Optimum Codes for FFH/BFSK Receivers with Self-Normalization Combining and Hard Decision Decoding in Fading Channels

X. Nikolakopoulos, T. T. Ha, and R. Clark Robertson
Naval Postgraduate School
Monterey, California 93943

Abstract

The application of forward error correction coding to a fast frequency-hopped binary frequency-shift keying (FFH/BFSK) noncoherent receiver with self-normalization combining and noise jamming is discussed in this paper. The performance of the receiver is examined both when data are encoded using Reed-Solomon codes and concatenated Reed-Solomon and convolutional codes, all with hard decision decoding. The effect of the transmission channel is considered, and results are derived for a Rayleigh fading channel and for Ricean fading channels with several ratios of direct-to-diffuse signal power. Only frequency-nonselective, slowly fading channels are considered. The combination of diversity and forward error correction coding is found to improve the performance of the receiver in the presence of jamming, and optimum codes for each coding scheme are discussed.

1 INTRODUCTION

The performance of a fast frequency-hopped BFSK receiver with self-normalization combining in a fading channel with partial-band interference has been previously examined [1]. The transmitter is assumed to send L hops per data bit. At the receiver (Fig. 1) the dehopped signals are demodulated by noncoherent matched filters with equivalent noise bandwidth of B Hz. Self-normalization combining is used to nonlinearly combine the outputs of the quadratic detectors of the two branches of the BFSK demodulator to form the L diversity signals which are then combined to obtain the decision statistics [1].

The channel for each hop is modeled as a frequency-nonselective, slowly fading Ricean process. Hence, the signal bandwidth is much smaller than the coherence bandwidth of the channel and the hop rate much greater than the Doppler spread of the channel [2]. In this case the dehopped signal can be modeled as the sum of a nonfaded (direct) and a Rayleigh-faded (diffuse) component.

Interference is considered to be both partial-band noise (caused by a jammer or any other source of narrowband interference) and wideband interference (i.e., thermal noise). Partial-band interference is modeled as additive Gaussian noise with power spectral density $\gamma^{-1}N_I/2$, where γ is the fraction of the bandwidth being jammed and $N_I/2$ is the average interference noise power spectral density over the entire bandwidth. Wideband interference is modeled as additive white Gaussian noise with noise power spectral density $N_0/2$.

The noise power for each hop k of a signal is

$$\sigma_k^2 = \left(\gamma^{-1}N_I + N_0\right) B \tag{1}$$

with probability γ when interference is present and

$$\sigma_k^2 = N_0 B \tag{2}$$

with probability $1-\gamma$ when interference is not present.

For a bit interval of T_b seconds, the bit rate is $R_b = 1/T_b$. The duration of the hop interval is $T_h = T_b/L$ for L^{th} order diversity, and the hop rate is $R_h = 1/T_h = LR_b$. If S is the average signal power, then

$$S = a^2 + 2\sigma^2 \tag{3}$$

where a^2 is the average power of the direct component of the signal, and $2\sigma^2$ is the average power of the diffuse component of the signal.

In this case the average energy per hop is $E_h = ST_h$ and the average energy per bit is $E_b = LE_h$. The signal power-to-noise power ratio is

$$\frac{S}{\sigma_k^2} = \frac{E_h R_h}{N_T B} = \frac{E_b R_b}{L N_T B} \tag{4}$$

where $N_T/2$ is the total noise power spectral density.

