COHERENT ADAPTIVE RADAR DETECTION IN NON-GAUSSIAN CLUTTER

Fulvio Gini[1], Maria V. Greco[1], Kevin J. Sangston[2], and Alfonso Farina[3]

[1]Dept. of Information Engineering, University of Pisa
Via Diotisalvi 2, I-56126, Pisa, Italy
{gini,greco}@iet.unipi.it

[2]Sensors and Electromagnetic Applications Lab
Georgia Tech. Research Institute, Atlanta, Georgia 30332-0800
jim.sangston@gtri.gatech.edu

[3]System Analysis Scientific Calculus Group, ALENIA
Via Tiburtina Km. 12.400, 00131 Roma, Italy
farina@alenia.finmeccanica.it

ABSTRACT

Adaptive detection of fluctuating radar targets in unknown correlated Gaussian disturbance has received considerable attention in the past. The Kelly's generalized likelihood ratio test (GLRT) is the preferred algorithm for detecting Swerling-I targets in Gaussian noise. Instead the problem of adaptive detection in non-Gaussian environment is still under investigation. In this paper we pursue two aims: (i) to investigate the performance of the Kelly's GLRT in non-Gaussian clutter; (ii) to derive a detection algorithm with constant false alarm rate (CFAR) behavior with respect to the amplitude probability density function (apdf) parameters and to the correlation structure of the disturbance that outperforms the Kelly's GLRT in non-Gaussian clutter. Performance analysis is presented using both simulated data and real sea clutter data.

1. INTRODUCTION

Adaptive detection is important for modern radar systems which are tasked to operate in non-homogeneous and non-stationary environment. A successful modern detection scheme should encompass the features of: (i) being adaptive to the interference spectral density, and more in general to the probability density function (pdf) of interference, (ii) maintaining CFAR conditions, and (iii) having a relatively simple processing scheme.

Adaptive radar detection against Gaussian noise has been largely investigated in the past (see, e.g., [8], [11], and [7]), the same detection problem in colored non-Gaussian clutter has been investigated only recently [9], [10]. With the support of experimental data the clutter process has been modeled as a *spherically invariant random process* (SIRP) [12], that includes the K-distributed and the Gaussian models as particular cases [1], [3], [13].

The objectives of the present paper are: (i) to derive a detection algorithm that is CFAR with respect to the amplitude pdf (apdf) and to the correlation structure of the disturbance, (ii) to investigate its performance in K-distributed clutter, and compare it with that of the Kelly's GLRT. The layout of this work is as follows. In Section 2 a brief introduction of the clutter model and the detection problem statement are provided. In Section 3 we reformulate the problem of adaptive detection under the compound-Gaussian clutter assumption, that encompasses the K-distributed model as a special case. The GLRT approach is considered and an approximate solution is derived. We will refer to this detector as the *adaptive linear-quadratic (ALQ) detector*. In Section 4, some results of our performance analysis with simulated data and real sea clutter data are reported and discussed. Some concluding remarks are reported in Section 5.

2. PROBLEM STATEMENT

Assume that the radar transmits a coherent train of m pulses. The m complex received samples can be assembled into the mx1 vector $\mathbf{z} = \mathbf{z}_I + j\mathbf{z}_Q = [z[1] \cdots z[m]]^T$, where $\mathbf{z}_I$ and $\mathbf{z}_Q$ represent the vectors of the in-phase (I) and quadrature (Q) components. The detection procedure is given by the decision between the hypotheses H_0 and H_1 after the vector $\mathbf{z}$ has been received. In the null hypothesis H_0 it is assumed that the data consist only of clutter $\mathbf{d}$. We assume here that the clutter follows the K-distribution, but the results are valid for each pdf belonging to the SIRP family. When the clutter is a SIRP [1], [12], [13], each element of the clutter vector $\mathbf{d}$ can be interpreted as the product of two independent random variables (rv's):

$$\mathbf{d} = \sqrt{\tau}\,\mathbf{x} \tag{1}$$

where $\mathbf{x} = \mathbf{x}_I + j\mathbf{x}_Q$ is an m-dimensional complex Gaussian circular vector, usually named *speckle*, that represents the properties of the coherent radar sensor. The in-phase (x_{Ii}) and quadrature (x_{Qi}) components are zero mean r.v.'s, with unit variance and normalized covariance matrix $\mathbf{M} = E\{\mathbf{x}\mathbf{x}^H\}/2$. The variable τ in (1), usually referred to as *texture*, represents the local clutter power of the clutter in the cell under test (CUT). It is characteristic of the observed scene and according to the K-model it is assumed to be Gamma distributed [1], [13], [9], [12], with mean value μ and order parameter ν:

$$p_\tau(\tau) = \frac{1}{\Gamma(\nu)}\left(\frac{\nu}{\mu}\right)^{\nu} \tau^{\nu-1} e^{-\frac{\nu}{\mu}\tau}, \qquad \tau \geq 0, \tag{2}$$

