation, with the quantity in each block being the gain factor applied to the various motion variables.

The complete simulation of the equations on the analog computer requires only fourteen amplifiers. Although no nonlinear equipment is used at the present, future work will require the addition of some function generators and multipliers.

Real-Time Simulator

In addition to analog computer studies which are often run in "slow time," a real-time simulator has been of material value in demonstrating the lateral motions caused by steering inputs. This simulator, shown in Fig. 9, is composed of a small, special-purpose analog computer which can be "steered" by turning a steering wheel attached to the computer. This motion causes "steering angle" voltages to be introduced into the analog computer circuit. The computer then solves the lateral equations of motion of the automobile. The voltages proportional to

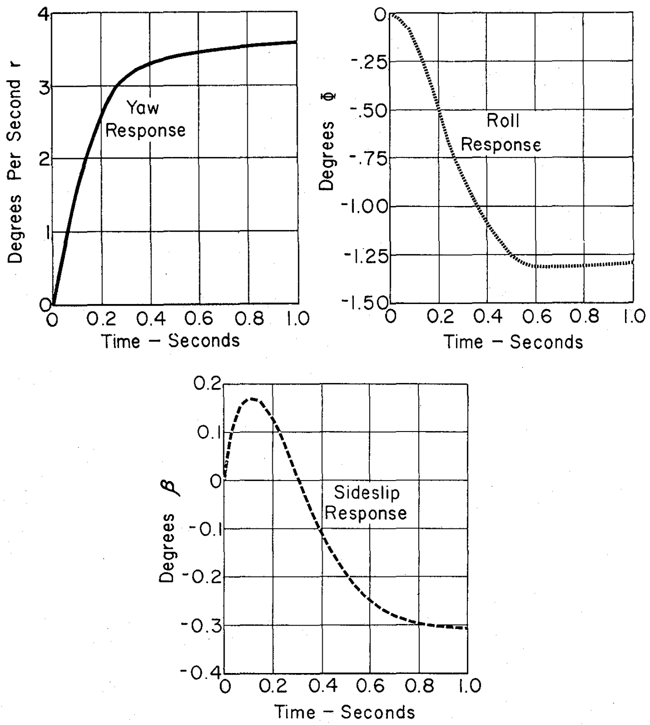

Fig. 10—Yaw, sideslip, and roll response of a 1953 Buick to a 1-degree step-steering input at 30 mph.

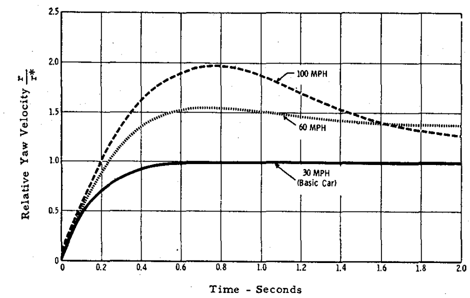

Fig. 11—Effect of speed on yaw.

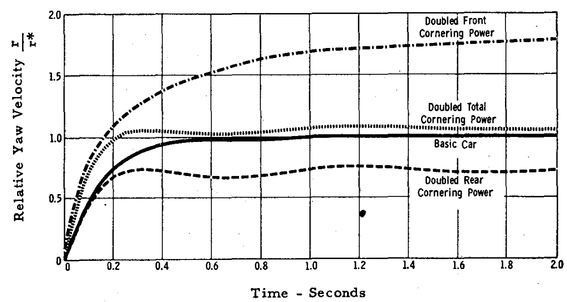

Fig. 12—Effect of tire cornering power on yaw.

yaw, roll, and sideslip are applied to a small servodriven model car. The model car can yaw and roll, and demonstrates sideslip by a pointer mounted on the hood to show the direction in which the car is actually going. The computer includes a "skid" meter, a "velocity" meter, and a meter which indicates the applied steer angle. By use of a ganged potentiometer which has an element in the input or feedback of certain of the amplifiers, it is possible to set the forward speed of the car at any point between 30 and 100 mph.

Digital Computer Studies

The digital computer was used to check the transient response obtained by the analog computer and also to obtain the frequency response and the roots of the characteristic equation of the automobile system. The transient response was determined by a Runge-Kutta method with values of the motion variables determined at each 1/40 of a second of problem time. The frequency response problem required the solution of three simultaneous, complex equations which result when the complex operator $j\omega$ is inserted in the equations. This solution utilized a matrix subroutine which converts a 3×3 complex matrix into a 6×6 real matrix. The roots of the characteristic equation were determined by first finding the characteristic equation by direct expansion and then solving the equation by a Newton-Raphson method.

A typical solution of the lateral response equations is shown in Fig. 10. The transient responses shown are the yaw, roll, and sideslip of a 1953 Buick traveling at 30 mph when the front wheels are steered suddenly to produce a step-steering input of 1 degree. The yaw and roll develop smoothly, but in the opposite sense, while the sideslip starts in one direction and then reverses. If these curves were normalized so that the steady state of each response tended toward unity with increasing time, the yaw would be shown clearly leading the roll and sideslip.

Fig. 11 shows the effect of increasing car speed on the lateral response. The curves in this figure have been normalized with respect to the steady-state yaw response of the car traveling at 30 mph. As the speed increases from 30 to 60 to 100 mph, the yaw response develops an overshoot. This is due to a decrease in the yaw damping of the tires as the car's speed increases.

