

# Open Loop Adaptive Filtering for Interference Excision in Spread Spectrum Systems

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## Abstract

*A new technique for interference excision in PN spread spectrum communications using time-frequency distributions is introduced. The excision filter coefficients under this technique depend on the jammer power and its the instantaneous frequency information, both values are gained in the time-frequency domain. The dependency of the excision filter characteristics on the interference power, which was absent in previous contributions in this area, is of significant importance, as it allows optimum trade-off between interference removal and the amount of the filter self-noise generated from the induced correlation across the PN chip sequence. This trade-off is bounded by the two extreme cases of no self-noise, which implies preprocessing disabled, and full interference excision, which the case previously considered. In this paper, we derive the FIR excision filters that maximize the receiver signal-to-noise ratio for a simple jammer case.*

## I. Introduction

Several past contributions deal with the suppression of narrowband interference [1,2,3,4]. For broadband interference, adaptive linear prediction filters have been commonly employed to track and remove the time-varying frequency characteristics of the interference [5]. Two different approaches for nonstationary interference excision in DS/SS communications based on time-frequency analysis have been recently considered [6,7,8,9]. One approach is linear and based on multiresolution analysis, whereas the second approach requires a bilinear transformation of the data. In linear transform interference exci-

sions, the data is processed using Fourier, Gabor, or wavelet transforms or M-band/subband filter banks. Excision of the correlated interference components of the received data is performed by clipping, or gating, the high coefficient values followed by inverse transformation to recover the desired signal.

In the recently developed open-loop adaptive filtering approach for interference excision [9, 11,12], a filter zero is put synchronous with the jammer instantaneous frequency (IF), which is estimated using Cohen's class [10] of time-frequency distributions (TFDs). This zero, which is placed on a unit circle with a phase equal to the jammer IF, causes an infinite deep notch, and thereby, effectively removes the jammer, causing the filter output to be essentially jammer free. However, this type of filter characteristics also create significant amount of self-noise, due to the correlation introduced across the different chips of the PN sequence. In a jammer free-environment, the filter self-noise reduces the receiver performance from the case when no preprocessing is applied as it limits the maximum attainable value of the correlator SNR. Depending on the filter characteristics, this value may very well be far below the spreading gain. Although in general, excision filters should be shut off if no jammer is present, the problem with the aforementioned interference excision system which is based solely on the IF information [9] is that even under significant jammer power, the performance is still worse than when preprocessing is disabled. This should not be the case if the filter coefficients are properly chosen.

In this paper, the TFD-based interference excision technique is further developed for application to a wide range of jammer to signal ratios (JSR). Since the TFD is the distribution of the signal power over time and frequency, then, in addition to the IF estimate, TFDs depict the instantaneous power of the different compo-

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nents of the received signal. The amplitude information of the interference gained in the t-f plane is used to control the notch of the excision filter, achieving a higher receiver SNR than the case of a fixed depth notch. Preprocessing the data before despreading produces a trade-off between the self-noise introduced by the filter and the effective removal of the jammer power. In order to account for this trade-off, the original open-loop adaptive interference excision system introduced in [9] is herein modified such that the filter notch location depends on the jammer IF, whereas the depth of the filter notch is controlled by a new variable, which is a function of jammer power. This variable is selected to achieve an optimum receiver performance for a given jammer environment.

It is noteworthy that the need to have both the instantaneous power and frequency to derive the proposed optimum excision of the interference advocates the use of TFD [10] and clearly distinguishes it from other estimators, which only give the IF. The analysis provided in this paper, however, can be used in conjunction with any detection scheme that provides the above two values [13]. Perfect knowledge of the jammer amplitude and IF is assumed. It is recognized that in using TFD, both of the above parameters will carry an error, depending on the jammer signal characteristics as well as the employed t-f kernel.