1058-6393/98 $10.00 © 1998 IEEE

where $\Gamma(v)$ is the Gamma function. Given a specific value of τ, $\mathbf{d}$ is a circular Gaussian random vector with conditional pdf given by:

$$p_{\mathbf{d}|\tau}(\mathbf{d}|\tau)=\frac{1}{(2\pi\tau)^m|\mathbf{M}|}\exp\left(-\frac{\mathbf{d}^H\mathbf{M}^{-1}\mathbf{d}}{2\tau}\right) \qquad (3)$$

The pdf of $\mathbf{d}$, that coincides with $p_{\mathbf{z}}(\mathbf{z}|H_0)$, is obtained by integrating $p_{\mathbf{d}|\tau}(\mathbf{d}|\tau)$ over τ.

In the alternative hypothesis H_1 it is assumed that the observed data consist of a sum of signal $\mathbf{s}=\alpha\mathbf{p}$ and clutter $\mathbf{d}$. α is a complex unknown parameter, while $\mathbf{p}$ is a perfectly known complex vector (the *steering vector*) with components $p_i=e^{j2\pi i f_D T_s}$, $1\le i\le m$. T_s is the radar pulse repetition time and f_D is the target Doppler frequency. We adopt the Swerling-I target model, i.e., α is a complex circular Gaussian random variable, with zero mean and variance $E\{|\alpha|^2\}=2\sigma_A^2$. The optimum strategy for detecting a Swerling-I target embedded in K-distributed clutter (in short *OKD*) has been derived in [6]. In order to implement the OKD detector the characteristic parameters of K-distributed clutter ($\mu, v, \mathbf{M}$) and of target signal (σ_A^2) should be known; for this reason, we will refer to the optimum detector as the *clairvoyant* OKD. In a realistic radar scenario these parameters are typically unknown, so they must be estimated from the data.

3. THE ADAPTIVE LINEAR-QUADRATIC (ALQ) DETECTOR

The goal is to derive a detection algorithm with CFAR behavior with respect to the texture parameters and to the clutter covariance matrix. As in [8], [11], we make use of K independent and identically distributed (i.i.d.) blocks of secondary data $\{\mathbf{z}_k\}_{k=1}^K$ from K adjacent range cells. Secondary data are supposed to have the same pdf and correlation properties of the disturbance in the cell under test. Denote by $\mathbf{Z}\equiv[\mathbf{z}\ \mathbf{z}_1\ \cdots\ \mathbf{z}_K]$ the $m\times(K+1)$ matrix of the totality of the data. In the hypothesis H_0 the joint pdf of the observed vectors results:

$$p_{\mathbf{z}}(\mathbf{Z}|H_0)=\frac{1}{(2\pi)^{m(K+1)}|\mathbf{M}|^{K+1}}\iint\cdots\int\frac{1}{\tau^m}\exp\left(-\frac{\mathbf{z}^H\mathbf{M}^{-1}\mathbf{z}}{2\tau}\right)$$
$$\times\prod_{k=1}^{K}\frac{1}{\tau_k{}^m}\exp\left(-\frac{\mathbf{z}_k^H\mathbf{M}^{-1}\mathbf{z}_k}{2\tau_k}\right)\ p_\tau(\tau)p_{\tau_1}(\tau_1)\cdots p_{\tau_K}(\tau_K)d\boldsymbol{\tau} \qquad (4)$$

where $\boldsymbol{\tau}\equiv[\tau\ \tau_1\ \cdots\ \tau_K]^T$ and $d\boldsymbol{\tau}\equiv d\tau d\tau_1\cdots d\tau_K$.