One particularly important vehicle parameter is the lateral stiffness or "cornering power" of the tires. Fig. 12 shows the effect of changes in the tire cornering power on the yaw response. When the cornering power of the front tires is doubled, the car is placed in an "over steer" condition characterized by the large steady-state response which is reached quite slowly. Doubling the rear cornering power reduces the steady-state response of the system, while doubling both the front and rear cornering power produces a quicker response with little change in the steady-state value.

All the other parameters in the system have been explored and their effects on the various lateral responses have been cataloged.

Results of the Study

As a result of the extensive examination of the solutions of the equations of motion, an understanding of the effects of the various car parameters on the car's lateral response has been obtained. In addition to this somewhat specialized information, a number of general observations regarding the car's lateral response has been made.

The first effect observed pertains to the coupling of the lateral response motions. Fig. 13 shows the relative strength of the coupling between the three types of vehicle motion. Yaw and sideslip are strongly coupled, but there is only a weak coupling linking the roll to the yaw and sideslip. However, this weak coupling is often the reason that a particular automobile is stable or unstable, particularly in yaw.

A second interesting aspect of the automobile pertains to its modes of motion when it is in a state of free lateral oscillation. Determination of the modes of motion was made with the anticipation that there would be a well-defined "directional" mode of motion consisting of yawing and sideslipping, and also a "rolling" mode of motion. The modes were found by calculating the roots of the characteristic equation of the system, then inserting these roots in the lateral-motion equations. This study showed that there are, generally, no distinct modes of motion, but under certain conditions it is possible to find modes of motion which are primarily "directional" and "rolling" in nature.

A third interesting result of this work is that the equations of motion predict lateral-motion responses which are compatible with the on-the-road observations of the "handling" engineer. Early work in this field has often concerned itself with the so-called "oversteer" or "understeer" characteristics of a given car.[4] These characteristics are shown in Fig. 14. Because of the nature of the pneumatic tires, they must run at slip angles β_1 and β_2 in order to produce side forces. If β_1 is greater than β_2, the path followed by the car curves away from the side force. This is called understeer. Conversely, if β_2 is greater than β_1, the path of the car curves toward the side force and this is called oversteer. The directional stability of an understeering car tends to increase with increasing speed, while the directional stability of the oversteering car tends to decrease. In fact, in the case of oversteer, the "handling" engineers found a critical speed at which the car becomes directionally unstable and at this speed only the most alert action of the driver will keep the car on the road. The equations of motion contain terms from which the "oversteer" or "understeer" characteristics may be computed, and also permit the calculation of a speed at which an "oversteer" car becomes unstable.

[4] M. Olley, "Road manners of the modern car," *Proc. Inst. Auto. Eng.*, vol. 15, no. 5, pp. 147-182; 1947.

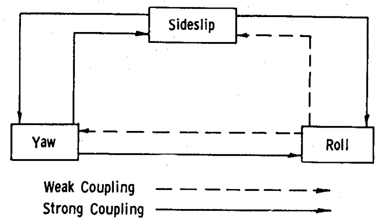

Fig. 13—Relative strength of coupling of automobile's motions.

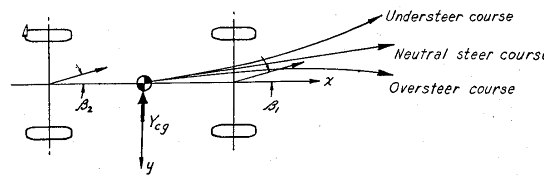

Fig. 14—Understeer and oversteer characteristics.

Conclusion

The result of this research program to date, then, consists of a verified linear model of the "automobile," shown in Fig. 1. This work represents only the first step in an over-all systems analysis of the general automobile-steering problem. In addition to a study of the nonlinearities encountered when the amplitudes of the lateral motion are large, further work will necessarily concern itself with the steering gear and the human operator. In the last analysis, the determination of desirable handling qualities, from the driver's point of view, is the most important result that will come from this research effort.

Discussion

O. J. Osofsky (Republic Aviation Corp.): What are the criteria for entering a skid?

Mr. Kohr: A skid occurs when a tire is in a region where it won't develop any more side force no matter how much more you turn it. If you would plot a curve of cornering force vertically vs slip angle horizontally, the place where the curve peaks and gets flat is generally called the skid region.

Mr. Osofsky: Has any work been done on the motion of an automobile while in a skid?

Mr. Kohr: To the best of my knowledge, no, there has not. The real difficulty is in determining the transient which occurs in going from linear to nonlinear motion. If we knew the complete nonlinear equations of motion we could then predict what the car would do in the skid when the front or rear end broke away first or if they happened to skid simultaneously.

R. J. Mead (McDonnel Aircraft): Can tire "thump" and power-steering dead-zone effects be studied easily?

Mr. Kohr: Tire thump is an acoustic phenomenon which enters the body and which has apparently nothing to do with steering and control. Consequently, we have not considered it in our studies.

As far as power-steering dead-zone effects are concerned, I don't foresee any difficulty in simulating these on either the analog or digital computer, although we have not included any such effect to date.

H. F. Meissinger (Hughes Aircraft Co., Culver City, Calif.): Do you consider emergency handling problems? For example, torque required to hold the car on course under blowout, etc.

Mr. Kohr: As you may recall, the equations that I showed were those which pertain when the force put into the system is only that introduced by the driver, *i.e.*, there are no road forces considered. If we could describe the system mathematically, we might be able to study it. The way this is done currently is as follows. If the car is suspicious in terms of stability during blowouts, it is taken out on a test and a forced blowout is made at a time unknown to the driver. The car's motion and stability following the blowout is then observed.