## II. Narrowband Interference Analysis

A simple jammer with a fixed frequency is now considered. The general expression of the receiver SNR using linear time-invariant interference excision filter of coefficients  $h_i$ ,  $i=1,2,\dots,N$ , is derived in [12] and given by

$$SNR_o = \frac{L^2 h_0^2}{L(1 + \sigma^2) \sum_{k=0}^N h_k^2 - Lh_0^2 + \sigma_j^2} \quad (1)$$

where  $L$  is the PN length,  $\sigma^2$  is the white noise variance, and  $\sigma_j^2$  is the jammer power. In the following analysis, we focus on the three-coefficient causal notch excision filter

$$H(z) = z^{-1}(z - ae^{-j\omega_0})(1 - az^{-1}e^{j\omega_0}) \\ = 1 - 2az^{-1}\cos\omega_0 + a^2z^{-2} \quad (2)$$

where the parameter  $a$  represents the amplitude of the filter zero that controls the depth of the filter notch at the jammer frequency  $\omega_0$ . The effect of this parameter on the filter frequency response is shown in Fig. 1(a). The filter impulse response comes directly from the definition of the Z-transform

$$h(n) = \delta(n) - 2a\delta(n-1)\cos\omega_0 + a^2\delta(n-2) \quad (3)$$

If we denote the filter coefficients by

$h_0 = 1$ ,  $h_1 = 2a\cos\omega_0$ ,  $h_2 = a^2$ , then the corresponding receiver SNR is a special case of (1) and is given by

$$SNR_o = \frac{L^2}{L[(1 + \sigma^2)(1 + a^4 + 4a^2\cos^2\omega_0) - 1] + \sigma_j^2} \quad (4)$$

The self-noise introduced by the filter  $H(z)$  is given by the first term in the denominator of equation (4), i.e., those terms dependent solely on the filter zero amplitude parameter  $a$ ,

$$\sigma_s^2 = L(a^4 + 4a^2\cos\omega_0). \quad (5)$$

The white noise sequence also becomes colored at the excision filter output and its contribution to the receiver SNR<sub>o</sub> in (4) is given by

$$\sigma_w^2 = L\sigma^2(1 + a^4 + 4a^2\cos\omega_0). \quad (6)$$

The quantity  $\sigma_T^2 = \sigma_s^2 + \sigma_w^2$  is the receiver noise in a jammer-free environment, so, we may refer to it as the jammer-free noise or the total self-noise (TSN). It is clear from equations (5) and (6) that the minimum value of the TSN occurs at  $a=0$ , and increases monotonically as a function of  $a$ . Figure 1(b) shows the zero-diagram of the notch filter for different values of  $a$ . For high jammer power,  $\sigma_j^2 \gg \sigma_T^2$ , the interference removal becomes more important than reducing the TSN. In this case, a high value of  $a$  should be chosen such that a deeper notch is introduced. The jammer is entirely removed in the extreme case of  $a=1$ , which is discussed in the original design [9]. On the other hand, as the jammer power decreases, the choice of  $a$  should tend towards favoring the reduction of the total self-noise over the jammer power, and to ultimately shut off the filter, for  $\sigma_s^2 + \sigma_w^2 \gg \sigma_j^2$ , disabling preprocessing of the received signal prior to despreading.

To obtain  $\sigma_j^2$ , it is prudent to first derive an expression of the jammer waveform that escapes the excision filter when  $a \neq 1$ . Consider a narrowband sinusoidal jammer of the form  $j(n) = A\sin(n\omega_0 + \varphi)$ , where  $A$  is the jammer amplitude and  $\varphi$  is its phase. The jammer at the filter output is given by

$$j_0(n) = A(1 - a)\sin(n\omega_0 + \varphi) + (a^2 - a)A\sin[(n-2)\omega_0 + \varphi]$$

The correlator output due to the jammer is

$$U_j = \sum_{n=1}^L j_0(n)p(n) \quad (7)$$

where the  $p(n)$  is the PN chip sequence. Accordingly,

$$\sigma_j^2 = E[U_j^2] - E^2[U_j] = \sum_{n=1}^L j_0^2(n). \quad (8)$$

It can be readily shown that for  $L \gg 1$ ,

$$\sigma_j^2 \approx LA^2(1-a)^2 \left[ \frac{1}{2} + \frac{a^2}{2} - a \cos 2\omega_0 \right] \quad (9)$$