As concerning the hypothesis H_1: by replacing $\mathbf{z}$ with $\mathbf{z}-\alpha\mathbf{p}$ in (4) we obtain $p_{\mathbf{z}}(\mathbf{Z}|\alpha,H_1)$; then, averaging with respect to $p_\alpha(\alpha)$ we finally obtain (at least in principle) $p_{\mathbf{z}}(\mathbf{Z}|H_1)$. Following the GLRT approach we should maximize $p_{\mathbf{z}}(\mathbf{Z}|H_0)$ and $p_{\mathbf{z}}(\mathbf{Z}|H_1)$ with respect to (w.r.t.) the unknown parameters ($\mu, v, \mathbf{M}$, and σ_A^2) and then take their ratio:

$$\frac{\max\limits_{\{\mu,v,\mathbf{M},\sigma_A^2\}}\{p_{\mathbf{z}}(\mathbf{Z}|H_1)\}}{\max\limits_{\{\mu,v,\mathbf{M}\}}\{p_{\mathbf{z}}(\mathbf{Z}|H_0)\}}\ \underset{H_0}{\overset{H_1}{\gtrless}}\ e^{\mathrm{T}} \qquad (5)$$

T is a threshold coefficient fixed according to the desired probability of false alarm (P_{FA}).

Posed in this way, no closed form solution seems to be available for our detection problem. An approximate solution can be found by considering α as a deterministic unknown parameter, i.e., neglecting the *a-priori* information on α represented by $p_\alpha(\alpha)$. This approach is reasonable and it has been followed also in [8], [11], [12]. Note that we make this assumption only to derive an *approximate* GLRT solution, but the performance will be tested under the Swerling-I target model assumption. An approximate solution of (5) has been derived (see [5] for details of the derivation), and is given by

$$\left|\mathbf{p}^H\hat{\mathbf{S}}^{-1}\mathbf{z}\right|^2-(1-e^{-T/m})(\mathbf{z}^H\hat{\mathbf{S}}^{-1}\mathbf{z})(\mathbf{p}^H\hat{\mathbf{S}}^{-1}\mathbf{p})\ \underset{H_0}{\overset{H_1}{\gtrless}}\ 0 \qquad (6)$$

where we defined the matrix

$$\hat{\mathbf{S}}=\sum_{k=1}^{K}\frac{\mathbf{z}_k\mathbf{z}_k^H}{\hat{\tau}_k}=2m\sum_{k=1}^{K}\frac{\mathbf{z}_k\mathbf{z}_k^H}{\mathbf{z}_k^H\mathbf{z}_k}, \qquad (7)$$

$\hat{\tau}_k=\mathbf{z}_k^H\mathbf{z}_k/(2m)$ represents the sample estimate of the power in the *k-th* reference cells. Note that the detection strategy of (6) depends on the data from the CUT only by means of a linear statistic ($\mathbf{p}^H\hat{\mathbf{S}}^{-1}\mathbf{z}$) and a quadratic statistic ($\mathbf{z}^H\hat{\mathbf{S}}^{-1}\mathbf{z}$), for this reason we will refer to the detector (6) as the *adaptive linear-quadratic (ALQ) detector*. No optimality properties can be claimed for the ALQ detector of (6), but this holds true also for the Kelly's GLRT in Gaussian clutter; on the other side a closed-form solution to the GLRT problem in (5) is not known. We will show that from an engineering standpoint the ALQ detector is eminently reasonable and that a substantial performance improvement is possible by using (6) in lieu of the Kelly's detector when the clutter is non-Gaussian distributed. First of all, it is worth observing that: (i) $\hat{\mathbf{M}}=\hat{\mathbf{S}}/(2K)$ represents an estimate of the normalized covariance matrix $\mathbf{M}$, (ii) $\hat{\mathbf{S}}$ is statistically equivalent to $\hat{\mathbf{S}}_x=2m\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^H/(\mathbf{x}_k^H\mathbf{x}_k)$, so it does not depend on the parameters μ and v of the texture pdf.

