1. The stabilization channel gain is high enough (approximately 2,000) to reduce drift and offset referred to the summing point of the computing amplifiers to less than 1 millivolt. No balancing control is incorporated in the computing amplifiers.

2. Virtually no crosstalk occurs between computing positions even if one amplifier is overloaded and its summing point is at a high potential with respect to ground.

3. An overload indication occurs at each computing position when that position overloads and at a master position when any amplifier overloads. The indicators, triggered, stay ignited until manually reset.

To the author's knowledge, a combination of high stabilization gain and no crosstalk has previously not been attained. Furthermore, this paper describes the first multichannel system with an overload indication that remains ignited after being triggered. The usefulness of this feature in finding and correcting overloads has been demonstrated during operation of the computer at the DACL. This overload system is also useful in preventing dielectric absorption in the stabilization filter capacitors and input coupling capacitor, and in reducing overload recovery time.

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Combined Analogue and Digital Computing Techniques for the Solution of Differential Equations

P. A. HURNEY, JR.

ONE of the difficulties in the use of an analogue computer for the solution of ordinary differential equations involving variable coefficients is its relative inability to perform certain multiplications rapidly and accurately. In instances where variables must be multiplied by a function of another variable, this difficulty is particularly apparent. This paper describes how a digitally stored table of functions may be used with an analogue computer to solve this general class of ordinary differential equations.

In the preparation of the machine for the problem, the functions of the variables are read into a magnetic drum by employing conventional digital techniques. During operation, the analogue inputs are compared continuously with their digital equivalents that are on the drum, and the digital functions of the input are read and digitally stored between readings. Digital-analogue multipliers are used to multiply the digital output by the appropriate analogue output. The advantages of the computer are (1) the shortness of the machine-preparation time, (2) the operating speeds which are faster than those of electromechanical multipliers using potentiometers or resolvers, and (3) the accuracy which is greater than that obtainable with electronic multipliers and function generators.

Introduction

The first part of this paper discusses a few of the limitations of analogue computers used in the solution of ordinary differential equations. Next are described several types of digital computers which are designed to perform certain operations more efficiently than the equivalent group of analogue components. Finally a specific analogue-digital computer is described which is currently under development at the Dynamic Analysis and Control Laboratory of the Massachusetts Institute of Technology.

Limitations of Analogue Computers

Analogue computers are capable of performing certain mathematical operations with rapidity and high accuracy. Some of these operations are addition, subtraction, multiplication by a fixed coefficient, and integration with respect to time. In general these operations permit solutions to problems involving ordinary differential equations at speeds many times that of the physical system under study and with accuracies often more than adequate for engineering purposes.

A second class of operations performed on analogue computers which are either less accurate or have slower solution times is multiplications of two or more variables. In general, two methods are used for the multiplication of independent variables. One is the electronic multiplier which may have a useful bandwidth of several hundred cycles but long-time accuracies seldom better than 0.25 per cent. A second method of multiplication is the servomultiplier. This multiplier is capable of static accuracies better than ten times that obtainable with the electronic multiplier but one input variable is restricted to a band width of several cycles per second if good accuracy is to be maintained. In addition, there are limitations on the maximum rate of change of the input.

A third class of operations which are restricted both in accuracy or bandwidth is the generation and multiplication of function of a variable. Electronic function generators have approximately the same bandwidth as the electronic multipliers but long-time stabilities are seldom better than 0.25 per cent. Additional errors are introduced in approximating complex functions by a series of straight lines. As a result, the product of one variable by a function of a second variable using electronic multipliers and function generators may have static errors as large as 0.5 per cent.

Servomultipliers with tapped potentiometers may also be used to obtain prod-

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ucts of the function of a variable. The tapped potentiometers are loaded with appropriate resistors to make the potentiometer conform to the desired function. The static accuracy and repeatability of the servo function generator are excellent but, as in the case of servomultipliers, the bandwidth is restricted to several cycles per second. In addition the tapped potentiometer also approximates the function with straight line segments.