The minimum equivalent noise bandwidth of the noncoherent matched filters is equal to the hop rate; i.e., $B = R_h$ [1] and (4) can be rewritten as:

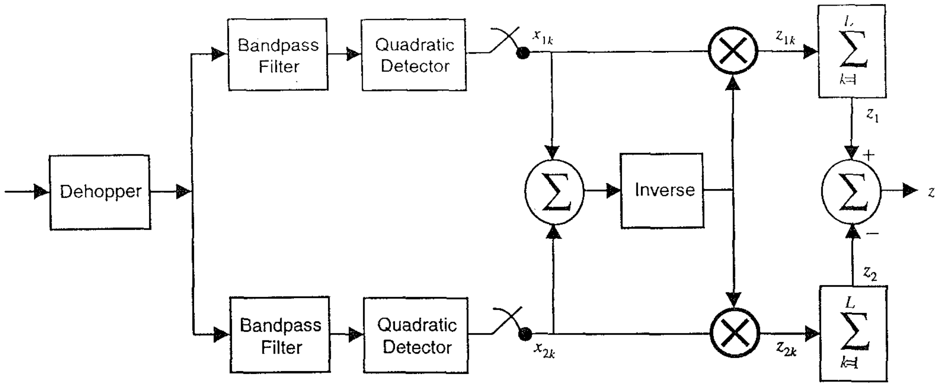

Figure 1: Self-normalization combining FFH/FSK receiver [1].

$$\frac{S}{\sigma_k^2} = \frac{E_h}{N_T} = \frac{E_b}{LN_T} \tag{5}$$

2 PREVIOUS RESULTS

The uncoded bit error probability for the receiver is

$$P_s = \sum_{i=0}^{L} \binom{L}{i} \gamma^i (1-\gamma)^{L-i} P_s(i) \tag{6}$$

where $P_s(i)$ is the conditional bit error probability given that i-hops are jammed and [1]

$$P_s(i) = \int_0^{L/2} f_{Z_1}(z_1|i)\, dz_1 \tag{7}$$

where

$$f_{Z_1}(z_1|i) = \left[f_{Z_{1k}}^{(1)}(z_{1k})\right]^{\otimes i} \otimes \left[f_{Z_{1k}}^{(2)}(z_{1k})\right]^{\otimes(L-i)} \tag{8}$$

and

$$f_{Z_{1k}}(z_{1k}) = \frac{\xi_k \lambda z_{1k} + (1+\xi_k)\left[1+\xi_k(1-z_{1k})\right]}{\left[1+\xi_k(1-z_{1k})\right]^3} \cdot e^{(-\xi_k\lambda(1-z_{1k})/1+\xi_k(1-z_{1k}))}, \quad \text{for } 0 \le z_{1k} \le 1 \tag{9}$$

In the preceding, $\rho_k = a^2/\sigma_k^2$ is the signal-to-noise ratio of the direct component of hop k of a bit, $\xi_k = 2\sigma^2/\sigma_k^2$ is the signal-to-noise ratio of the diffuse component of hop k of a bit, and $\lambda = \rho_k/\xi_k = a^2/2\sigma^2$ the ratio of the direct-to-diffuse power components. From (5):

$$\rho_k + \xi_k = \frac{a^2+2\sigma^2}{\sigma_k^2} = \frac{S}{\sigma_k^2} = \frac{E_b}{LN_T} \tag{10}$$

and

$$\xi_k\left(1+\frac{\rho_k}{\xi_k}\right) = \frac{E_b}{LN_T} \tag{11}$$

Thus

$$\xi_k = \frac{1}{1+\lambda}\frac{E_b}{LN_T} = \frac{1}{(1+\lambda)L}\frac{E_b/N_0}{1+\frac{E_b/N_0}{E_b/N_I}\gamma^{-1}} \tag{12}$$

In (8), $f_{Z_{1k}}^{(1)}(z_{1k})$ is used with σ_k^2 in (1), and $f_{Z_{1k}}^{(2)}(z_{1k})$ is used with σ_k^2 in (2).

3 REED-SOLOMON CODING

The performance of a fast frequency-hopped BFSK receiver with self-normalization combining in a fading channel with partial-band interference and with Reed-Solomon error correction coding is now examined.