Substituting (9) in (4), the receiver SNR becomes

$$SNR_o = L / \left\{ (1 + \sigma^2)(1 + a^4 + 4a^2 \cos^2 \omega_0) - 1 + A^2(1-a)^2 \left[ \frac{1}{2} + \frac{a^2}{2} - a \cos 2\omega_0 \right] \right\} \quad (10)$$

The jammer power given by (9) has a minimum value at  $a=1$ , and monotonically increases for both increased and reduced values of  $a$ . Equation (9) can be rewritten as

$$\sigma_j^2 = \frac{A^2}{2} [(1-a)^4 + 2a(1-a)^2(1 - \cos 2\omega_0)] \quad (11)$$

Careful study of the above equation reveals that the value of  $\sigma_j^2$  increases faster for  $a>1$  than for  $a<1$ . This is because both factors  $(1-a)^4$  and  $(1-a)^2$  are invariant for  $a = 1 \pm \Delta$ . Due to the appearance of  $a$  as a multiplicative factor in the second term,  $\sigma_j^2$  will be greater for  $+\Delta$  than for  $-\Delta$ . Negative values of  $a$  change the filter notch position and move it away from  $\omega_0$ , and therefore should be avoided. Since the self-noise increases for increased value of  $a$ , as stated earlier, one can conclude that the minimum value of the denominator in the  $SNR_o$  expression (10),  $\sigma_T^2$ , should occur for  $a$  in the range  $[0,1]$ .

If  $f(a)$  represents the denominator in (10), then in order to find the maximum  $SNR_o$ , we simply differentiate  $f(a)$  with respect to  $a$ ,

$$f'(a) = a^3(4 + 4\sigma^2 + 2A^2) + a^2(-3A^2 - 3A^2 \cos 2\omega_0) + a[8(1 + \sigma^2)\cos^2 \omega_0 + 2A^2 + 4A^2 \cos 2\omega_0] - (A^2 + A^2 \cos 2\omega_0) \quad (12)$$

and sets  $f'(a)=0$ . Since  $f'(0) < 0 \quad \forall \omega_0$ , except at  $\omega_0 = \pi/2$  where  $f'(0) = 0$ , and  $f'(1) > 0$  for all values of  $\omega_0$ , then  $f'(a)$  must have a real root in the range  $[0,1]$ , which represents the optimal  $a$ . For the specific value of  $\omega_0 = \pi/2$ ,  $f'(0) = 0$  and  $f''(0) = -2A^2 < 0$ . This means  $f'(0) < 0$ . With  $f'(1) > 0$ ,  $f(a)$  must then intersect the  $f'(a) = 0$  axis in the range  $[0,1]$ , and the same conclusion can be drawn for  $\omega_0 = \pi/2$  as for  $\omega_0 \neq \pi/2$ .

To solve for  $a$ , we rewrite the polynomial  $f'(a)=0$  as

$$ba^3 + ca^2 + da + e = 0 \quad (13)$$

where

$$\begin{cases} b = 4 + 4\sigma^2 + 2A^2 \\ c = -3A^2 - 3A^2 \cos 2\omega_0 \\ d = 8(1 + \sigma^2)\cos^2 \omega_0 + 2A^2 + 4A^2 \cos 2\omega_0 \\ e = -(A^2 + A^2 \cos 2\omega_0) \end{cases} \quad (14)$$

and substitute  $b = a - \frac{d}{3c}$ ,

$$a^3 + pa + q = 0 \quad (15)$$

where

$$p = \frac{3bd - c^2}{3b^2}, \quad q = \frac{e}{b} - \frac{cd}{3b^2} - \frac{2c^3}{27b^3} \quad (16)$$

The roots of polynomial (15) are given in [14],

$$\begin{cases} a_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ a_2 = \gamma \left( \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \right) + \gamma^2 \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \\ a_3 = \gamma^2 \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \gamma \left( \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \right) \end{cases} \quad (17)$$

where  $\gamma = \frac{-1 + i\sqrt{3}}{2}$ .