Then, taking into account that the Gamma rv τ is always positive, we have that the statistic of the ALQ test is statistically equivalent to

$$\left|\mathbf{p}^H\hat{\mathbf{S}}_x^{-1}\mathbf{x}\right|^2 - (1-e^{-T/m})(\mathbf{x}^H\hat{\mathbf{S}}_x^{-1}\mathbf{x})(\mathbf{p}^H\hat{\mathbf{S}}_x^{-1}\mathbf{p}),$$

so the test (6) is CFAR with respect to the texture statistics, i.e., with respect to clutter apdf. The ALQ test is not invariant w.r.t. the actual covariance matrix $\mathbf{M}$ due to the presence of the terms $\mathbf{z}_k^H\mathbf{z}_k$ at the denominator of (7), but when $\mathbf{z}_k^H\mathbf{z}_k/(2m) \to \tau_k$, i.e., for increasing m, the detector (6) becomes CFAR w.r.t. $\mathbf{M}$ (in fact if $\{\hat{\tau}_k\}_{k=1}^K$ were known or perfectly estimated the CFAR property would derive directly as in [8]). This point has been checked by numerical simulation.

For the sake of comparison, we report also the Kelly's GLRT algorithm in a form similar to (6):

$$\left|\mathbf{p}^H\hat{\mathbf{M}}_z^{-1}\mathbf{z}\right|^2 - \lambda\left[1+\frac{1}{K}(\mathbf{z}^H\hat{\mathbf{M}}_z^{-1}\mathbf{z})\right](\mathbf{p}^H\hat{\mathbf{M}}_z^{-1}\mathbf{p}) \underset{H_0}{\overset{H_1}{\gtrless}} 0 \quad (8)$$

where λ is a threshold coefficient and $\hat{\mathbf{M}}_z$ is the sample estimate of the clutter covariance matrix $\mathbf{M}_z$ (note that $\mathbf{M}_z \equiv E\{\mathbf{z}\mathbf{z}^H\} = 2\mu\mathbf{M}$):

$$\hat{\mathbf{M}}_z = \frac{1}{K}\sum_{k=1}^{K}\mathbf{z}_k\mathbf{z}_k^H \quad (9)$$

With the covariance matrix estimate of (9), the decision strategy (8) is CFAR with respect to the covariance matrix [8], but not w.r.t. the clutter apdf parameters, as it will be shown in the following. The ALQ detector differs from the Kelly's GLRT for: (i) the way the covariance matrix is estimated, and (ii) the way the quadratic term in $\mathbf{z}$ takes part in the decision strategy. The first difference is responsible for the fact that the ALQ is CFAR w.r.t. the apdf parameters μ and ν, whereas the Kelly's GLRT is not. The second difference is due to the fact that the quadratic statistic of $\mathbf{z}$ takes part by means of the term $\mathbf{z}^H\hat{\mathbf{S}}^{-1}\mathbf{z}$ in (6), and by means of the term $[1+(\mathbf{z}^H\hat{\mathbf{M}}_z^{-1}\mathbf{z})/K]$ in (8). In the Kelly's GLRT, for large number of secondary data ($K \to \infty$), the term in brackets goes to unity and $\hat{\mathbf{M}}_z$ approaches to $\mathbf{M}_z$, so the test reverts to the whitening matched filter, optimum in Gaussian clutter but with large performance loss in K-distributed clutter [1], [4]. On the other side, when $K \to \infty$ the ALQ detector reverts to the linear-quadratic CFAR detector proposed in [4], that in non-Gaussian clutter largely outperforms the whitening matched filter. For this reason we expect the ALQ detector to outperform the Kelly's GLRT in non-Gaussian clutter, even for finite *K*.

4. PERFORMANCE ANALYSIS

Performance comparison has been carried out via Monte Carlo simulation in terms of P_D versus the signal-to-clutter-power ratio (SCR), defined as $SCR = E\{|\alpha|^2\}/(2\mu)$. We made the following assumptions: (i) the product $f_D T_s$ was set equal to 0.5, (ii) the mean value of the clutter Doppler spectrum has been assumed to be zero, (iii) the clutter samples were generated assuming an exponential correlation structure, i.e., Lorentzian spectrum [4], so that the matrix $\mathbf{M}$ has elements: $[\mathbf{M}]_{ij} = \rho^{|i-j|}$, $1 \le i, j \le m$, where ρ is the one-lag correlation coefficient.

CFAR properties. First of all we investigated the CFARness of the Kelly's GLRT and of the ALQ detectors to changes of the texture pdf (i.e., changes of the shape parameters ν), and changes of the correlation structure (i.e., changes of the one-lag correlation coefficient ρ).