For Reed-Solomon codes with code words of length n symbols, each with k information symbols, the number of errors t that the code can correct and the minimum distance of the code $d_{\min}$ are given as $t = (n-k)/2$ and $d_{\min} = n-k+1$ [3]. For the analysis that follows, it is assumed that the transmitted power P as well as the bit rate R_b are fixed. In this case, the bit energy E_b is also fixed since $P = E_b R_b$. Therefore, the variable code rate $\tau = k/n$, and the coded bit energy E_c will vary for constant transmitted power since $E_c = \tau E_b$. The performance of this coding/modulation scheme with hard decision decoding is given in [2] as:

$$P_b = \frac{1}{n}\sum_{i=t+1}^{n} i \binom{n}{i} p_s^i (1-p_s)^{n-i} \tag{13}$$

where p_s is the channel transition probability given by (6) and E_b/N_0 replaced by E_c/N_0.

4 CONCATENATED CODING

Concatenated coding with Reed-Solomon codes as outer codes and convolutional codes as inner codes are discussed in this section. The outer code has a code rate $R_0 = k_0/n_0$ and the inner code has a code rate $R_i = k_i/n_i$. The resulting code has a code rate $R = R_0R_i = k_0k_i/n_0n_i$.

Since the inner decoder performs hard decision decoding for a convolutional code, the union bound for the coded bit channel transition probability is given as [4]

$$P_i \leq \sum_{j=d_f}^{\infty} w_j P_j \qquad (14)$$

where P_j is given by

$$P_j = \sum_{d=(d_f+1)/2}^{d_f} \binom{d_f}{d} p^d (1-p)^{d_f-d} \qquad (15)$$

if d is odd, and

$$P_j = \sum_{d=d_f/2}^{d_f} \binom{d_f}{d} p^d (1-p)^{d_f-d} + \frac{1}{2}\binom{d_f}{d} \cdot p^{d_f/2}(1-p)^{d_f/2} \qquad (16)$$

if d is even. In (14) w_j is the total information weight of a code path of weight j.

In (15) and (16) p is the uncoded bit error probability of a fast frequency-hopped BFSK receiver with self-normalization combining in a fading channel with partial-band interference and is given by (6) and the weight structure w_j is found in [4] for codes of different rates and constraint lengths.

The symbol channel transition probability P_s at the input of the Reed-Solomon decoder is given as a function of P_i as

$$P_s \approx 1 - (1 - P_i)^m \qquad (17)$$

where $2^m - 1 = n_0$ and n_0 is the length of the Reed-Solomon code used as outer code.

The union bound for the bit error probability at the output of the outer (Reed-Solomon) decoder is [2]–[4]

$$P_b \leq \frac{2^{m-1}}{2^m - 1} \sum_{j=t+1}^{n_0} \frac{j}{n_0} \binom{n_0}{j} P_s^j (1-P_s)^{n_0-j} \qquad (18)$$

where P_s is given by (17) and $t = (n_0 - k_0)/2$ is the number of errors the Reed-Solomon code can correct.

Given that $2^m - 1 = n_0$ and $2^{m-1} = (n_0+1)/2$, (18) can be rewritten as:

$$P_b \leq \frac{n_0+1}{2n_0^2} \sum_{j=t+1}^{n_0} j \binom{n_0}{j} P_s^j (1-P_s)^{n_0-j} \qquad (19)$$

The assumption that the transmitted power as well as the bit rate are fixed is made in this case too. Therefore, the signal-to-noise ratio in the uncoded symbol error probability is scaled by the code rate $R = k_0k_i/n_0n_i$.