If  $q < 0$ , then  $a_2$  and  $a_3$  are complex due to the presence of  $\gamma$  and its complex value  $\gamma^2$ . Accordingly,  $a_1$  should be real. On the other hand, if  $q=0$  as in the case of  $\omega_0 = \pi/2$ , then  $a_1$  becomes zero and  $p$  is given by

$$p = \frac{d}{b} = \frac{-2A^2}{4 + 4\sigma^2 + 2A^2} < 0, \quad (18)$$

In this case, it can be easily shown that  $a_2$  and  $a_3$  are both real, and have opposite signs with magnitude smaller than one. Accordingly, there is only one positive real root which should be in the range  $[0,1]$ , as argued before.

### III. Performance Improvement

A fixed frequency jammer excised by a three-coefficient filter is considered. The optimal value of  $a$  is computed from equation (12) and substituted back into the SNR expression to provide the maximum SNR for a given jammer power and frequency. Figure 2 compares the correlator SNRs using the original excision filter, no excision filter, and the optimal excision filter. It is evident that for low jammer-to-signal ratio, shutting off the excision filter, leads to a higher receiver SNR than processing the data with the original excision filter, which only depends on the IF information. This situation is reversed for a high jammer power where the original filter outperforms the case when preprocessing is disabled. But, the performance of the proposed optimum interference excision filter asymptotically reaches the desired performance of both the IF-based excision and no excision for high and low jammer power, respectively. In between these two extreme cases, the proposed excision filter, which is based on both the amplitude and frequency of the jammer, gives

clear improvement over the other two techniques. That is, the optimal notch filter gives an excellent trade-off between filtering and no filtering and as expected,  $a$  increases with increased jammer power.

In Figures 3-4, we compare the receiver performance versus the interference IF. The optimal excision filtering curve depicting the change in SNR vs frequency is not only above the other curves, correspondingly to the two alternative aforementioned techniques, but also it becomes much flatter than the receiver SNR curve of the original filter. This means that a by-product of the proposed approach is to make the receiver SNR less dependent on the jammer IF.

Figure 5 shows the computer simulation results of bit error curve. A chirp jammer whose frequency changes from 0 to  $\pi$  in every bit duration and a white noise sequence with 0 dB relative to the signal are both added to the PN sequence of length  $L=64$ . One million bits are tested at every 2 dB JSR increment from -10dB to 10dB. Consistent with the result in SNR analysis, for low jammer to signal ratio, applying no filter leads to lower bit error rate than processing the data with the original excision filter, whereas for a high jammer power, the original filter has lower errors than the case when preprocessing is disabled. Because the optimum notch, adaptive coefficient filter outperforms both cases above. It gives a bit error rate of  $10^{-5}$  at 10 dB JSR and smaller rates at lower JSR.

## Conclusions

An optimum open-loop adaptive notch filtering approach for interference excision in PN spread spectrum communications has been developed and discussed in this paper. The FIR filter with variable depth notch that partially removes the jammer achieves optimum receiver SNR over both extreme cases of full jammer excision and no excision. The optimum performance is reached by trading-off the jammer power and filter self-noise. The filter notch is controlled by a new variable whose optimum value is a function of the jammer power, the jammer instantaneous frequency, and the white noise power. Several examples have been presented which show the improvement in the receiver signal-to-noise ratio achieved by using the optimum excision filter over both cases of preprocessing disabled and preprocessing enabled, but only based on the IF information. This improvement is exhibited over a wide range of JSR, and is shown using exact value of the interference amplitude and instantaneous frequency.