Figure 1 shows the curves of P_{FA} as a function of the threshold coefficient T, for different values of the correlation coefficient ρ. The order parameter was $\nu = 0.1$ (very spiky clutter). It is evident that the ALQ is not perfectly CFAR w.r.t. ρ but it is very robust: P_{FA} does not change significantly when ρ changes from *0.2* to *0.9*, and this has been checked for different values of *m* and *K*. As concerning the Kelly's GLRT, our numerical results show that its false alarm rate degrades considerably in non-Gaussian clutter, both when $m=4, K=8$, and when $m=16$, $K=24$, as it is evident in Fig. 2.

Detection performance. In Fig. 3 we compare the detection performance of the Kelly's GLRT and the ALQ detector. The probability of detection, P_D, of the *clairvoyant* OKD, the optimum detector that knows the clutter apdf parameters and its covariance matrix, is also reported as a benchmark. It is worth observing that the detection loss of the Kelly's GLRT w.r.t. the ALQ is considerable, this loss tends to decrease when ν increases. For $\nu = 4.5$ they have almost the same performance, when $\nu \to \infty$ (Gaussian clutter) the Kelly's GLRT performs sligthly better than the ALQ detector. Similar results have been observed for $m = 4$ and $K = 8$.

Results with IPIX sea clutter data. We also checked our performance prediction with real sea clutter data collected at Osborne Head Gunnery Range in November 1993, with McMaster University IPIX radar [2]. The radar pulse repetition frequency (PRF) is *2 KHz*, the pulse width is *200 ns*, thus the range resolution is *30 m* and the range acquisition window is *210 m* wide. There are *7* range cells, and the number of time samples per cell is *131,072*. The beamwidth is of *0.9°*. Statistical analysis of these data is reported in [3]. For the data set we analyzed, the 16 frequencies in the X-band (*9.4 GHz*) are transmitted with the frequency agility mode; the azimuth is fixed at *79.753°* and the grazing angle is *0.945°*. The analyzed data set was stored with a sea state 3 (Beaufort scale), a wind speed of *22 km/h*, from a direction of *40°* with respect to the looking radar direction and a significant wave height of *1.42 m*.

The analysis of amplitude histograms, of cumulants and moments has demonstrated that for VV polarization the data distribution approaches the K model [3]. To test the receivers performance we used the K-distributed VV polarized data, with shape parameter $\nu = 1.25$, and scale parameter $\mu = 4.44 \cdot 10^{-3}$ (estimated on the entire set of data). Because of the small number of range cells (only 7), to estimate the performance of the two adaptive detectors with the real data, we could not collect the K secondary vectors from K range cells surrounding the CUT, as described in Sect. 3, but from the same range cell by making use of data recorded at successive sampling times, through the entire data set. In this way, the differences between the K vectors are not due to the space variations but to the movement of the sea waves in the same cell. Naturally, with this procedure, the textures of different vectors are partially correlated, so the secondary vectors are not independent.

In Figs. 4-5 we report the performance of the adaptive detectors obtained by processing real data and simulated data generated according to the K model, with the covariance matrix and ν estimated on the entire set of data. A synthetic target has been generated according to the Swerling-I model and added to the clutter data, in order to obtain different values of SCR. Due to the limited amount of available data the plots were derived by setting $P_{FA} = 10^{-3}$ when *m=4* and *K=8* (Fig. 4), and $P_{FA} = 10^{-2}$ when *m=16* and *K=24* (Fig. 5). The results show a small disagreement between performance prediction based on the K model and the performance obtained by processing the real data. This is probably due to the fact that the texture is strongly correlated among different cells, so the hyphotesis that the $\{\tau_k\}_{k=1}^{K}$ are i.i.d. is violated. In [5] it was also found that the performance of the ALQ detector are very robust w.r.t. texture correlation among different range cells, while the performance of the Kelly's GLRT heavily depends on it (it performs better when the $\{\tau_k\}_{k=1}^{K}$ are strongly correlated). These results show that for *m=4* the Kelly's GLRT performs better than the ALQ detector, whereas for *m=16* they are almost the same. We expect that if data with spikier behavior ($\nu < 1.25$) or with i.i.d. textures were available the superior performance of the ALQ detector would result.