5 NUMERICAL RESULTS

From the analysis performed in [1] for the uncoded performance of fast frequency-hopped BFSK receiver with self-normalization combining in a fading channel with partial-band interference, it is clear that for a relatively large number of diversity levels ($L > 3$) worst case performance (assuming the same jammer power for both broadband and partial-band cases) corresponds to broadband interference (i.e., the whole bandwidth is jammed) or, equivalently, when all hops of a bit are jammed ($\gamma = 1$ or $i = L$, respectively). This assumption has been made for the purposes of the numerical analysis of the coded performance of the system, and a diversity level of $L = 4$ is considered. The probability of a bit error as a function of the bit energy-to-interference noise density ratio E_b/N_I is examined for Reed-Solomon codes of length $n = 15$ and 31 and concatenated codes with a convolutional code rate 3/4 and constraint length $v = 8$ used as an inner code and various length Reed-Solomon codes used as outer codes. The ratio of bit energy-to-thermal noise density E_b/N_0 is taken to be 13.35 dB in the first case and 18 dB in the study of concatenated codes. The value of $E_b/N_0 = 13.35$ dB corresponds to $P_b = 10^{-5}$ when there is no fading and $L = 1$ [1].

The effect of the channel for a Rayleigh channel model ($a^2/2\sigma^2 = 0$) for both coding cases is examined.

5.1 Reed-Solomon Coding

Results are obtained for codes of length $n = 15$ as can be seen in Fig. 2. For E_b/N_I greater than approximately 9 dB, the code (15,7) for the Rayleigh channel outperforms the other codes of the same length.

Among all codes with length $n = 31$ for a Rayleigh channel model, the (31,15) code has the best performance (Fig. 3). To achieve this improvement, the ratio E_b/N_I must exceed approximately 9 dB.

For error correction codes, it seems reasonable that the redundancy provided by the parity bits improves the performance of any system. This means that we expect that the fewer the information symbols used in a code word, the better the performance. This is not always the case. When the transmitted power and the bit rate are fixed, the bit energy is constant, but the coded bit energy increases as we increase the number of information symbols. This increase in coded bit energy can, in some circumstances, overcome the advantage that additional redundant bits provide, giving higher rate codes with better performance.

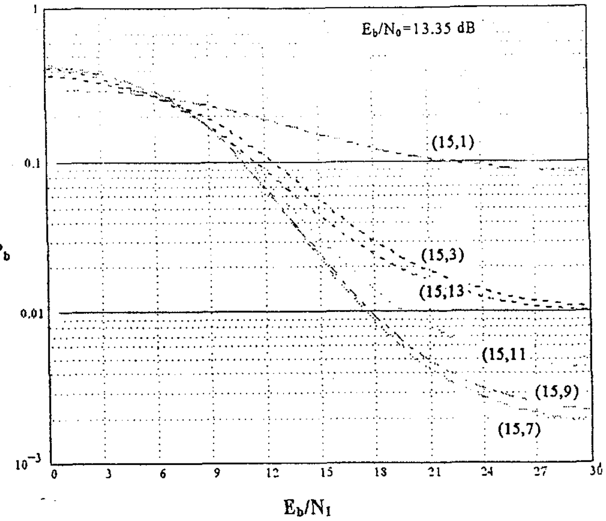

Figure 2: Performance of fast frequency-hopped BFSK receiver with self-normalization combining in a Rayleigh fading channel ($a^2/2\sigma^2 = 0$) with Reed-Solomon coding, diversity $L = 4$, and broadband interference.

5.2 Concatenated Coding

The performance of the system with concatenated codes is now examined. A convolutional code of rate 3/4 and constraint length of $v = 8$ is used as the inner code. Various Reed-Solomon codes are used as outer codes so that the overall code rate is approximately 1/2. Since the probability of bit error is calculated using union bounds for both the inner and outer codes, the ratio of bit energy-to-thermal noise E_b/N_0 is taken to be 18 dB to resolve numerical problems in the analysis. The probability of bit error as a function of the bit energy-to-interference noise ratio E_b/N_I is examined for a Rayleigh channel model.

In Fig. 4 the performance of the system using Reed-Solomon codes of length 15 is examined for a Rayleigh fading channel and broadband interference. Concatenated codes with overall rate of approximately 0.55 give the best performance.

