2-D DISCRETE ORTHOGONAL TRANSFORMS BY MEANS OF 2-D LMS ADAPTIVE ALGORITHMS

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Abstract

This paper [1] establishes a general relation between the two-dimensional Least Mean Square (2-D LMS) algorithm and 2-D discrete orthogonal transforms. It is shown that the 2-D LMS algorithm can be used to compute the forward as well as the inverse 2-D orthogonal orthogonal transforms in general for any input by suitable choice of the adaptation speed.
Simulations are presented to verify the general relationship results.

I. INTRODUCTION

The Least-Mean-Square (LMS) algorithm has been utilized to compute the continuous-flow discrete Fourier transform (DFT) of an input signal [1]. This was described as the LMS Spectrum Analyzer. The DFT has wide applications in several areas of signal processing. For example, this method of computing the DFT has been applied in transform-domain LMS adaptive filtering [4].

Recently, the procedure has been generalized to compute any discrete orthogonal transform [3] like the discrete cosine transform (DCT), discrete sine transform (DST) etc. Also the method described in [1] has been adapted to two-dimensional signals. Now the 2D-DFT can be computed by means of the 2-D LMS algorithm.

In this paper, we show that it is possible to utilize the 2-D LMS algorithm to compute any 2-D orthogonal discrete transform. In section II, we present the method for the computation of the two-dimensional discrete orthogonal transform by using the 2-D LMS algorithm. The next section extends this result to any orthogonal 2-D discrete transform. The last section discusses some simulation results.

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II. 2-D ORTHOGONAL TRANSFORM

The one dimensional case in [3] can be extended to two dimensional case. A two dimensional arbitrary sequence $\{f(n_1,n_2)\}, 0 \le n_1 \le N_1-1, 0 \le n_2 \le N_2-1$, can be represented as a superposition of combination of members of two orthogonal families of same type. Let $\{\phi_{N1,r1}(k_1)\}$, $r_1 = 0,1,\ldots\ldots,N_1-1$ and $\{\phi_{N2,r2}(k_2)\}$, $r_2 = 0,1,\ldots\ldots,N_2-1$ be orthogonal family of N_1 and N_2 linearly independent sequences respectively satisfying the orthogonality relation

$$\sum_{k_i=0}^{N_i-1} \phi_{N_i,r_i}(k_i)\,\phi^*_{N_i,s_i}(k_i) = \delta_{r_i-s_i} = \left\{ \begin{array}{l} 0, r_i \neq s_i \\ 1, r_i = s_i \end{array} \text{ for } i = 1,2 \right\}$$

where $*$ denotes complex conjugate. Hence $\{f(n_1,n_2)\}$ can be uniquely represented as

$$f(n_1,n_2) = \sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} F(k_1,k_2)\,\phi_{N_1,k_1}(n_1)\,\phi_{N_2,k_2}(n_2) \quad (6)$$

where the forward transforms $\{F(k_1,k_2)\}$, $k_1 = 0,\ldots\ldots,N_1-1$ and $k_2 = 0,\ldots\ldots,N_2-1$ are given by

$$F(k_1,k_2) = \sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} f(n_1,n_2)\,\phi^*_{N_1,k_1}(n_1)\,\phi^*_{N_2,k_2}(n_2) \quad (7)$$

The 2-D discrete orthogonal transformations could be expressed in matrix form as

$$F = \phi_{N_1} f \phi_{N_2} \quad (8)$$

where

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$$\phi_{N_1} = \begin{bmatrix} \phi^*_{N_1,0}(0) & \cdots\cdots & \phi^*_{N_1,0}(N_1-1) \\ \phi^*_{N_1,1}(0) & \cdots\cdots & \phi^*_{N_1,1}(N_1-1) \\ \vdots & \vdots & \vdots \\ \phi^*_{N_1,N_1-1}(0) & \cdots\cdots & \phi^*_{N_1,N_1-1}(N_1-1) \end{bmatrix}$$

and

$$\phi_{N_2} = \begin{bmatrix} \phi^*_{N_2,0}(0) & \cdots\cdots & \phi^*_{N_2,N_2-1}(0) \\ \phi^*_{N_2,0}(1) & \cdots\cdots & \phi^*_{N_2,N_2-1}(1) \\ \vdots & \vdots & \vdots \\ \phi^*_{N_2,0}(N_2-1) & \cdots\cdots & \phi^*_{N_2,N_2-1}(N_2-1) \end{bmatrix}$$