For certain classes of nonlinear functions special techniques are used. The trigonometric functions are common to problems. The most widely used method for generation of sines and cosines is the servoresolver. This unit is similar to the servomultiplier except that special nonlinear multiplier potentiometers are used. These potentiometers generate sine and cosine functions to good accuracies. However, for static accuracies better than 0.1 per cent the bandwidth of the servoresolver is even less than the servomultiplier. This is because accurate sine-cosine potentiometers are large, having high inertias and friction levels.

One device used in the resolution of vectors in analogue computers is the a-c resolver. These electromagnetic units have small inertias, low friction levels, high accuracies, and infinite resolution. As a result a servo using a-c resolvers has a bandwidth superior to the servoresolver using potentiometers. However, a serious limitation in the use of the a-c resolver is the a-c carrier needed in the operation of these units. The a-c carrier system generates many special problems. One difficulty is in the combined use of a-c and d-c computing equipment with the present lack of accurate high-speed modulation and demodulation apparatus. It is often desirable in a large problem to perform a portion of the computation on d-c analogue equipment and the trigonometric computation on the a-c section of the computer. It is not economical to use only an a-c computer in the solution of many problems because of the complexity of a-c equipment needed to perform many computations which are basic to the d-c analogue computer.

The foregoing brief and incomplete description indicates some of the limitations of the modern analogue computer. There is a need for better methods for the generation of functions of variables, multiplication by functions of variables, in particular, the trigonometric functions, and, to a lesser extent, multiplication of two variables. The need for faster, accurate computation methods arises from two sources. First, complex systems involving a number of independent variables may be studied rapidly to determine effects of parameter variation. Secondly, simulation of systems involving physical components requires the computer to operate in real time. For example, the study of a guidance and control system for a missile involves simulation in 3-dimensional space. This requires a number of geometric resolutions and multiplication by arbitrary functions. A study of a complete missile system may involve hundreds of solutions to include the many possible initial conditions and parameter variations. Very often, components of the guidance and control system are included in a final study of an over-all system. The section of the computer simulating the aerodynamic and space geometry must be capable of operation in real time to be able to test the system components. In the case of missiles where the actual flight tests are expensive, it is desirable to be able to check the effect of the operation of the missile components on the over-all system. The accuracy of computation of space geometry is very important in a problem of this type. Since these computations are mathematical in nature rather than a simulation of physical equipment, small errors in geometry may result in large over-all problem errors.

Digital Auxiliaries to Analogue Computers

The digital computer which may be used with analogue computers is naturally a specialized device and may differ in many respects from conventional equipment. Input-output conversion is needed which will allow the digital machine to operate simultaneously with the analogue computer. For convenience in programming, the digital equipment must be capable of operating much as an analogue element in the complete system. The digital computer must operate at a time scale compatible with the analogue computer so that the lags in the digital computer will not effect the accuracy of the over-all problem solution.

Several types of digital computers are useful in the applications described. The first of these is a computer consisting primarily of a multiplier. With a multiplier and suitable programming, problems involving multiplication of variables, trigonometric function generation, reciprocals, and powers, and, to a lesser extent, arbitrary functions may be solved. The second type of digital computer consists of a storage element in which is stored an input and a function of that input. This type of digital computer may be called a function table. The stored function of the input may be any function of a single variable. The function table may have stored trigonometric, inverse trigonometric, or any arbitrary relationship between the input and output. With external multipliers, products of variables involving any of the stored functions are obtainable.

The basic difference between the two digital methods described is that in the case of the multiplier or polynomial generator the relationship between the input and output is stored as a series of instructions in the program. The function table has stored all the relationships between the input and output. Each method has advantages and the choice of either depends upon the type of problems to be solved.