5. CONCLUDING REMARKS

In this work we have addressed the problem of coherent adaptive radar detection of fluctuating targets against correlated K-distributed clutter. The contribution of this paper can be summarized as follows: (i) derivation of a new adaptive detection algorithm for Swerling-I targets embedded in non-Gaussian clutter, (ii) analysis of the CFAR property and performance comparison of the new adaptive detector with the Kelly's GLRT in K-distributed clutter. The analysis has been based both on simulated data and live recorded sea clutter data, and it suggests the following conclusions:

- the Kelly's GLRT is CFAR w.r.t. the covariance matrix, also in K-distributed clutter, but it is not CFAR w.r.t. the clutter apdf, its P_{FA} strongly depends on the clutter spikiness;
- the ALQ detector is CFAR w.r.t. the clutter apdf and very robust w.r.t. changes of the covariance matrix;
- the choice of the best strategy in K-distributed clutter depends on the number m of integrated samples and on the spikiness of the clutter: for m>8 and spiky clutter (say $\nu \geq 4.5$) the the ALQ detector performs better than the Kelly's GLRT;
- the Kelly's GLRT has considerable performance loss when the clutter is spiky ($\nu < 1$), while the performance of the ALQ detector are really close to that of the *clairvoyant* OKD whatever is the value of ν, provided that the number m of integrated pulses is large enough (e.g. *m=16*).

ACKNOWLEDGEMENTS. The authors thank Dr. Anastasios Drosopoulos for kindly providing the database and the reference [2].

REFERENCES

[1] Aluffi Pentini F., Farina A., Zirilli F., "Radar Detection of Targets Located in a Coherent K-Distributed Clutter Background," *IEE Proceedings-F*, Vol.139, No.3, pp. 239-245, June 1992.

[2] Drosopoulos A., *Description of the OHGR Database*, Tech. Note N°94-14, Defence Research Establishment Ottawa, December 1994.

[3] Farina A., Gini F., Greco M.V., Verrazzani L., "High Resolution Sea Clutter Data: A Statistical Analysis of Recorded Live Data," *IEE Proceedings-F*, vol.144, No 3, pp. 121-130, June 1997.

[4] Gini F., "Sub-optimum Coherent Radar Detection in a Mixture of K-Distributed and Gaussian Clutter," *IEE Proceedings-F*, vol.144, No.1, pp. 39-48, February 1997.

[5] Gini F., Greco M.V., *Coherent Adaptive Radar Detection in Non-Gaussian Disturbance: A Suboptimum GLRT Approach*, Tech. Report, Editrice Tecnico Scientifica, Pisa, June 1997.

[6] Greco M.V., Gini F., Farina A., Lombardo P., "Coherent Radar Detection of Signals with Unknown Parameters Against a Combination of K-Distributed and Gaussian Clutter," Proc. of *GRETSI Symposium on Signal and Image Processing*, Juan-Les-Pins, France, September 1995.

[7] Haykin S., Steinhardt A., *Adaptive Radar Detection and Estimation*, Wiley series in Remote Sensing, New York: Wiley, 1992.

[8] Kelly E.J., "An Adaptive Detection Algorithm," *IEEE Trans. on Aerospace and Electronic Systems,* Vol.23, No.1, pp. 115-127, November 1986.

[9] Rangaswamy M., Michels J.H., "A Parametric Multichannel Detection for Correlated Non-Gaussian Random Processes," Proc. of *1997 National Radar Conf.*, Syracuse, pp. 349-354, NY, USA, May 1997.

[10] Richmond C.D., "A Note on Non-Gaussian Adaptive Array Detection and Signal Parameter Estimation," *IEEE Signal Processing Letters*, Vol.3, No.8, pp. 251-252, August 1996.

[11] Robey F.C., Fuhrmann D.R., Kelly E. J., Nitzberg R., "A CFAR Adaptive Matched Filter Detector," *IEEE Trans. on Aerospace and Electronic Systems*, Vol.28, No.1, pp. 208-216, January 1992.

[12] Sangston K. J., Gerlach K.R., "Coherent Detection of Radar Targets in a Non-Gaussian Background," *IEEE Trans. on Aerospace and Electronic Systems*, Vol.30, No.2, pp. 330-340, April 1994.

[13] Ward K.D., Baker C.J., Watts S., "Maritime Surveillance Radar. Part 1: Radar Scattering from the Ocean Surface," *IEE Proceedings-F*, Vol.137, No.2, pp. 51-62, April 1990.