while the inverse transform is given by

$$f = \psi_{N_1} F \psi_{N_2} \qquad (9)$$

where

$$\psi_{N_1} = \phi^*_{N_1} = \begin{bmatrix} \phi_{N_1,0}(0) & \cdots\cdots & \phi_{N_1,0}(N_1-1) \\ \phi_{N_1,1}(0) & \cdots\cdots & \phi_{N_1,1}(N_1-1) \\ \vdots & \vdots & \vdots \\ \phi_{N_1,N_1-1}(0) & \cdots\cdots & \phi_{N_1,N_1-1}(N_1-1) \end{bmatrix}$$

and

$$\psi_{N_2} = \phi^*_{N_2} = \begin{bmatrix} \phi_{N_2,0}(0) & \cdots\cdots & \phi_{N_2,N_2-1}(0) \\ \phi_{N_2,0}(1) & \cdots\cdots & \phi_{N_2,N_2-1}(1) \\ \vdots & \vdots & \vdots \\ \phi_{N_2,0}(N_2-1) & \cdots\cdots & \phi_{N_2,N_2-1}(N_2-1) \end{bmatrix}$$

It is interesting that in equation (6), the product $\{\phi_{N_1,r_1}(k_1)\phi_{N_2,r_2}(k_2)\}$ $r_1 = 0,1,.....,N_1-1$ and $r_2 = 0,1,.....,N_2-1$ form an orthogonal family of $N_1 x N_2$ linearly independent sequences.
Considering the following

$$\sum_{k_1=0}^{N_1-1}\sum_{K_2=0}^{N_2-1}[\phi_{N_1,r_1}(k_1)\phi_{N_2,r_2}(k_2)][\phi_{N_1,s_1}(k_1)\phi_{N_2,s_2}(k_2)]^*$$

$$= \left[\sum_{k_1=0}^{N_1-1}\phi_{N_1,r_1}(k_1)\phi^*_{N_1,s_1}(k_1)\right]\left[\sum_{k_2=0}^{N_2-1}\phi_{N_2,r_2}(k_2)\phi^*_{N_2,s_2}(k_2)\right]$$

from (5),

$$= \delta_{r_1-s_1}\delta_{r_2-s_2}$$

$$= \begin{Bmatrix} 1 & \text{if } r_1 = s_1 & \text{and} & r_2 = s_2 \\ 0 & \text{if } r_1 \neq s_1 & \text{or} & r_2 \neq s_2 \end{Bmatrix} \qquad (10)$$

Equation (10) is the two dimensional extension of the result in [3]. Therefore, the product $\{\phi_{N_1,r_1}(k_1)\phi_{N_2,r_2}(k_2)\}$ $r_1 = 0,1,.....,N_1-1$ and $r_2 = 0,1,.....,N_2-1$ form an orthogonal family of $N_1 x N_2$ linearly independent sequences. To derive this two dimensional extension of the 1-D orthogonality relation in [3], substituting (9) in (8) gives

$$F = (\phi_{N_1}\psi_{N_1})F(\psi_{N_2}\phi_{N_2}) \qquad (11)$$

For $i = 1$, from (5),

$$(\phi_{N_1}\psi_{N_1}) = (\phi_{N_1}\phi^*_{N_1}) = I \qquad (12)$$

leads to

$$(\psi_{N_2}\phi_{N_2}) = (\phi^*_{N_2}\phi_{N_2}) = I \qquad (13)$$

which gives

$$\sum_{k_2=0}^{N_2-1}\phi_{N_2,k_2}(r_2)\phi^*_{N_2,k_2}(s_2) = \delta_{r_2-s_2} = \begin{Bmatrix} 0 & r_2 \neq s_2 \\ 1 & r_2 = s_2 \end{Bmatrix} \qquad (14)$$

Similarly, if we substitute the transpose of (8) in (9),

$$f^T = (\psi^T_{N_2}\phi^T_{N_2})f^T(\phi^T_{N_1}\psi^T_{N_1}) \qquad (15)$$

For $i = 2$, from (5),

$$(\psi^T_{N_2}\phi^T_{N_2}) = (\phi_{N_2}\psi_{N_2})^T = (\phi_{N_2}\phi^*_{N_2})^T = I \qquad (16)$$

leads to

$$(\phi^T_{N_1}\psi^T_{N_1}) = (\psi_{N_1}\phi_{N_1})^T = (\phi^*_{N_1}\phi_{N_1})^T = I = (\phi^*_{N_1}\phi_{N_1}) \qquad (17)$$

where the last equality gives

$$\sum_{k_1=0}^{N_1-1}\phi_{N_1,k_1}(r_1)\phi^*_{N_1,k_1}(s_1) = \delta_{r_1-s_1} = \begin{Bmatrix} 0 & r_1 \neq s_1 \\ 1 & r_1 = s_1 \end{Bmatrix} \qquad (18)$$