The polynomial generator is primarily a serial type of computer, i.e.; for one multiplier only one operation or product may be computed at one time. However, if the multiplier is fast enough, the unit may be time-shared among many inputs. The analogue input is first converted into a digital number by a time-shared analogue-to-digital converter (ADCON). Next the operation is performed on the input number according to the instructions in the problem control. Operations involving trigonometric functions are computed with several successive multiplications and additions until the required accuracy is obtained. Several registers are generally needed for internal storage. Arbitrary functions may be computed from one or more polynomial approximations, although very often this type of function is very difficult to program to the required accuracy. In addition, programming time for even simple arbitrary functions may be excessive. More accurate and convenient function generation may often be obtained using conventional electronic analogue function generators. However, if a problem involves a number of trigonometric functions and multiplications, the polynomial generator provides a fast accurate method for performing these computations. The output digital number is converted into analogue form by the digital-to-analogue converter (DACON) which is time-shared among the various outputs. The output information is stored in an analogue storage device between successive digital solutions.

The function table computer which involves digital storage of each value of the input and the function of the input offers a method of computing any function of a single input variable. It is this type of computer which is under development at

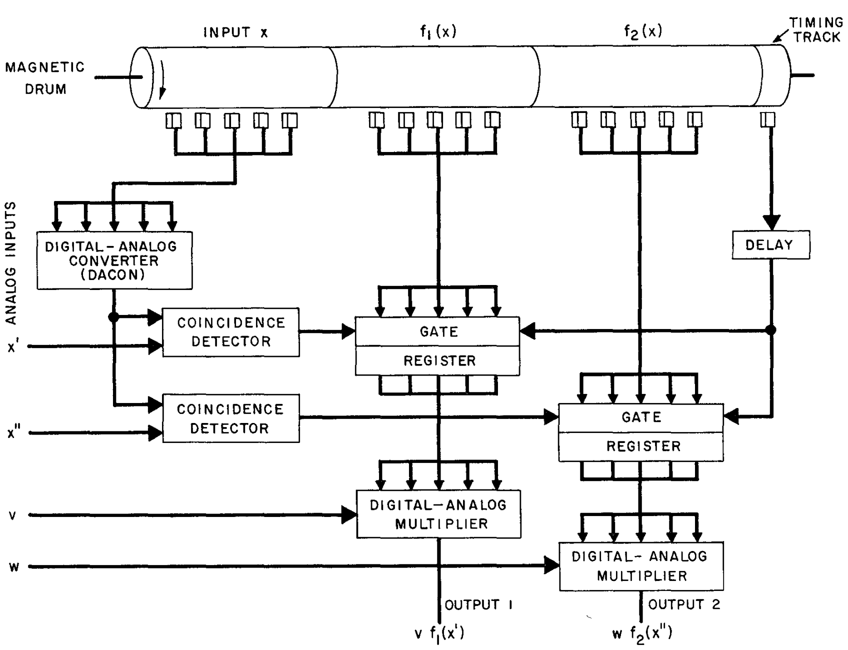

Fig. 1. Digital function table

the Dynamic Analysis and Control Laboratory (DACL) at the Massachusetts Institute of Technology. The function table computer, together with a digital-analogue multiplier, provides a general-purpose auxiliary to an analogue computer which results in an extremely flexible over-all machine. Not only does the system allow rapid and accurate solution to many type of problems, but it also results in a simple programming procedure.

The system described in the next part of this paper will allow the solution of large-scale problems. It may be used in solving problems involving resolutions, function generation, and multiplication. The system bandwidth should be higher than conventional d-c electromechanical computing elements, and compatible with most electronic analogue computers. Over-all reliability should be greater than an equivalent electromechanical system because of the elimination of the servos with their associated potentiometers, gearing, and motors.

Description of the Function Table Computer

The basic element in the system is the storage device, a magnetic drum. On this drum are stored columns of tabulated numbers similar, for instance, to a table of sines. In the first column are the values of the angle, and adjacent to this, a column with the values of the sines of these angles. On the magnetic drum, not one, but a set of desired functions may be stored. Each function shares a single column of nondimensionalized inputs. As the drum is rotated, the input column is scanned, and when the desired value of input is reached, a function of the input is read from the proper column and stored. Fig. 1 is a block diagram of a drum and associated equipment. In this example the drum has three columns of numbers. The first is the input x, the second $f_1(x)$, and the third $f_2(x)$. As the drum rotates, the output of the magnetic reading heads which scan the drum read a particular value of x, $f_1(x)$, and $f_2(x)$.