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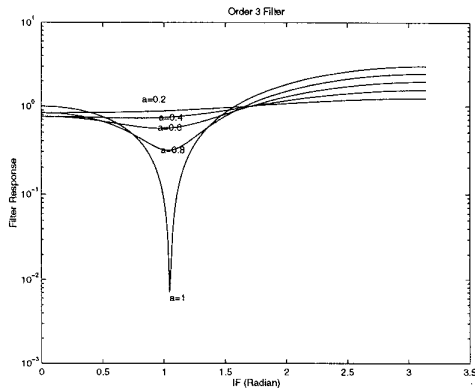


Fig. 1(a) Frequency responses of the three-coefficient adaptive notch filters with  $IF=\pi/3$ .

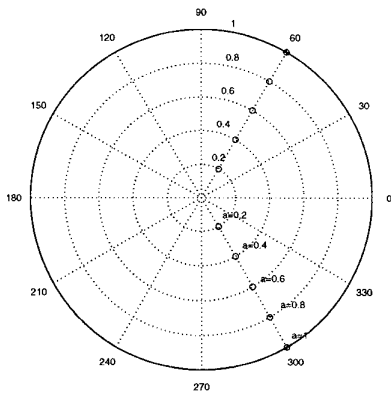


Fig. 1 (b) Zero diagram of the adaptive notch filter and the exact jammer frequency. 'o', zero position, '+', exact jammer frequency.

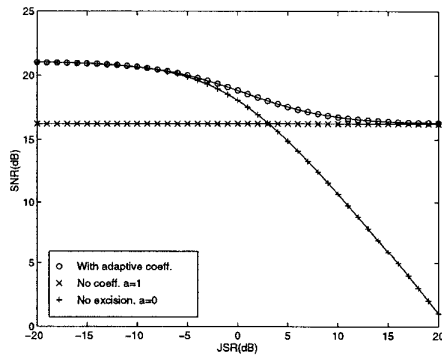


Fig.2  $IF=\pi/2.1$ , 0dB white noise, PN length=128. Correlator SNR with 3 coefficient filters and filter off.

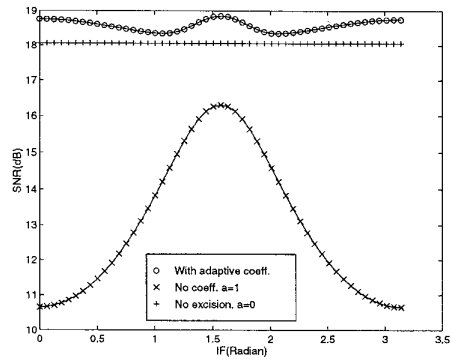


Fig.3 JSR=0dB, 0dB white noise, PN length=128. Correlator SNR with 3 coefficient filters and filter off versus jammer IF.

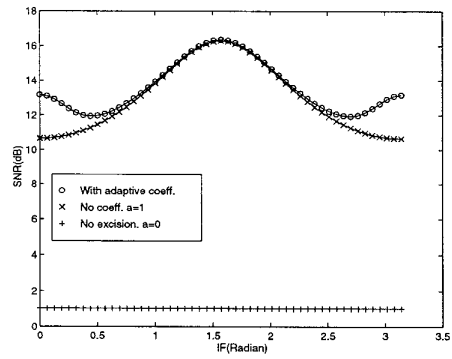


Fig.4 JSR=20dB, 0dB white noise, PN length=128. Correlator SNR with 3 coefficient filters and filter off versus jammer IF.

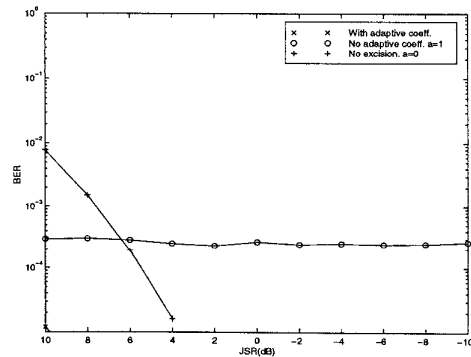


Fig.5 Bit error rate of the three-coefficient filter with adaptive filter coefficient, fixed coefficient and filter off. 0dB white noise, PN length=64.