A comparison between the performance of different lengths and rate approximately 1/2 Reed-Solomon codes (that are the optimum Reed-Solomon codes in this case) and the concatenated code with a (63,45) outer Reed-Solomon code is made in Fig. 5 for a

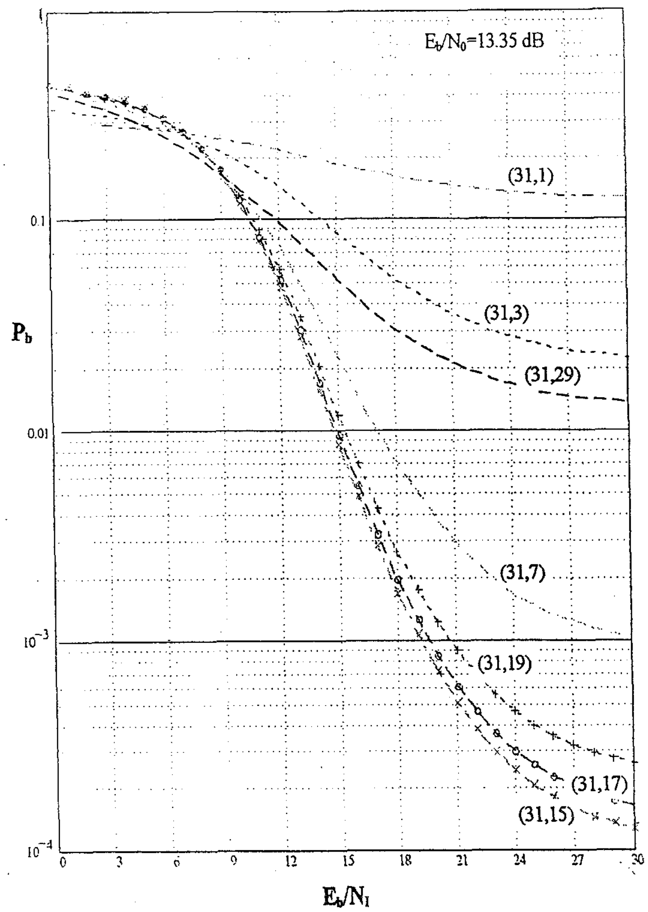

Figure 3: Performance of fast frequency-hopped BFSK receiver with self-normalization combining in a Rayleigh fading channel ($a^2/2\sigma^2 = 0$) with Reed-Solomon coding, diversity $L = 4$, and broadband interference.

Rayleigh fading channel and broadband interference. The use of the concatenated code clearly outperforms shorter Reed-Solomon codes of the same rate. Only the (255,133) Reed-Solomon code can give better performance than the concatenated code.

6 CONCLUSIONS

An analysis has been presented of the uncoded and coded performance of fast frequency-hopped BFSK receiver with self-normalization combining in a fading channel. Previously derived expressions in [1] and [2] have been used and the performance of such a system has been examined for Reed-Solomon codes and concatenated codes, all with hard decision decoding, for a fading channel model and partial-band interference.

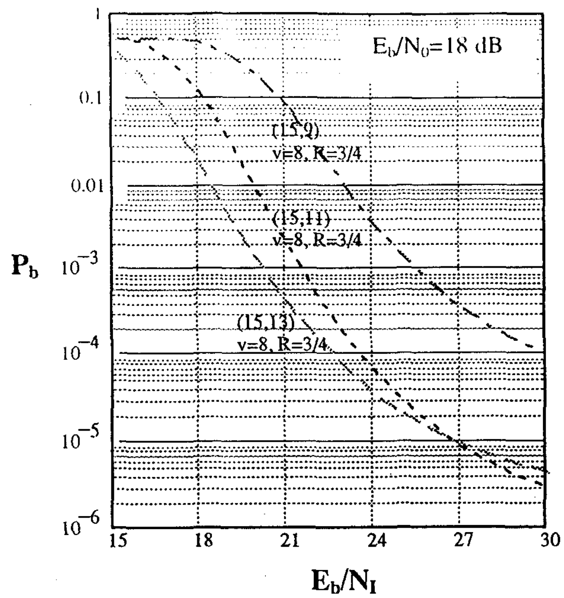

Figure 4: Performance of a fast frequency-hopped BFSK receiver with self-normalization combining in a Rayleigh fading channel ($a^2/2\sigma^2 = 0$) with concatentated coding, diversity $L = 4$, and broadband interference.