The digital numbers representing the input are converted by DACON into an analogue voltage. This voltage is fed into the coincidence detector along with the analogue input x'. When the analogue voltage representing the digital drum input equals the analogue input x' the coincidence detector opens the gate which reads the instantaneous value of $f_1(x')$ on the drum and puts it into a storage register. When the second analogue input x'' equals the drum input, in like manner, $f_2(x'')$ is put into another storage register. If the products of these functions with other input variables are desired, the outputs of these registers are fed into digital-analogue multipliers along with the analogue variables v and w, resulting in the products

$$\text{Output}_1 = vf_1(x')$$
$$\text{Output}_2 = wf_2(x'')$$

The system may be expanded in an obvious manner to obtain additional products and functions of other input variables.

The basic elements of the system are:

1. Magnetic drum.
2. Digital-to-analogue converter.
3. Coincidence detector.
4. Gate.
5. Storage register.
6. Digital-analogue multiplier.

THE MAGNETIC DRUM

The magnetic drum is the information storage unit and must be large enough to hold the required number of functions. It must have around its periphery enough values of any particular function to represent this function to the desired precision. The speed of the drum should be as high as possible in order to obtain the shortest possible sampling interval, which results in the widest bandwidth. Presuming that a basic precision of 0.1 per cent of full scale is desired, the input which is a linearly increasing ramp must be divided into 1,000 parts for the positive voltage range and 1,000 parts for the negative voltage range or a total of 2,000 parts. Therefore, there should be 2,000 different digitally coded values of the ramp around the periphery of the drum. Since the ramp is coded in binary form, an 11-bit word is the minimum necessary if 2,000 different values are to be represented (2 inches = 2,048). Therefore the input ramp consists of 2,000 11-bit words equally spaced around the drum. For each value of the input there is a corresponding value for each function. The speed of the drum determines the number of times per second that a new value of function appears at the output of the system. At present, a speed of approximately 125 revolutions per second appears feasible. In this case the basic output frequency is 125 cycles per second and the corresponding sampling interval is 8 milliseconds. The equivalent lag is 4 milliseconds because the error is zero at each sampling time and builds up to a maximum just before the next sample. Phase compensation circuits may be used on the input analogue signal. This compensation effectively cancels the drum lag and results in an equivalent phase lag of the over-all system of less than 1 degree up to 10 cycles per second. In addition, there are no restrictions on the maximum rate of change of the input variable.

Information is read into and out of the drum through magnetic heads and read-write amplifiers. The "write" equipment may be as simple as a keyboard, but for convenience in programming, punched cards or tape should be used. The "read" amplifiers must be capable of driving several gates in parallel.

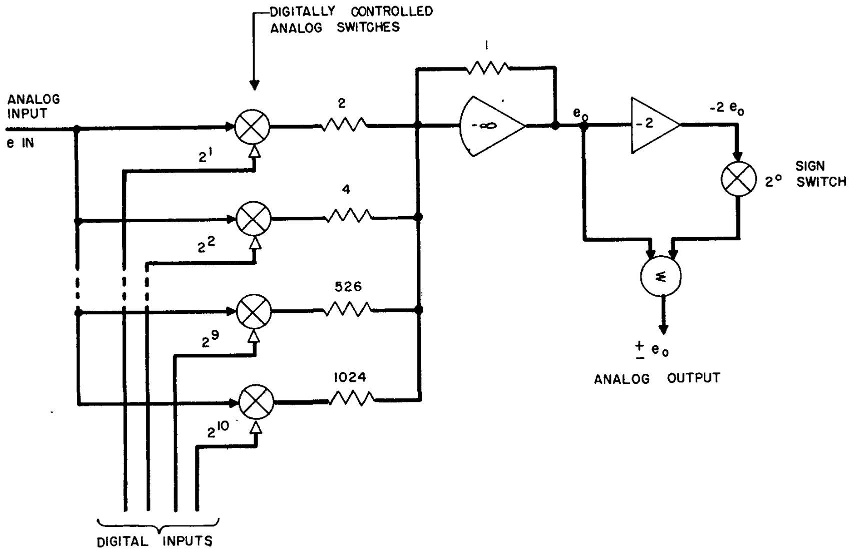

Fig. 2. Digital-analogue multiplier

A typical drum would have provision for the storage of seven functions plus the input ramp function. In addition several tracks are used as timing tracks. On these tracks are stored timing pulses which are used to synchronize the write and read gates.