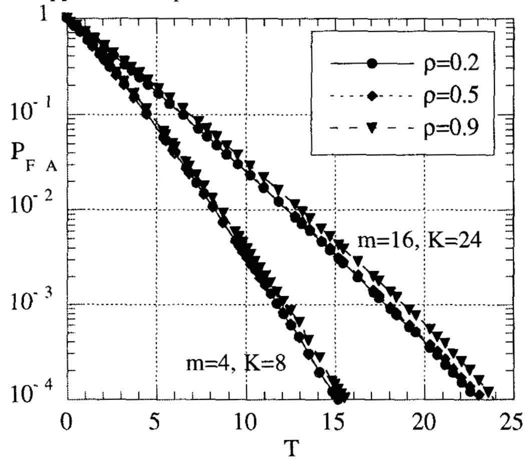

Fig.1 - P_{FA} *vs.* T for the ALQ detector, $\nu = 0.1$.

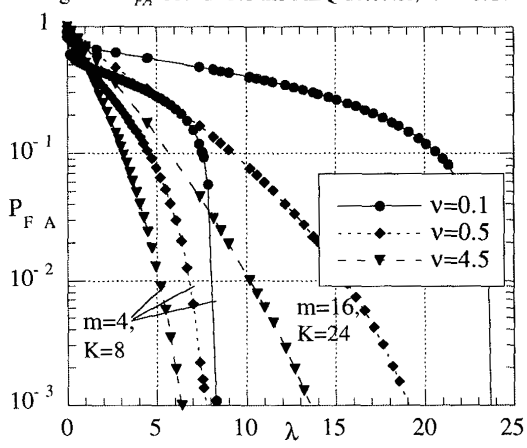

Fig.2 - P_{FA} *vs.* λ for the Kelly's GLRT, $\rho = 0.9$.

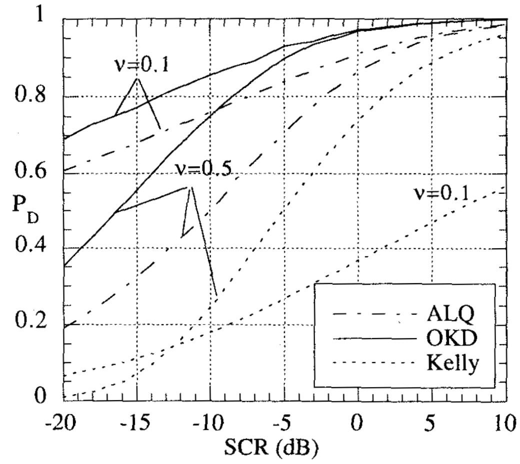

Fig.3 - P_D *vs.* SCR, $m = 16$, $K = 24$, $\rho = 0.9$.

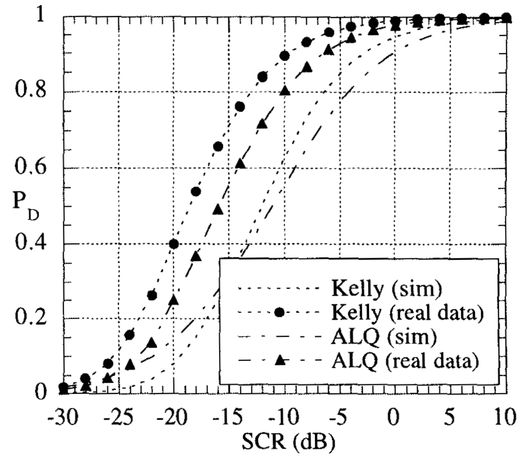

Fig.4 - P_D *vs.* SCR, $m = 4$, $K = 8$, $\nu = 1.25$.

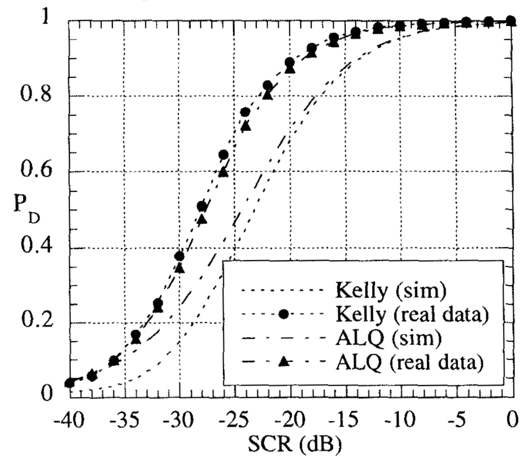

Fig.5 - P_D *vs.* SCR, $m = 16$, $K = 24$, $\nu = 1.25$.