With (14) and (18), it follows that

$$\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1}\left[\phi_{N_1,k_1}(r_1)\,\phi_{N_2,k_2}(r_2)\right]\left[\phi_{N_1,k_1}(s_1)\,\phi_{N_2,k_2}(s_2)\right]^*$$

$$=\left[\sum_{k_1=0}^{N_1-1}\phi_{N_1,k_1}(r_1)\,\phi^*_{N_1,k_1}(s_1)\right]\left[\sum_{k_2=0}^{N_2-1}\phi_{N_2,k_2}(r_2)\,\phi^*_{N_2,k_2}(s_2)\right]$$

$$=\delta_{r_1-s_1}\delta_{r_2-s_2}$$

$$=\left\{\begin{array}{ll} 1 & \text{if } r_1=s_1 \text{ and } r_2=s_2 \\ 0 & \text{if } r_1\neq s_1 \text{ or } r_2\neq s_2 \end{array}\right\} \qquad (19)$$

This two dimensional extension is crucial in later derivation. A 2-D sequence $x(n_1,n_2)$ where $0\leq n_1<\infty$, $0\leq n_2\leq N_2-1$, can be uniquely mapped to a 1-D sequence by a transformation such that

$$i=n_1N_2+n_2 \qquad (20)$$

as described in [2],[5]. Similarly a 2-D infinite pixel by pixel strip representation of 2-D images can be treated as a 1-D infinite strip as each pixel $x(n_1,n_2)$ is uniquely mapped to $x(i)$ where $i=n_1N_2+n_2$.

Also an N_1 by N_2 2-D matrix $W(n_1,n_2)$ can be uniquely mapped to a 1-D vector of N_1N_2 elements as described in [2], [5].

III. 2-D LMS ORTHOGONAL TRANSFORM

The 2-D version of LMS spectrum analyzer, as shown in figure 1, can be used to evaluate any 2-D orthogonal transforms that are in the general forms as in (6) and (7). The input signal of the analyzer is a $N_1 x N_2$ 2-D matrix $U(n_1,n_2)$ given by

$$U(n_1,n_2)=P1(n_1)\,P2^T(n_2) \qquad (22)$$

where

$$P1(n_1)=\left[\phi_{N_1,0}(n_1 mod N_1)\ \ldots\ \phi_{N_1-N_1-1}(n_1 mod N_1)\right]^T$$

$$P2(n_2)=\left[\phi_{N_2,0}(n_2)\qquad \ldots \qquad \phi_{N_2,N_2-1}(n_2)\quad\right]^T$$

The (desired input) signal $x(n_1,n_2)$ is a pixel stream of 2-D images. The weighted sum is

$$\sim x(n_1,n_2)=\sum_{m=0}^{N_1-1}\sum_{n=0}^{N_2-1}u_{m,n}(n_1,n_2)\,w_{m,n}(n_1,n_2) \qquad (23)$$