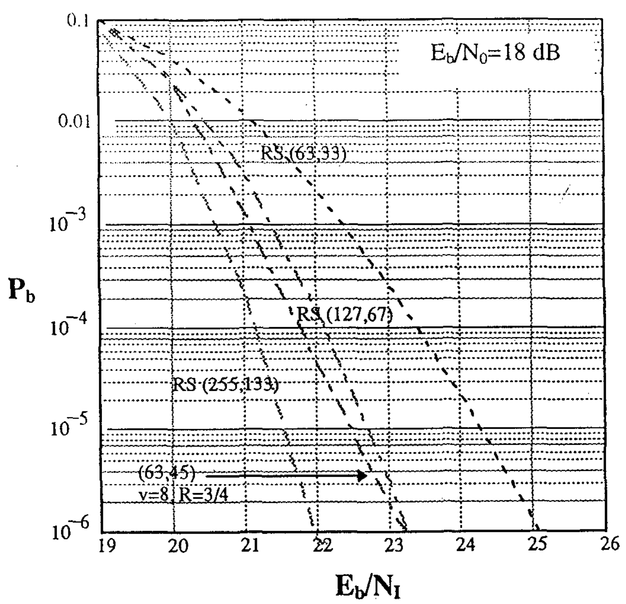

Figure 5: Performance comparison of concatenated coding and same rate Reed-Solomon codes for a fast frequency-hopped BFSK receiver with self-normalization combining in a Rayleigh fading channel ($a^2/2\sigma^2 = 0$) with diversity $L = 4$, and broadband interference.

The application of forward error correction coding improves the performance of such a system and optimum codes have been obtained for the case of Reed-Solomon coding. The optimum Reed-Solomon codes are consistent with the optimum codes obtained in [3] where the coded performance for M-ary FSK modulation in fading channels for a conventional receiver has been examined.

In the analysis performed, the transmitted power and the bit rate are kept fixed and the bit energy is considered to be constant. As a result, an increase in the number of information symbols per code word increases the coded bit energy as well. The increase in coded bit energy allows the use of higher rate codes with better performance in some cases as it overcomes the advantage of the better error correction that additional redundant bits provide to lower rate codes.

Concatenated codes with convolutional inner codes, Reed-Solomon outer codes and overall a rate of approximately 1/2 have also been examined. The performance of the system using concatenated coding dramatically improves. Probability of bit error as low as 10^{-6} or 10^{-7} can be obtained. Concatenated codes of relatively small length are found to outperform Reed-Solomon codes.

In conclusion, the application of forward error correction coding in systems with diversity improves the performance of the system and should be used if possible. Concatenated codes give the best performance as compared to Reed-Solomon of similar code rates, especially for Rayleigh fading channels with broadband interference.

REFERENCES

[1] Robertson, R.C., and Ha, T.T., "Error probabilities of fast-frequency hopped FSK with self-normalization combining in a fading channel with partial band interference," *IEEE J. Select. Areas in Commun.*, vol. 10, no. 4, pp. 714–722, May 1992.

[2] Proakis, J.G., *Digital Communications*, 3rd Ed., New York: McGraw-Hill, 1995.

[3] Khalona, R.A., "Optimum Reed-Solomon codes for M-ary FSK modulation with hard decision decoding in Ricean fading channels," *IEEE Trans. Commun.*, vol. 44, no. 4, pp. 409–412, April 1996.

[4] Clark and Cain, J.B., *Error-Correction Coding for Digital Communications*, New York: Plenum Press, 1981.