The Digital-Analogue Converter (DACON)

The digital-analogue converter is an electronic device which converts paralleled binary-coded information to analogue form. Basically the unit consists of a series of accurate current switches driving a resistance ladder. As many current switches are needed as there are bits in the coded information. A converter of this type can be built to respond in less than a microsecond; in this system several microseconds are available. The drum rotates at about 125 cycles per second and there are 2,000 words in each revolution; therefore, a new word occurs about every 4 microseconds. Approximately half this time would be available to operate the DACON associated with the drum input circuitry.

The input DACON is isolated from the coincidence detectors by an amplifier which prevents loading of the converter by the detectors. The amplifier must have a wide bandwidth and an accurate gain. Only one such amplifier of this type is needed.

The Coincidence Detector

The coincidence detector is essentially a zero-sensing detector. When the analogue input to the system and the analogue signal from the input DACON have the same value, the output of the detector changes sign rapidly. If the output of the DACON is less than the analogue input signal the output of the detector is at a negative potential. As the ramp signal from the drum increases, the output from the DACON increases in a positive direction until the voltages are equal. At this time the sign of the output of the coincidence detector reverses and a positive gating signal is generated.

The Gate

The gate is used to control the transfer of information from the drum into the storage register. The gates are controlled by signals from the coincidence detector and from the timing unit. See Fig. 1. The timing signals from the drum are used to prevent the gating of a signal into the register if the number on the drum is changing value. The timing signals are spaced so that the gate may be opened only when the output from the coincidence detector and the input DACON have reached a steady state value.

The gate is associated with the register and therefore is logically a part of the register. However, it is discussed separately here because of its function in the operation of the system.

The Storage Register

The function of the storage register is to store information from the drum between samples. The input to the register is a binary coded parallel signal from the drum which is switched into the register by the gates from the readout amplifiers. The register is reset just before the new information is to be stored. The reset function is accomplished by the timing circuit. When the gate associated with a particular register opens, information is supplied to each of the flip-flop elements in the register simultaneously. The presence of a pulse on any line sets the flip-flop associated with that line. Because of the low repetition rate of the registers, nondestructive readout magnetic cores may be used in the construction of the register.

Digital-Analogue Multiplier

The digital-analogue multiplier is a unit which has one analogue and one digital input and an analogue output which is a product of the two inputs. Basically the multiplier consists of ten parallel conductances arranged in binary ratios. The conductances are switched into the circuit according to commands from the register. The ten switches and ten conductances can be made to have 1,028 different values of total conductance. These conductances form the input resistor to a feedback amplifier resulting in an amplifier which has a gain between 0 and 1 depending upon the condition of the switches. See Fig. 2 for a simplified schematic of the multiplier. The 11th bit from the register controls the sign of the digital multiplication. It operates an electronic switch at the output of the multiplier which changes the sign of the multiplication. Since the analogue input may have either sign the over-all multiplication is in four quadrants.

Since the digital input to the multiplier changes in a stepwise manner the gain of the unit also changes in increments. Therefore the output wave form contains components of the sampling frequency. The wave form is similar to that obtained from a servomultiplier which uses a wire-wound potentiometer. The switching time of the digital-analogue multiplier which has been developed at Dynamic Analysis and Control Laboratory has a switching times of a few microseconds and a change in the digital code does not introduce spurious signals into the output. The output wave form therefore has small components of the switching frequency which are easily filtered.

Operation of Function Table

The computer would consist of the magnetic drum, the input digital-to-analogue converter, one coincidence detector for each input signal, a gate and register for each desired function, and a digital-analogue multiplier for each product. The analogue inputs and outputs would be handled on the analogue computer patchboard in the standard manner.