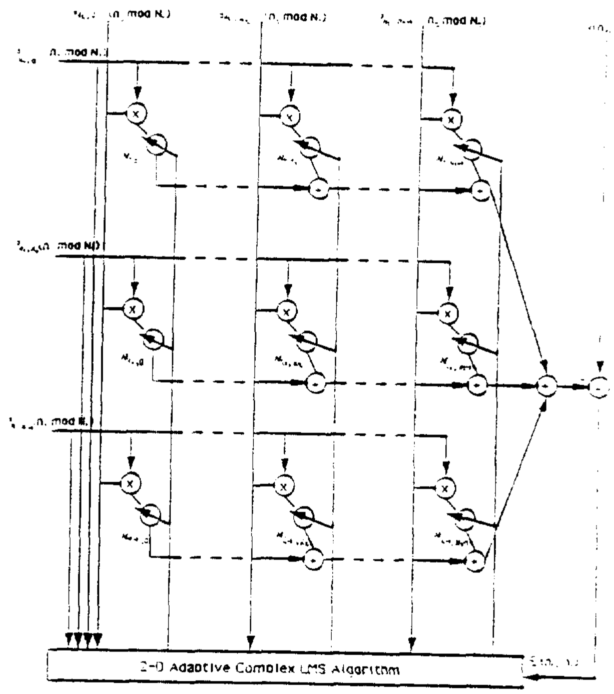

Figure 1: The 2-D LMS Spectrum analyzer

The error signal required for adaptation is defined as

$$\varepsilon(n_1,n_2)=x(n_1,n_2)-\sim x(n_1,n_2) \qquad (24)$$

The 2-D complex LMS algorithm in matrix form [2] is

$$W'=W(n_1,n_2)+2\mu\varepsilon(n_1,n_2)\,U^*(n_1,n_2) \qquad (25)$$

Using the approach in [5], we can map a 2-D matrix to a 1-D vector by setting

$$i=n_1N_2+n_2 \qquad \text{and} \qquad M=N_1N_2$$

$$\varepsilon(i)=x(i)-W^T(i)\,U^*(i) \qquad (29)$$

The update equation (25) may be expressed as a 1-D algorithm:

$$W(i+1)=W(i)+2\mu\varepsilon(i)\,U^*(i) \qquad (30)$$

Substituting (29) in (30),

$$W(i+1)=\left\{I-2\mu U^*(i)\,U^T(i)\right\}W(i)+2\mu x(i)\,U^*(i) \quad (31$$

From (19),

$$U^*(R)\,U^T(S)=\delta_{R-S} \qquad (32)$$

Assuming $W(0) = 0$ and $x(m) = 0$ for $m < 0$, we obtain a general formula applicable for all $i \geq 1$

$$W(i) = 2\mu \sum_{k=0}^{\infty} (1-2\mu)^k \sum_{m=i-(k+1)M}^{i-kM-1} x(m)\, U^*(m) \quad (33)$$

If we use $\mu = \frac{1}{2}$, then all but the $k = 0$ term are zero. Consequently, $W(n_1, n_2)$ is the 2-D forward orthogonal transform of the trailing block of $x(n_1, n_2)$. Importantly, if $(n_1, n_2) = (n_F N_1, 0)$, then $W(n_F N_1, 0) =$ 2-D forward orthogonal transform of frame number $(n_F - 1)$.
Considering

$$\psi_{N_1} = \phi^*_{N_1} \quad \text{and} \quad \psi_{N_2} = \phi^*_{N_2} \qquad (35) \text{ and } (36)$$

The inverse 2-D orthogonal transforms can also be evaluated in a similar manner by setting

$$U(n_1, n_2) = P1(n_1)\, P2^T(n_2) \qquad (37)$$

where

$$P1(n_1) = \left[\phi^*_{N_1,0}(n_1 mod N_1) \; \ldots \; \phi^*_{N_1,N_1-1}(n_1 mod N_1)\right]^T$$

$$P2(n_2) = \left[\phi^*_{N_2,0}(n_2) \quad \ldots \quad \phi^*_{N_2,N_2-1}(n_2)\right]^T$$

IV. SIMULATION RESULTS

From [2], a frame of a test signal used in testing the 2-D DFT LMS algorithm is used here with $N_1 = N_2 = 32$. This is fed to a 2-D spectrum analyzer for the 2-D discrete cosine transform (DCT).
The results of this transform using LMS and the definition are plotted in Figure 2. It is shown that both methods give same result. This is also demonstrated for other transforms such as the 2-D discrete Fourier transform (DFT) and the cas-cas version of the 2-D discrete Hartley transform (DHT) in [5].

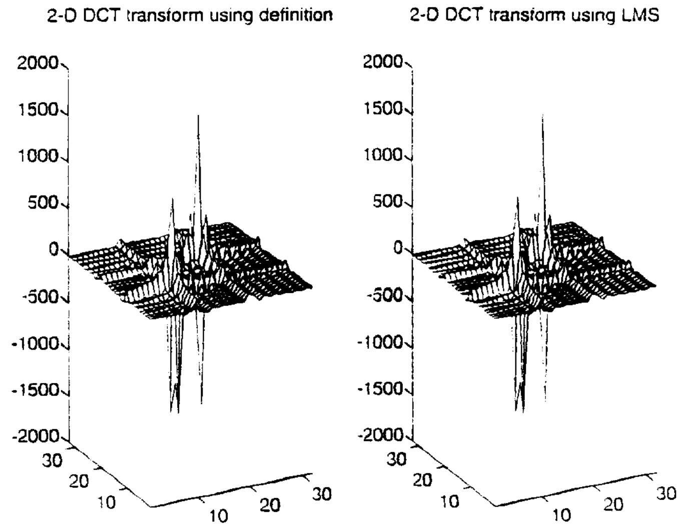

Figure 2: Simulation result for 2-D DCT

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