The interconnection of the digital components could be accomplished with multiconductor patching or switching within the digital computer itself. The actual computer setup time for interconnection of units would be approximately the same as a conventional machine.

The information to be stored on the drum would best be handled with punched cards. Functions such as sines or cosines could be calculated and punched on cards using standard digital computers. Empirical function could be either punched into cards using a manual keyboard, or if desired, curve followers with digital outputs could be used to encode the desired information automatically. Once the set of data has been placed on the cards the information is read into the drum using well known techniques. If standard equipment were used each function would require approximately 10 minutes to be read into the drum. Once the function is on cards it could be stored and would be available at any future time for use on the machine.

CONCLUSIONS

The function table computer together with a suitable analogue computer would be a machine capable of solving many types of problems faster than is now possible with analogue computers. The accuracy of the proposed system would be comparable with present computers. Programming is simplified because of the ease of computing with any function of a variable. The combined computer should be easier to maintain because of the absence of servomultipliers and resolvers. Checking the operation of the digital function table may be accomplished by standard analogue techniques. At present the machine is feasible because it uses available equipment and techniques. Future development in components may reduce the cost of the computer and enhance its usefulness.

An Experimental Monitoring Routine for the IBM 705

H. V. MEEK

THERE is a pressing need for aid in the knotty business of checking a code for a large digital computer. What better instrument is to be used than the computer itself? Because an automonitoring feature is absent from the circuitry of most computers, the monitoring operation should be programmed and will furnish an effective means of detecting many coding errors.

A routine for the International Business Machines Corporation (IBM) *705* has been prepared which monitors the instructions of a code being tested, and gives a complete history of the computer action as a result of that code.

Before the format of the output of the monitoring routine is discussed in detail, a brief description of the *705* internal nature should be given. Also, some of the questions which arose during the monitor planning phase and the decisions which were made should be mentioned.

The *705* is a high-speed stored-program electronic data-processing machine. Its main memory is either 20,000 or 40,000 character positions of magnetic core storage. Each character consists of seven binary digits; a 4-bit "numeric" part, a 2-bit "zone" part, and one check bit, and may be a decimal digit, a letter, a punctuation mark, or a special symbol. A 256-position accumulator, and 15 auxiliary storage units which together comprise 256 positions, provide temporary working storage for arithmetic and logical operations. Instructions have five characters each: one operation-code character and a 4-decimal digit address part. The thousands position of the address part must be appropriately zoned to specify a location with address greater than 9999. If one of the 15 auxiliary storage units is to be specified, the tens and hundreds positions of the address part are zoned so that the four zone bits together form a binary number equal to the address of the desired auxiliary storage. There is no fixed word length in the *705;* the length of an operand depends on the current condition of the specified accumulator or storage, the type of instruction being executed, and/or the characters of the operand and its adjacent fields.

Now then, what exactly should the monitor tell about a code? It was decided that each executed instruction, its location, its interpretation, the operand it calls for, and the results it produces would be given. The instruction interpretation consists of a 3-character mnemonic operation symbol, a 5-decimal digit address part, and a 2-digit reference to the accumulator or auxiliary storage unit. Each operand, including its length and sign, each change in main memory content, and the length, content, and sign of the accumulator or an auxiliary storage unit after each reference will be noted alongside the corresponding instruction. Flags will call attention to coding irregularities, such as specifying an auxiliary storage unit for a "multiply" instruction, or allowing the character to the left of a field stored to be signed. Also, before any monitoring commences, the initial length, sign, and contents of the accumulator and each auxiliary storage unit will be given.

Experience with the routine may show that some of the output is not useful enough to warrant the memory space and additional execution time required, and a subsequent version of the monitor may be written which does not include such features. Meanwhile, starting with the assumption that it will be easier to trim than to add, an effort was made to make the monitoring routine output a really complete record of the computer activity.

Paradoxically, a prime requirement of a monitoring routine is that it have the ability to execute sections of a code without monitoring and at full speed. There was no question that this feature would be included in the *705* monitoring routine, the problem was how to specify those sections. The method chosen is simply this: The coder prepares sequence cards, each specifying the location of the first

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